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STABILITY PROBLEM OF A SHALLOW CONICAL SHELL UNDER LATERAL PRESSURE

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1. Stability Equations

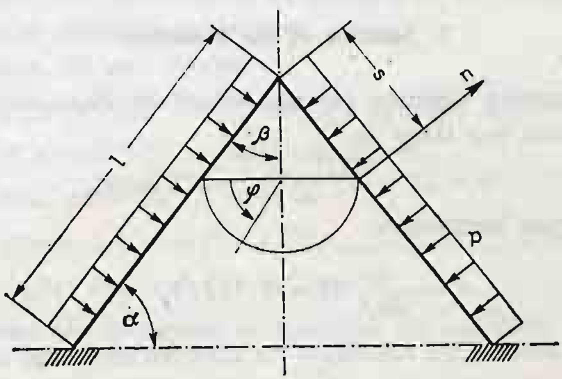

Fig. 1

The set of stability equations for a conical shell under external pressure is of the form:

$$\text{(1)}\quad \nabla^2\nabla^2 F - Eh\left\{\operatorname{ctg}\beta \frac{l}{x}\frac{\partial^2 w}{\partial x^2} + \left[\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial w}{\partial \varphi_1}\right)\right]^2 - \frac{\partial^2 w}{\partial x^2}\left(\frac{1}{x^2}\frac{\partial^2 w}{\partial \varphi_1^2} + \frac{1}{x}\frac{\partial w}{\partial x}\right)\right\} = 0,$$

$$\text{(2)}\quad D\nabla^2\nabla^2 w + \operatorname{ctg}\beta \frac{l}{x}\frac{\partial^2 F}{\partial x^2} - \frac{\partial^2 w}{\partial x^2}\left(\frac{1}{x}\frac{\partial F}{\partial x} + \frac{1}{x^2}\frac{\partial^2 F}{\partial \varphi_1^2}\right) - \frac{\partial^2 F}{\partial x^2}\left(\frac{1}{x}\frac{\partial w}{\partial x} + \frac{1}{x^2}\frac{\partial^2 w}{\partial \varphi_1^2}\right) +$$
$$+2\left(\frac{1}{x^2}\frac{\partial w}{\partial \varphi_1} - \frac{1}{x}\frac{\partial^2 w}{\partial x \partial \varphi_1}\right)\left(\frac{1}{x^2}\frac{\partial F}{\partial \varphi_1} - \frac{1}{x}\frac{\partial^2 F}{\partial x \partial \varphi_1}\right) +$$
$$+ pl^3 \operatorname{tg}\beta \cdot \left(\frac{1}{2}\frac{\partial^2 w}{\partial x^2} + \frac{1}{x}\frac{\partial w}{\partial x} + \frac{1}{x^2}\frac{\partial^2 w}{\partial \varphi_1^2}\right) = 0,$$

where: $x = \frac{s}{l}$, $\varphi_1 = \varphi \sin\beta$ (see Fig. 1),

w — shell deflection,
F — force function,

$$\nabla^2\nabla^2 = \frac{\partial^4}{\partial x^4} + \frac{2}{x^2}\frac{\partial^4}{\partial x^2 \partial\varphi_1^2} + \frac{1}{x^4}\frac{\partial^4}{\partial\varphi_1^4} + \frac{2}{x}\frac{\partial^3}{\partial x^3} - \frac{2}{x^3}\frac{\partial^3}{\partial x \partial\varphi_1^2} + \\ + \frac{4}{x^4}\frac{\partial^2}{\partial\varphi_1^2} - \frac{1}{x^2}\frac{\partial^2}{\partial x^2} + \frac{1}{x^3}\frac{\partial}{\partial x}.$$

Equations (1), and (2), given here in a transformed form, were derived for the conical shell of an arbitrary shape, c.f. [1]. In equation (2) $pl^4\cos^4\alpha$ should be substituted instead of the underlined term for the stability problem of a shallow conical shell (for a shallow shell $\mathrm{tg}\,\alpha < 0.2$).
In this paper the solution of the shallow conical shell stability problem is presented, where the equation (2) in a "full" (with under lined term included) and in a "simplified" form are used. It can be concluded from the analysis which of the equations of (1) and (2) are better in use. The analysis of the influence of shell dimensions on the critical load is also presented.

