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ON A METHOD OF FORMULATION OF TWO-DIMENSIONAL THEORIES FOR ELASTIC SHELLS

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The purpose of the paper[1)] is to present a certain method of construction of such theories of shells, in which the two-dimensional solutions can be treated as the approximations (with a defined error) for three-dimensional displacements and stresses describi the state of equilibrum of an elastic body. The method is based on the assumption that the error of replacing the three-dimensional solution by a two-dimensional one should be the smallest in a certain domain of the tolerance solutions. In other words, the shell solution should minimize the error in this domain. We shall call this assumption the error minimization principle [1]. Two simplified two-dimensional approaches to elastic shells have been constructed on the basis of the assumption. Within these frames one is able to find tolerance solutions for displacements and stresses.

1. The error functional.

Consider the system

$$\mathcal{M} = \langle V^1, V^2, F, T, K\rangle, \tag{1.1}$$

where V^1 is a space of the vector functions

$$v^1: \Omega \to R^3, \quad \Omega \subset R^3,$$

including a displacements of the body, V^2 is a space of the symmetric tensor functions

$$v^2: \Omega \to R^{3\times 3},$$

including stresses, F is a space of pairs (b, p), where b is a body force and p is a surface traction

$$b: \Omega \to R^3, \quad p: \partial_1\Omega \to R^3$$

and $\partial_1\Omega \subset \partial\Omega$, whereas the operators T: $V^1 \to V^2$ and $K: V^2 \to F$ are taken in the form

$$T(v^1) = \frac{1}{2} C(\nabla v^1 + \nabla v^{1T}), \quad K(v^2) = (\operatorname{div} v^2, v^2|_{\partial_1\Omega} n), \tag{1.2}$$

[1)] This paper is an extended version of a lecture [1] delivered at the Third Conference „Shell structures — theory and applications", Opole 1982.

and $C = (C_{ijkl})$ is the tensor of elastic moduli. The superposition KT will be refered to by M.

The system $\mathcal{M}$ will be called *the equilibrium linear elasticity theory system* (or *structure*) [1] - [3].

Within the structure $\mathcal{M}$ we are able to formulate the following problem:

Let $f_0 = (-b, p)$ be a known element in F; find such $v^K \in V^K$, $K = 1, 2$, that

$$v^1|_{\partial_2\Omega} = u_0, \quad v^2 = T(v^1), \quad f_0 = K(v^2) \tag{1.3}$$

where u_0 is the given function, and $\partial_2\Omega$ is the subset, such that $\partial_1\Omega \cup \overline{\partial_2\Omega} = \partial\Omega$, $\partial_1\Omega \cap \partial_2\Omega = \emptyset$.

The elements v_0^K, $K = 1, 2$, which satisfy the above conditions, will be called the solutions corresponding to f_0. The solution v_0^1 is a solution in displacements and v_0^2 in stresses.

We introduce now the concept of an error functional.

For the sake of simplicity let W stands for V^1 or V^2.

Let be known the functional

$$\delta: 2^{W\times W} \to \langle 0, +\infty)$$

such that

$$\begin{gathered}
\operatorname{dom}\delta \neq \emptyset, \\
(\forall \alpha \in \operatorname{dom}\delta)(\forall w_1, w_2 \in W)[(w_1, w_2) \in \alpha \Rightarrow (w_2, w_1) \in \alpha, \\
(w_1, w_1) \in \alpha, \quad (w_2, w_2) \in \alpha], \\
(\forall \alpha_1, \alpha_2 \in \operatorname{dom}\delta)[\alpha_1 \cup \alpha_2 \in \operatorname{dom}\delta, \quad \alpha_1 \cap \alpha_2 \in \operatorname{dom}\delta], \\
(\forall \alpha \in \operatorname{dom}\delta)[\alpha = \mathrm{id} \Rightarrow \delta(\alpha) = 0], \\
(\forall \alpha_1, \alpha_2 \in \operatorname{dom}\delta)[\alpha_1 \subset \alpha_2 \Rightarrow \delta(\alpha_1) \leqslant \delta(\alpha_2)].
\end{gathered} \tag{1.4}$$

The condition $(1.4)_2$ states, that each element in the domain of functional δ is a reflexive and symmetric relation whereas condition $(1.4)_3$ says that domain δ constitutes a lattice. Hence the elements of domain δ are tolerances [6] - [7].

