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STRESS-ASSISTED CORROSION OF REINFORCED CONCRETE

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Nowadays there is no need to underline the importance of considera-tion of concrete corrosion in engineering structures subjected to agres-sive surrounding.

It is also understandable that diffusion process is strongly dependent on the mechanical state and other constitutive details of the solid. These are always implicit in all classical diffusion theories and based on intuitive modification of first Fick's law for pure diffusion. Clas-sical theories propose simple formulae to quantify a stress-assisted dif-fusion situation. These formulae, which are not unified in nature are extensively used to model physical processes like embrittlement mechanism and stress corrosion cracking .

In the latter part of seventies Aifantis did not regard Fick type po-stulation as statement of universal validity and he decided to seek a basis for diffusion on physical principles of conservation.

Due to the fact that balance equations are meaningful, when a diffusing substance can be considered as a continuously distributed matter, it has been reasonable to assume the existence of the stress tensor T supported by the diffusing species alone. It was also reasonable to assume the existence of diffusive force p describing local interaction between in-terdiffusing materials.

The fact that only classical diffusion is considered, allows to assume that the solid matrix remains macroscopically rigid with respect to dif-fusion. So, the macroscopic response of the solid is determined indepen-dently of the diffusion process. On the other hand, as it has been men-tioned, the diffusion process is strongly dependent on the mechanical state of the solid matrix.

In many practical cases the deformation of the solid can not be determined within an uncoupled theory for the solid alone because of some conceptual and practical difficulties involved in mixture theories (Truesdell (1962)). We view the interdiffusing materials as a single body whose mechanical response is defined by a total stress and total strain. However, in the constitutive equation the stress does not depend only on the strain but also on the density ρ and flux $\mathbf{j}$ of the diffusing species, so ρ and $\mathbf{j}$ are viewed as internal variables whose behavior is governed by differential expressions. The stress represents the total stress and is not treated as a preassigned function but as an unknown quantity to be determined from the proper coupled problem.

Aifantis formulates the problem for homogeneous, isotropic, linear and slow diffusion processes. It can be shown that the local forms of mass and momentum balances are:

$$\dot{\rho} + \mathrm{div}\,\mathbf{j} = q \quad , \tag{1}$$

$$\mathrm{div}\,\mathbf{T} + \mathbf{p} = \dot{\mathbf{j}} \quad , \tag{2}$$

where $\mathbf{j}$ is diffusive flux vector,

$$\mathbf{j} = \rho\,\mathbf{v} \quad , \tag{3}$$

q - denotes the rate of mass of diffusing substance that iscreated within the unit volume due to chemical reaction.

Classical diffusion theories usually neglect inertia effects. If we do so, the right side of the second equation vanishes.

Constitutive equations for the stress $\mathbf{T}$ and diffusive force $\mathbf{p}$ in general are

$$\mathbf{T} = \mathbf{T}(\rho,\ \mathbf{v}-\mathbf{v}_s,\ \mathrm{grad}\rho;\ \alpha^i,\ \mathbf{a}^j\mathbf{A}^k;\ \beta^r,\ \mathbf{b}^s,\ \mathbf{B}^t;\ \mathbf{X}) \ , \tag{4}$$

$$\mathbf{p} = \mathbf{p}(\rho,\ \mathbf{v}-\mathbf{v}_s,\ \mathrm{grad}\rho;\ \alpha^i,\ \mathbf{a}^j\mathbf{A}^k;\ \beta^r,\ \mathbf{b}^s,\ \mathbf{B}^t;\ \mathbf{X}) \ . \tag{5}$$

The first three variables are basic diffusion variables, the next three model specyfic features of the diffusing species and the last three are associated with those parameters of the state of solid which influence the diffusion process.

Further, we will be concerned with linear diffusion process where the dependences of T and p on the diffusion variables

$$v, \ \mathrm{grad}\ \rho, \ a, \ A^k ,$$

is linear.

Furthermore, we will accept for the consideration of stress - assisted diffusion the simplest possible constitutive equations in the form:

$$T = - \Gamma(\rho, B{=}S) , \tag{6}$$

$$p = -A(\rho,S) j - B(\rho,S)\ \mathrm{grad}\rho , \tag{7}$$

where S is the stress field supported by the solid and Γ, A, B are linear, isotropic function of S meeting the representation

$$\Gamma_i = \gamma_{11}\ 1 + \gamma_{12}\ 1 + \gamma_{13}\ S . \tag{8}$$

The principal reason to interpret the stress state by identifying with stres state for the solid alone is that the scientists dealing with material do not have apparatus for determining extra strain and stress fields induced by diffusion.

From the equation of balance momentum and making use of the fact that inverse of an isotropic function is isotropic we may write:

$$j = -(D1 + N\sigma 1 + KS)\ \mathrm{grad}\rho + (L + M\sigma)\ \mathrm{grad}\sigma . \tag{9}$$

We assume also that stress affects diffusion only through its hydrostatic components, so, $K = 0$.