2. Solution of the Equations.

The strain compability equation (1) was solved by Papkowicz — type procedure. The deflection function was taken as

$$w = (x^2-1)^2 f + x^4(x^2-1)^2 f_1 \cos n\varphi_1, \tag{3}$$

where: f, f_1 — unknown parameters,

$$n = \frac{k}{\sin\beta} \quad (k = 0, 1, 2, 3, \ldots).$$

The function (3) satisfies the conditions for clamped shell edge at $x = 1$, i.e.:

$$w = 0; \quad \frac{\partial w}{\partial x} = 0. \tag{4}$$

When the deflection function (3) is introduced into right-hand side of equation (1), this can be written as follows:

$$\nabla^2\nabla^2 F = \mathrm{E}h(A_0 + A_n \cos n\varphi_1 + A_{2n}\cos 2n\varphi_1), \tag{5}$$

where A_0, A_n, A_{2n} are the functions of x.

The parameters of deflection function and shell dimensions are also included in these functions. The equations are of the form given in ref. [3]. The solution of equation (5) we accept in the form of power series

$$F(x_1\varphi_1) = \sum_{m=1}^{\infty} F_m(x)\cos m\varphi_1. \tag{6}$$

The coefficients in equation (6) can be determined when the set of four differential equations, obtained by substituting the function (6) into equation (5) and comparing by identity the corresponding terms of the left — and right-hand side, is solved. Thus the force function takes the form of

$$F(x_1\,\varphi_1) = F_0 + F_n \cos n\varphi_1 + F_{2n}\cos 2n\varphi_1 . \tag{7}$$

F_0, F_n, F_{2n} are functions of x and of deflection function parameters and they are of a complex structure. When the force function is known, then we can approximately solve the equilibrium equation (2) assuming a deflection function w.

A Bubnov-Galerkin-type procedure is used for solving the equation (2). The "full" and also the "simplified" equations are solved. Orthogonalization of equation (2) requires

$$\begin{aligned} &\int_0^{2\pi}\int_0^{1} K(x,\varphi_1)x(x^2-1)^2 dx d\varphi \sin\beta = 0, \\ &\int_0^{2\pi}\int_0^{1} K(x,\varphi_1)x^5(x^2-1)^2 \cos n\varphi_1 dx d\varphi \sin\beta = 0, \end{aligned} \tag{8}$$

where: $K(x, \varphi_1)$ is left — hand side of equation (2).

When the conditions (8) are expanded we obtain a set of two algebraic equations in the vector of deflection functions parameters.

For the "full" equation (2) one obtains

$$\begin{aligned} &A_1 p^* \zeta_1 + A_2 \zeta_1 + A_3 \zeta_1^2 + A_4 \zeta_1^3 + A_5 \zeta_1 \zeta_2^2 + A_6 \zeta_2^2 = 0, \\ &B_1 p^* + B_2 + B_3 \zeta_1 + B_4 \zeta_1^2 + B_5 \zeta_2^2 = 0, \end{aligned} \tag{9}$$

and for the "simplified" equation (2) there is

$$\begin{aligned} &A_1^1 p^* + A_2 \zeta_1 + A_3 \zeta_1^2 + A_4 \zeta_1^3 + A_5 \zeta_1 \zeta_2^2 + A_6 \zeta_2^2 = 0, \\ &\zeta_2 (B_2 + B_3 \zeta_1 + B_4 \zeta_1^2 + B_5 \zeta_2^2) = 0. \end{aligned} \tag{10}$$

The next quantities are introduced in equations (9) and (10):

$$\zeta_1 = \frac{f}{h}, \quad \zeta_2 = \frac{f_1}{h}, \quad p^* = \frac{p}{\mathrm{E}}.$$