In the special case when W is a set with a norm $\|\cdot\|$, functional δ can be defined as

$$\delta(\alpha) = \sup_{(w_1, w_2)\in\alpha} \|w_1 - w_2\|. \tag{1.5}$$

The set $\alpha \in \operatorname{dom}\delta$ will be referred to as $A[w_1, w_2]$, so that $(w_1, w_2) \in \alpha$.

The number

$$\varepsilon(w_1, w_2) = \inf_{\alpha \in A[w_1, w_2]} \delta(\alpha), \tag{1.6}$$

is said to be the error of identification of the elements w_1 and w_2. The relation (1.6) then determines the functional ε: $W \times W \to \langle 0, +\infty)$ which will be called the error functional. In particular we can assume that $\varepsilon(w_1, w_2) = \|w_1 - w_2\|$. However, the error does not have to be determined by norm. In mechanics there are many cases for which we are not interested in an error in the sense of the distance (as an example we can take the case in which only some components of the stress tensor have to be compared).

2. Error minimization principle

Let us assume that an elastic body constitutes a shell and that in system $\mathcal{M}$ the reference configuration is of the form $\Omega = \pi \times (-h, h)$, $\varphi(\pi)$ being the midsurface of the shell and φ is the diffeomorfism of $\pi \times (-h, h)$ into the physical space. Apart from the system $\mathcal{M}$ we consider two systems of the form

$$\mathcal{N}^K = \langle Y^K, G^K, N^K \rangle, \qquad K = 1, 2,$$

which will be called *shell systems*. In these systems Y^K, G^K are the spaces of functions determined on π with the values in R^n, and N^K are operators $N^K\colon Y^K \to G^K$.

Moreover, let us assume that the structures $\mathcal{N}^K$ are interrelated with system $\mathcal{M}$ in such a way, that the spaces Y^K are the spaces of generalized coordinates for V^K, and G^K are the spaces of generalized forces for F. The operators

$$A^K\colon Y^K \to V^K, \qquad B^K\colon F \to G^K,$$

have to be defined, (Fig. 1). The operators A^K determine the constraints in spaces V^K [5].

$$\begin{tikzcd}
V \arrow[r, "T"] & S \arrow[r, "K"] & F \arrow[d, "B^1"] \arrow[dd, bend left, "B^2"] \\
Y^1 \arrow[u, "A^1"] \arrow[rr, "N^1"] & & G^1 \\
 & Y^2 \arrow[u, "A^2"] \arrow[r, "N^2"] & G^2
\end{tikzcd}$$

Fig. 1

In systems $\langle Y^K, G^K, N^K \rangle$, $K = 1, 2$ the problem of finding solutions can also be formulated:

Given $g_0^K \in G^K$, *find* y^K, $K = 1, 2$ *such that*

$$g_0^K = N^K(y^K). \tag{2.1}$$

Each y_0^K satisfying the above conditions will be called a solution corresponding to g_0^K.

If $g_0^K = B_0^K(f_0)$ and v_0^K are solutions (in displacements and stresses corresponding to f_0, then the solutions y_0^K and the constraints A^K determine in spaces V^K certain elements $v^K = A^K(y_0^K)$. The set of those elements will be denoted by $\tilde{V}^K = \{v^K; (\exists y_0^K, v_0^1)[v^K = A^K(y_0^K), N^K(y_0^K) = B^K(M(v_0^1))]\}$.

Let us define some non — empty sets of the solutions of problem (1.3) (sets of the exact solutions): $V_0^1 \subset \operatorname{dom} M \subset V^1, V_0^2 \subset \operatorname{dom} K \subset V^2$ and assume, that the structures $\mathcal{N}^K$ together with operators A^K, B^K, $K = 1, 2$ satisfy the following conditions

$$\begin{gathered}
(\forall v_0^K \in V_0^K)(\exists y_0^K \in Y^K)[N^K(y_0^K) = B^K(N(v_0^1)), \\
\varepsilon^K(A^K(y_0^K), v_0^K) = \min_{v^K \in \tilde{v}^K} \varepsilon^K(v^K, v_0^K), \qquad K = 1, 2].
\end{gathered} \tag{2.2}$$

Condition (2.2) introduced for shell systems in the form of a postulate will be called *the error minimization principle*. This principle states that for the given structure $\mathcal{M}$ and the given errors ε^K, the structures $\mathcal{N}^K$, together with the operators A^K, B^K should be such

that the solution y_0^K to problem (2.1) corresponding to generalized forces $B^K(M(v_0^1))$ has to minimize the error of identification of the elements $v^K \in \tilde{V}^K$ and the solutions v_0^K.