We also neglect the influence of diffusive stress on diffusion process ($L = M = 0$).

Unlike Aifantis we assume that diffusion coefficient may be expressed by an arbitrary function of time and coordinates.

Substituting diffusion flux into mass balance equation and $\nabla^2\sigma = 0$ (trace of stress tensor is a harmonic function) we obtain the boundary problem governing the solution of stress- assisted, two-dimensional diffusion process in the following form

$$\dot{\rho} = \sum_i \left[\partial_i (D+N\sigma)\partial_i \rho\right] + q , \qquad i=1,2,3 , \tag{10}$$

with initial and boudary conditions

$$\rho(x_i,0) = \rho_o , \tag{11}$$

$$\rho|_\Gamma = \rho_{ext} , \tag{12}$$

or

$$\left[D(\frac{\partial\rho}{\partial n}) + \beta\rho \right]_\Gamma = \beta\rho_{ext} , \tag{12a}$$

where β is coefficient of surface conductance, if β >>D then (12a)$\rightarrow$(12).

Due to our above mentioned assumption of dependence of coefficients of effective diffusion on time and coordinates eq.(10) takes another form than presented by Aifantis.

To obtain the solution of the boundary problem the finite difference method with irregular mesh has been adopted.

The set of nonlinear equations has been solved by using PEACEMAN - RACHFORD method where each numerical time step is divided into two intervals. The first of them includes a implicit scheme for one coordinate and the second for the other one.

As it has been proved by SAMARSKI the method is absolutely convergent if the second order approximation of ρ is adopted. The method can be generelized for the case where the diffusivity D is a nonlinear function of concentration ρ (Zaborski (1986)).

The proposed approach to concrete corrosion gives a possibility to consider a wide class of problems.

The principal task we meet nowaday is to evaluate suitable material constants. Existing experimental data published in some papers are based on intuitive simple formulae to quantify the stress - assisted diffusion process. These data have been used in our numerical examples as hypothetical material constants due to the fact that the principal aim of the presented examples is to give a qualitative illustration of the adopted method. The process parameters and material constants ,adopted in calculation, have been evaluated on the basis of experimental data published by Kubik & Zybura (1980). We assumed that the depth of the corroded layer of reinforcement may be expressed in form:

$$\frac{d\lambda}{dt} = P \left[e^{-Rt} + S \right] \rho \quad .$$

Fig. 1 presents the distribution of the solute concentration along the symmetry axis of the square cross section of the beam subjected to pure bending as time goes on.

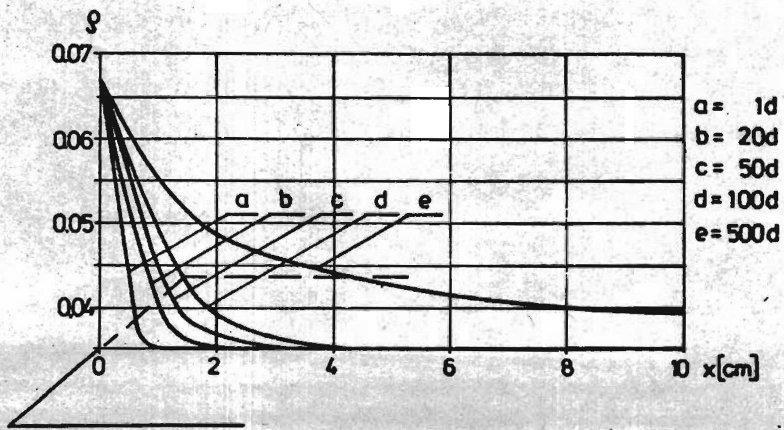

Fig. 1. Distribution of solute concetration.

Fig. 2 presents, let's call it a "corner effect". Usually the solution of the one-dimensional flow is adopted for a two-dimensional problem. So, it is not possible to show that the corrosion process proceeds far quicker in the corners. The isolines which join the points with a constant solute concentration are rounded near the corner. With time the cross section core takes the form more and more circular.

Fig. 2 shows also the symmetry and asymmetry of core shape when the diffusion process of beam subjected to pure bending is influenced by stress *(b)* and is not *(a)*. This asymmetry qualitatively coincides with Moskwin's experimental results.