The coefficients A_i and B_i include shell dimensions and parameter n. Their structure is very complicated. When parameter ζ_2 is eliminated from equations (9) we obtain an expresion form which we calculate the pressure

$$p^* = K_0 \frac{\zeta_1^3 + K_2 \zeta_1^2 + K_3 \zeta_1 + K_4}{\zeta_1 + K_1} . \tag{11}$$

The same operation made on equations (10) gives

$$p^* = H_1 + H_2 \zeta_1 + H_3 \zeta_1^2 + H_4 \zeta_1^3 . \tag{12}$$

Since the directions of the pressure and the deflection (see Fig. 1) in equation (11), and (12) are opposite one has to put $\zeta_1 \leqslant 0$.

3. Analysis of the Solution

The analysis has been performed for shells with $\frac{l}{h} = 100, 200, 300$ and with angle α varied ($\operatorname{tg}\alpha$ was from 0.1 trough 0.5 by step of 0.1).

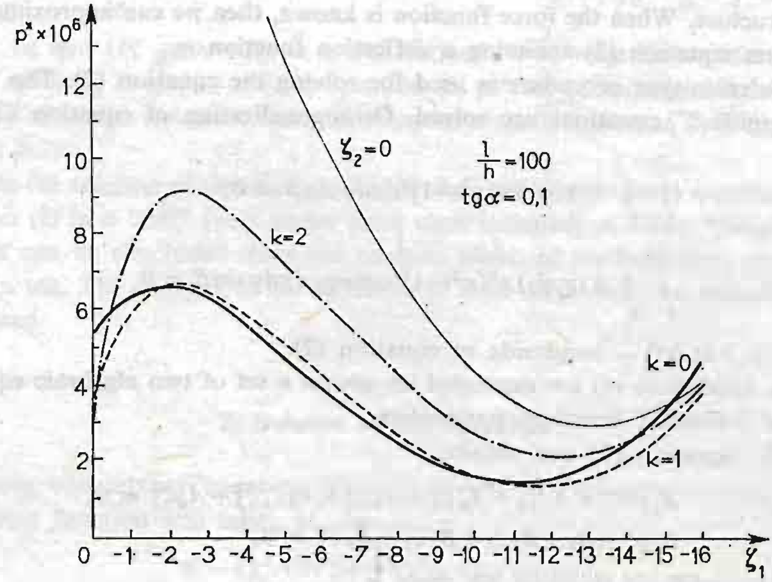

Fig. 2

From equations (11) and (12) for each pair of $\frac{l}{h}$ and $\operatorname{tg}\alpha$ one obtains an infinite number of solutions, because they both include the parameter $n\left(n = \frac{k}{\sin\beta}\right)$. The only significant solution is the solution which gives a minimum p^* value.

Fig. 2 is a plot of curves obtained from the solutions of equation (11). They refer to a shell for which $\frac{l}{h} = 100$ and $\operatorname{tg}\alpha = 0.1$. Each of the solutions brings two extremal values of the pressure. The lowest from maximum pressures is the upper critical load, signed p_g^*, the lowest taken from minimum pressures is the lower critical load p_d^*.

The lowest pressures were obtained at $k = 1$. These are $p_g^* = 6.6489 \cdot 10^{-6}$ and $p_d^* = 1.4374 \cdot 10^{-6}$. The line for $\zeta_2 = 0$ is also presented.

It represents a symmetrical form of buckling and it is of a first approximation of the solution. The minimum value is $2.859 \cdot 10^{-6}$.

Change of dimensions and angle α do not influence the quality changes. The critical load is then obtained from the equation at $k = 0$.

The solutions of equation (12) are of the same form. However the buckling critical loads are much higher (for $k = 0$) here then buckling loads obtained from equation (11).