Each element $\tilde{v}^K$, $K = 1, 2$ belonging to the domains of operators K and T, respectively, and such that for known $a^K \geqslant 0$, $K = 1, 2$ we have

$$\varepsilon^K(\tilde{v}^K, A^K(y_0^K)) \leqslant a^K, \tag{2.3}$$

will be called *a tolerance solution* of problem (1.3). In particular $A^K(y_0^K)$ are tolerance solutions to this problem.

3. The error minimization principle in the displacement spaces

Let us define in space V^1 (in what follows we drop the index 1) the scalar product in the form

$$v_1 \cdot v_2 = \int_\Omega \nabla v_1^T C \nabla v_2 d\Omega + \sum_i v_1^i v_2^i \tag{3.1}$$

where $v_1^i = v_1(x^i)$, $v^2 = v_2(x^i)$, $x^i \in \partial\Omega$, $i = 1, 2, \ldots, n \geqslant 3$ and x^i are non-collinear. Let us take the error in V as $\varepsilon(v_1, v_0) = ||v_1 - v_0||$ where the norm $||\cdot||$ is defined by the scalar product (3.1).

The error minimization principle (2.2) will have the form

$$(\forall v_0 \in V_0)(\exists y_0 \in Y)[||A(y_0) - v_0|| = \min_{v \in V} ||v - v_0||] \tag{3.2}$$

where $\tilde{V}$ is determined by the structure V^1 and $N(y_0) = B(M(v_0))$. Moreover $v^i = v_0^i$, $i = 1, 2, \ldots, n$.

Let us denote by $V(\tilde{v})$, $\tilde{v} \in \tilde{V}$ the set of displacements admissible by the constraints. Now, condition (3.2) will be represented in the form

$$(\forall v_0 \in V_0)(\exists y_0 \in Y)(\forall v \in V(A(y_0))[\partial_F ||A(y_0) - v_0||^2(v) = 0] \tag{3.3}$$

Since

$$[\min_{v \in V} ||v - v_0||]^2 = \min_{v \in V} ||v - v_0||^2,$$

where ∂_F stands for the Frechet derivative.

Since $||v + h||^2 - ||v||^2 = 2(v, h) + ||h||^2$, then $[\partial_F ||\tilde{v}||^2](v) = 2(\tilde{v}, v)$, Using the last equation and (3.1) we have

$$(\tilde{v} - v_0, w) = \int_\Omega (T(\tilde{v}) - T(v_0))\nabla(w) d\Omega = \int_\Omega (T(\tilde{v})\nabla(w) + \operatorname{div} T(v_0) w) d\Omega -$$

$$- \int_{\partial\Omega} (T(v_0) nw) d\partial\Omega = \int_\Omega T(\tilde{v})\nabla(w) d\Omega - \int_\Omega bw d\Omega - \int_{\partial\Omega} pnw d\partial\Omega.$$

Substituting the right-hand side of the foregoing into (3.3) we have

$$\begin{aligned}(\forall v_0 \in V_0)(\exists y_0 \in Y)(\forall w \in V(A(y_0)))\Big[\int_\Omega (T(A(y_0)))\nabla(w) d\Omega - \\ - \int_\Omega bw d\Omega - \int_{\partial\Omega} pnw d\partial\Omega = 0\Big]\end{aligned} \tag{3.4}$$

where $p = T(v_0)|_{\partial\Omega}$.

The obtained relation represents the principle of virtual work. Thus the following conclusion can be drawn:

The principle of virtual work is a particular case of the error minimization principle (3.2), *such that the norm defining the error* (1.6) *is defined by the scalar product* (3.1).

Let us return to the shell structure $\mathcal{N}^1$. Firstly we define space Y^1 as space of functions

$$q^\alpha : \pi \to R^3, \quad \alpha = 0, 1, \ldots, l.$$

Space Y^1 is space of generalized coordinates and can be interpreted for instance as a space isomorfic to space $V^1/\approx$ where $\approx$ is an equivalence relation in V^1

$$v_1 \approx v_2 \Leftrightarrow v_1|_{\pi_\alpha} = v_2|_{\pi_\alpha}$$

and $\pi_\alpha \in \{\pi_\beta : \pi_\beta = \varphi(\pi \times \{y_\beta\}), -h = y_0 < y_1 < \ldots < y_l = h\}$ whereas mapping φ is a diffeomorphism of the domain Ω into the physical space. In thr case discussed above

$$q^\alpha(z^K) = v^1(z^K, y^\alpha), \quad v^1 \in V^1, \quad z^K \in \pi, \quad y^\alpha \in (-h, h).$$