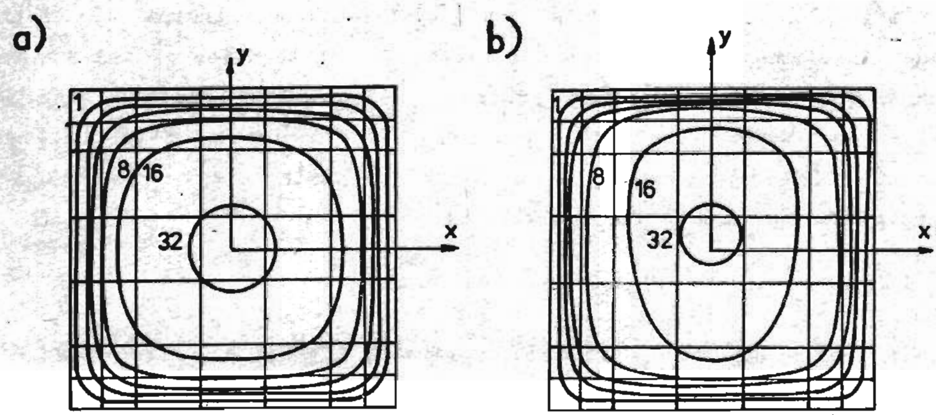

Fig.2. Distribution of solute concentration in cross-section.

Fig.3a presents the results of another considered example of reinforced beam under corrosion power after some of time lapse.

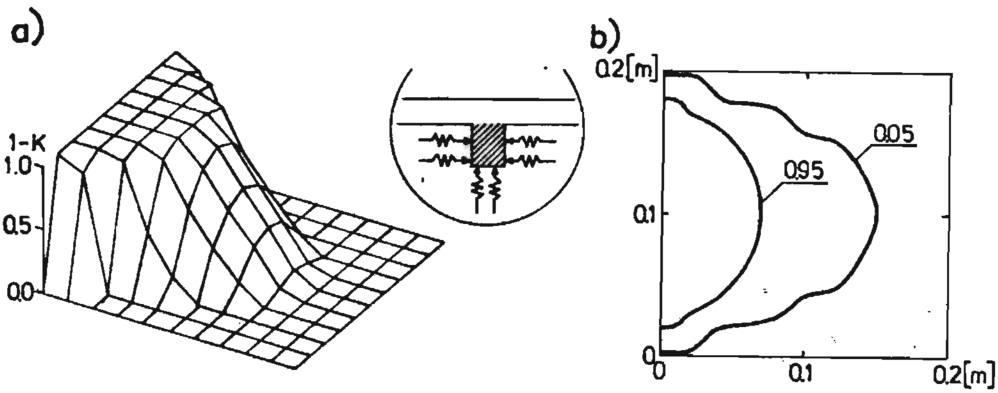

Fig.3 Corrosion of beam cross section.

In fig. 3b one can see the core of the cross section and corrosive wear, presented using isolines.

In the last case we have considered the diffusion process of concrete as well as reinforcement. The time redistribution of concentration and of stresses when reinforcement is subjected to corrosion power which growths from zero to ρ_{ext} and when reinforcement is not influanced by corrosion is of some interest.

Fig. 4 shows how strongly the cross-section stiffness depends on the corrossion of reinforcement.

At the end it is proper to add that all of existing intuitive theories, among others Moskwin's theory, are special cases of the above presented approach.

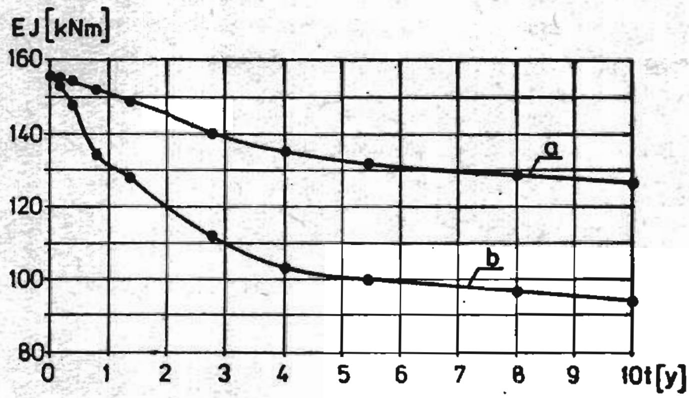

Fig. 4. Change of cross-section stiffness:
a - reinforcement does not corrode,
b - giving consideration to reinforcement corrossion.

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Summary

KOROZJA NAPRĘŻENIOWA ŻELBETU

W niniejszej pracy wykorzystano model korozji naprężeniowej zaproponowany przez Aifantisa. Dokonano uogólnienia modelu na przypadek efektywnego współczynnika dyfuzji, będącego dowolną funkcją zarówno współrzędnych jak i stopnia koncentracji roztworu. Ponadto w równaniu bilansu masy uwzględniono zmianę stężenia wynikającą z przebiegającego procesu chemicznego.

W rozwiązaniu numerycznym sformułowanego zagadnienia początkowo - brzegowego wykorzystano schemat Peacemana - Rachforda metody różnic skończonych. Pokazano wyniki obliczeń przykładów prętowych elementów konstrukcji żelbetowych.

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