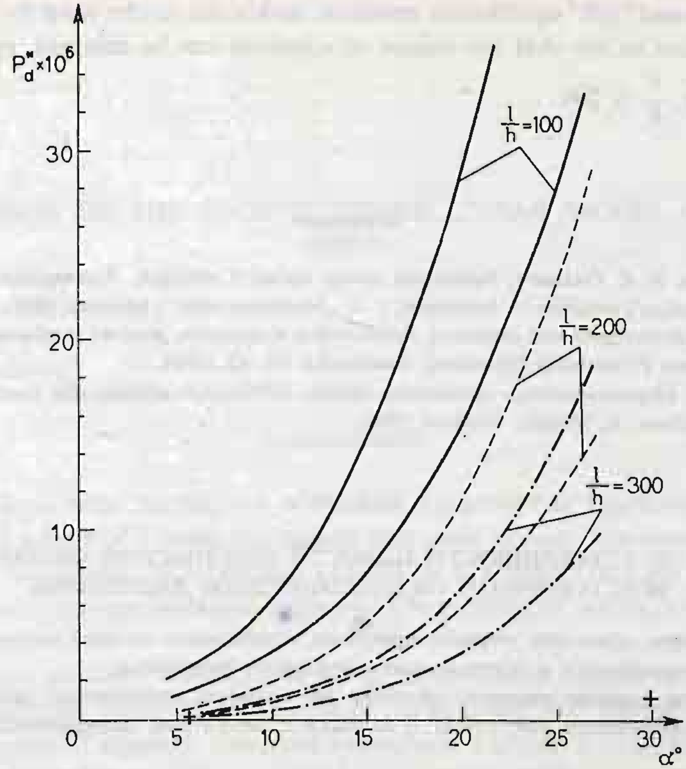

Fig. 3

Fig. 3 presents the lines of lower critical load p_d^* versus angle α for three different values of $\frac{l}{h}$. The lower of the two lines presented by the same type of line is referred to equation (11), the upper line is referred to equation (12). It is worthenoting to show that by using the "full" equilibrium equation (2) one obtains in each case, the lower critical load smaller than the critical load of the "simplified" equation. The decrease is as much as 50% of the pressure obtained from "simplified" equation. The critical load increases rapidly with the increase of angle α but the increase is not so rapid when the $\frac{l}{h}$ ratio is larger.

To evaluate theoretical results the use is made of the experimental data given in ref. [4].

These data are pointed aut by crosses in Fig. 3, and they refer to shells of $\frac{l}{h} = 200$, $\operatorname{tg}\alpha = 0.1$ and of $\frac{l}{h} = 300$ and $\alpha = 30°$.

The experimental result for a shallow shell is contained within the solutions of equations (11) and (12), but the result for a shell of $\alpha = 30°$ differs very much from the theoretical predictions (when the latter are extrapolated for the angle of 30°). Since the other experimental data are not a vailable the range of valid solutions is not resolvable correctly.

One may say with cortainty that the accepted deflection, while using a Papkowicz-

-type procedure and "full" equilibrium equation, makes the results valid for shells of small angle α; it is also to say that the regime of solutions can be enlarged up to tg $\approx$ 0.3, especially when $\frac{l}{h} > 200$.

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Р е з ю м е

ЗАДАЧА ОБ УСТОЙЧИВОСТИ ПОЛОГОЙ КОНИЧЕСКОЙ ОБОЛОЧКИ СО ВСЕСТОРОННИМ ГИДРАВЛИЧЕСКИМ ДАВЛЕНИЕМ

Работа содержит сравнение решений проблемы устойчивости пологой конической оболочки с применением упрощенного и неупрощенного уравнения равновесия.

Анализируется влияние размеров оболочки на стоимость критических давлений. Сравниваются также теоретические результаты с взятыми с литературы экспериментальными результатами.

S t r e s z c z e n i e

ZAGADNIENIE STATECZNOŚCI MAŁO WYNIOSŁEJ POWŁOKI STOŻKOWEJ POD DZIAŁANIEM CIŚNIENIA

W pracy dokonano porównania rozwiązań zagadnienia stateczności powłoki stożkowej o małej wyniosłości przy zastosowaniu uproszczonego i nieuproszczonego równania równowagi. Przeanalizowano wpływ wymiarów i kształtu powłoki na wartość obciążeń krytycznych. Oceniono również przydatność otrzymanych wyników na podstawie danych doświadczalnych wziętych z literatury.

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