Now we define the constraints i.e. operator A^1. We consider the constraints representing certain given a priori kinematic hypotheses of the form

$$v^1(x) = \Phi(z^K, y, q^\alpha(z^K)), \quad x = (z^K, y).$$

The basic system of equations for the vector of the generalized coordinates is determined by the error minimization principle (3.4) (the principle of the virtual work). Applying the know procedure we arrive at Euler's system of equations in the form of equilibrium equations and constitutive equations. From this principle we obtain also the form of operator B^1 determining generalized forces f

$$\text{(3.5)} \qquad \begin{gathered} \overset{*}{\mathrm{Div}}\, H^\alpha + h^\alpha + f^\alpha = 0, \\ H^\alpha = \frac{\partial \varepsilon}{\partial \overset{*}{\nabla} q^\alpha}, \quad h = -\frac{\partial \varepsilon}{\partial q^\alpha}, \\ \varepsilon = \int_{-h}^{h} \sigma(\nabla \Phi) dy, \\ f^\alpha = \int_{-h}^{h} b \partial \Phi / \partial q^\alpha dy + p \left.\frac{\partial \Phi}{\partial q^\alpha}\right|_{y=-h} + p \left.\frac{\partial \Phi}{\partial q^\alpha}\right|_{y=h}, \end{gathered}$$

where $\overset{*}{\mathrm{Div}}$ is the divergence in π and σ the strain energy function. Equations (3.5) should be fulfilled for each $z \in \pi$. On the other hand the geometric boundary conditions on $\partial\pi$ have to be satisfied

$$\text{(3.6)} \qquad H^\alpha \overset{*}{n} = \int_{-h}^{h} p \frac{\partial \Phi}{\partial q^\alpha} dy,$$

where $\overset{*}{n}$ is a unit normal vector to $\partial\pi$.

Functions q_0^α which satisfy (3.5), (3.6) are the solution to problem (2.1), whereas $\Phi(q_0^\alpha)$

are the tolerance solutions in V. Solution $\Phi(q_0^\alpha)$ minimizes the error, which is defined by the scalar product (3.1).

Stresses $T(\Phi(q_\alpha^0))$ which correspond to solution q_0^α in general do not minimize the error in V^2. Thus we have to look for the tolerance solutions also in stresses. This problem will be the subject of the following Section.

4. Error minimization principle in the space of stresses

Notice that in the Hilbert space W for every $w_0 \in W$ two mutually orthogonal linear manifolds can be determined, such that for every $w_1 \in W_1$, $w_2 \in W_2$ we have

$$(w_1 - w_0)(w_2 - w_0) = 0. \tag{4.1}$$

At the same time each element $w \in W$ can be represented in the form $w = \frac{1}{2}(w_1 + w_2)$ where w_1, w_2 fulfill (4.1). Equality (4.1) is equivalent to the equality

$$||w - w_0|| = \frac{1}{2}||w_1 - w_2||, \tag{4.2}$$

where $w = \frac{1}{2}(w_1 + w_2)$.

Now determine in the space of stresses V^2 (which is referred to as S) the scalar product in the form [4]:

$$s_1 \cdot s_2 = \int_\Omega s_1^T A s_2 d\Omega, \tag{4.3}$$

where $A = C^{-1}$ is the matrix of elastic moduli.

Let S_1 be a set of kinematically admissible stress, i.e.

$$s_1 \in S_1 \Leftrightarrow K(s_1) = (-b, p), \tag{4.4}$$

and S_2 be a set of statically admissible stresses

$$s_2 \in S_2 \Leftrightarrow (\exists v \in V_0)\,[s_2 = T(v)] \tag{4.5}$$

where V_0 is a set of displacements fulfilling the boundary conditions in displacements. Using (4.3) it is easy to verify that:

$$(s_1 - s_0)(s_2 - s_0) = 0,$$

holds, where $s_2 = T(v)$, $s_0 = T(v_0)$ is the exact solution in displacements of problem (1.3).

Error (1.6) will be assumed in the form

$$\varepsilon(s, s_0) = \frac{1}{2}||s - s_0||. \tag{4.6}$$

Novertheless, according to (4.2) the error is also given by

$$\varepsilon(s, s_0) = \frac{1}{2}|s_1 - s_2||$$

where $s = \frac{1}{2}(s_1 + s_2)$ and s_0, s_1, s_2 satisfy condition (4.1).

The error minimization principle (2.2) in this case will have the form

$$||\tilde{s}_1 - \tilde{s}_2|| = \min_{\substack{s_1 \in \tilde{S}_1 \\ s_2 \in \tilde{S}_2}} ||s_1 - s_2||$$

or, equivalently

$$||\tilde{s}_1 - T(\tilde{v})|| = \min_{\substack{s_1 \in \tilde{S}_1 \\ v \in \tilde{V}}} ||s_1 - T(v)|| \tag{4.7}$$

where $\tilde{S}_1$, $\tilde{S}_2$ are determined by structure $\mathcal{N}^2$ in the terms of the subsets in S_1, S_2 and $\tilde{V} \subset V^1$, $\tilde{V} = \{v; T(v) \in \tilde{S}_2\}$, respectively.

Using (4.3), principle (4.7) can be written down in the form

$$\begin{aligned} V_p(\tilde{v}) &= \min_{v \in \tilde{V}} V_p(v), \\ V_k(\tilde{s}_1) &= \min_{s_1 \in \tilde{S}_1} V_k(s_1), \end{aligned} \tag{4.8}$$

where

$$V_p(v) = \int_{\Omega} \frac{1}{2} T(v)^T A T(v) d\Omega - \int_{\partial_1 \Omega} v p d\partial\Omega,$$

$$V_k(s_1) = \int_{\Omega} \frac{1}{2} s_1^T A s_1 d\Omega - \int_{\partial_2 \Omega} s_1 n u_0 d\partial\Omega.$$

Relations (4.8) describe the well known principles of the minimum of potential and complementary energy. Hence the conclusion:

The principle of minimum of potential and complementary energy (4.8) *is the error minimization principle* (2.2), *in which the error is defined in form* (4.3).

Now we proceed to structure $\mathcal{N}^2$. Let us define operator B^2 as assigned to loading p (the body forces are to be neglected) the triplets of functions p^+, p^-, P where p^+, p^- are the loadings acting at the upper and lower surfaces of the shell, respectively, on the other hand $P: \partial_1\pi \to R^3$, $\partial_1\pi \subset \partial\pi$ is a mean loading

$$P = P(p|_{\partial_1\pi \times (-h, h)}),$$

defined by

$$P = \int_{-h}^{h} p|_{\partial_1\pi \times (-h, h)} dy.$$

Let the space Y^2 be a space of functions

$$m^\beta: \pi \to R^3, \quad \beta = 0, 1, \ldots, t$$

and the constraints for statically admissible stresses will be defined by the operator $\Psi: Y^2 \to S$

$$s = \Psi(m^\beta). \tag{4.9}$$

Stresses (4.9) have to satisfy Eqs. (4.4), which characterize the interrelation between the forces $f \in F$ (it means that p^+, p^-, P) and the generalized coordinates m^β as well as

their gradients

$$
\begin{aligned}
&d_\gamma(m^\beta, \nabla m^\beta; p^+, p^-) = 0, \quad \gamma = 1, 2, \ldots, r \\
&b_\varrho(m^\beta, \mathrm{P}) = 0, \quad \varrho = 1, 2, \ldots, s
\end{aligned}
\tag{4.10}
$$

where $(4.9)_1$ are defined in π and $(4.9)_2$ in $\partial_1\pi$

Substituting (4.9) into $(4.8)_2$ and assuming that the known displacements u_0 are defined on $\partial_2\Omega \subset \partial_2\pi \times (-h, h)$ we obtain

$$
v_k(m^\beta) = \int_\pi \varepsilon(m^\beta) d\pi - \int_{\partial_2\pi} \omega(m^\beta) d\partial\pi,
$$

where

$$
\begin{aligned}
\varepsilon(m^\beta) &= \int_{-h}^{h} \frac{1}{2} \; {}^T(m^\beta) A \Psi(m^\beta) dy, \\
\omega(m^\beta) &= \int_{-h}^{h} \Psi(m^\beta) n u_0 dy.
\end{aligned}
\tag{4.11}
$$

Functions (4.10), (4.11) are differentiable provided that the Lagrange multipliers theorem can be applied, the condition (4.8) will take the form

$$
\delta v_k(m^\beta) + \int_\pi \lambda^\nu \left(\frac{\partial d_\nu}{\partial m^\beta} m^\beta + \frac{\partial d_\nu}{\partial \nabla m^\beta} \nabla m^\beta \right) d\pi + \int_{\partial_1\pi} \mu^\varrho \frac{\partial b_\varrho}{\partial m^\beta} m^\beta d\partial\pi = 0
$$

where δv_k is a variation of functional v_k and λ^ν, b_ϱ are Lagrange multipliers.

Applying appropriaete calculations of the variational aaproach, we obtain

$$
\begin{aligned}
&-\overset{*}{\operatorname{div}} \left(\lambda^\nu \frac{\partial d_\nu}{\partial \nabla m^\beta} \right) + \frac{\partial \varepsilon}{\partial m^\beta} + \lambda^\nu \frac{\partial d_\nu}{\partial m^\beta} = 0, \\
&\lambda^\nu \frac{\partial d_\nu}{\partial m^\beta} n + \mu^\varrho \frac{\partial b_\varrho}{\partial m^\beta} = 0, \\
&\frac{\partial \omega}{\partial m^\beta} = 0.
\end{aligned}
\tag{4.12}
$$

Eqs. $(4.12)_1$ should be fulfilled almost every in π, whereas $(4.12)_2$ in $\partial_1\pi$ and $(4.12)_3$ in $\partial_2\pi$. Together with equations (4.10) they constitute the system of equations for the unknown functions. After solving the system of equations we find functions m_0^β which after substituting to (4.9) determine the admissible equilibrium stresses $\tilde{s}_1 = \Psi(m_0^\beta)$. In general this is not a solution that minimizes error (4.6). By virtue of the foregoing assumptions, the solution minimizing error (4.6) is $\tilde{s} = \frac{1}{2}(\tilde{s}_1 + \tilde{s}_2)$ where $\tilde{s}_2$ should fulfill condition $(4.7)_1$. For such a solution of s_2 we can take the solution of system (3.5) transferred to the space S such that is $T(\Phi(q_0^\alpha))$ it minimizes the potential energy in $\tilde{V}$. Thus the tolerance solution that minimizes the error (4.6) will be

$$
\tilde{s} = \frac{1}{2} \left(\psi(m_0^\beta) + T(\Phi(q_0^\alpha)) \right).
\tag{4.13}
$$

Moreover, it should be noted that the error (4.6) can be effectively calculated without the knowledge of the exact solutions $v_0, s_0 = T(v_0)$ that is the solutions of the problem (1.3). According to (4.2)

$$\varepsilon(\tilde{s}, s_0) = \frac{1}{2}\|\Psi(m_0^\beta) - T(\Phi(q_0^\alpha))\|$$

and $\tilde{s}$ are determined by (4.13).

5. Final ermarks

In the paper a method of formulations of the basic system of equations for displacements and stresses which depend on certain functions defined on the midsurface of a shell heve been discussed. The method is based on the error minimization principle resulting from replacing of an exact (three-dimensional) solution by a shell solution. In the case of displacements the principle is of the form of the principle of virtual work, and in the case of stresses has the form of the principle of minimum of potential and complementary energy. On the basis of these principles we have introduced the systems of equations for the two-dimensional generalized displacements and for the two-dimensional generalized stresses.

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Резюме

О НЕКОТОРОМ МЕТОДЕ КОНСТРУКЦИИ ТЕОРИИ УПРУГИХ ОБОЛОЧЕК

Целью настоящей работы является представление некоторого метода конструкции таких теории оболочек, в которых трехмерные решения будут приближениями с определенной ошибкой для трехмерных перемещений и напряжений описывающих состояние равновесия упругого тела. Этот метод основан на предпосылке, что ошибка изменения трехмерного решения — двухмерным решением должна быть самой маленькой в некоторой области толеранционных решений.

Streszczenie

O PEWNEJ METODZIE KONSTRUKCJI TEORII POWŁOK SPRĘŻYSTYCH

Celem pracy jest przedstawienie pewnej metody konstrukcji takich teorii powłok, w których rozwiązania dwuwymiarowe będą przybliżeniami, z określonym błędem, dla trójwymiarowych przemieszczeń i naprężeń opisujących stan równowagi ciała sprężystego. Metoda ta oparta jest na założeniu, że błąd zastąpienia rozwiązania trójwymiarowego rozwiązaniem dwuwymiarowym powinien być najmniejszy w pewnej dziedzinie rozwiązań tolerancyjnych.

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