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TOROIDAL SHELL STRUCTURES - GOVERNING EQUATIONS AND COMPUTERIZED ANALYSIS OF CURVED TUBES, ELBOWS AND BELLOWS (A SURVEY)

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1. Introduction

1.1 Geometric and material nonlinearities. Collapse and bifurcation buckling analysis needed for design and evaluation of nuclear power plant piping systems, offshore pipelines, flexible piping components such as elbows, bellows and expansion joints is the most important motivation for recent works on the nonlinear analysis of thin-walled toroidal shells. Both, geometric and material nonlinearities interact to precipitate limit loads and critical states of these structures when subject to various combinations of external forces. These are: in-plane and/or out-of-plane bending of elbows, in-plane bending of pressurized curved tubes, bending of large diameter curved pipelines in the presence of external hydrostatic pressure, thermal and pressure expansion of bellows of various shape, bending deformation of bellows. In all cases under consideration the theory of rotationally symmetric toroidal shells with either closed or open-cross section is applicable for either curved pipes and elbows or toroidal-shape bellows, respectively.

1.2 Critical states and collapse in the presence of changes of geometry. The classification of the possible critical states of elastic-perfectly plastic toroidal shells, if geometric and/or material nonlinearities are accounted for, was discussed by Bielski and Skrzypek (1989). In a general sense, critical states may correspond either to instability or to decohesion (failure). Instability may occur if the *bifurcation point* (BP) or the *limit point* (LP) is reached. Bifurcation may correspond either to the *rotationally symmetric buckling* (SB) of a cross-section or to the longitudinal *rotationally nonsymmetric buckling* (NB). Meridional buckling of toroidal shells under predominant outer hydrostatic pressure or longitudinal buckling of subsea pipelines under predominant bending may illustrate both cases considered. The limit point may be understood as the exhaustion of the *maximal carrying capacity* connected with: *i*) the *flattening* of the cross section (MCCf), *ii*) the *bulging of the wall* (MCCb), *iii*) the *snap through* (MCCs). Beyond the elastic limit other critical states can be defined. These may correspond either to the classical *limit carrying capacity* (LCC), or to the formation of the local displacement discontinuity, if the 'stress profile'reaches one of two parabolic points at the HMH yield ellipse. In the latter case the corresponding loading parameters may be recognized as the *decohesive carrying capacity* (DCC) in a sense proposed first by Szuwalski and Życzkowski in 1973. For a more realistic hardening model the phenomenon of decohesion yields to formation of plastic hinge and collapse mechanism; Winter (1981), Lang (1984, 1985).

2. Toroidal shell analysis

2.1 Elastic models.

(a). Reissner's models. The geometrically nonlinear theory of elastic toroidal shells, considered as a particular case of the rotationally symmetric shell problem, Fig. 1, was formulated and explored during three decades by Reissner (1949a, 1949b, 1950), Clark and Reissner (1951), Clark (1958), Reissner (1958, 1963a, 1963b, 1969, 1972, 1974, 1981). Governing equations, which describe the problem of bending of a torus, retain the 'rotational symmetry in a broader sense' (ordinary differential equations) even in the case if a displacement field is rotationally nonsymmetric of a type

$$U_r = U_r(\Phi), \quad U_z = U_z(\Phi), \quad U_\theta = k\theta R(\Phi). \tag{2.1}$$

U_r, U_z, U_θ denote radial, axial and circumferential displacements, whereas Φ, θ, angular, meridional (hoop) and longitudinal coordinates, respectively. Tueda, in 1934, was the first to recognize this fact, also explored later by Reissner (1949b), Clark and Reissner (1951), Clark (1958), Reissner (1981).

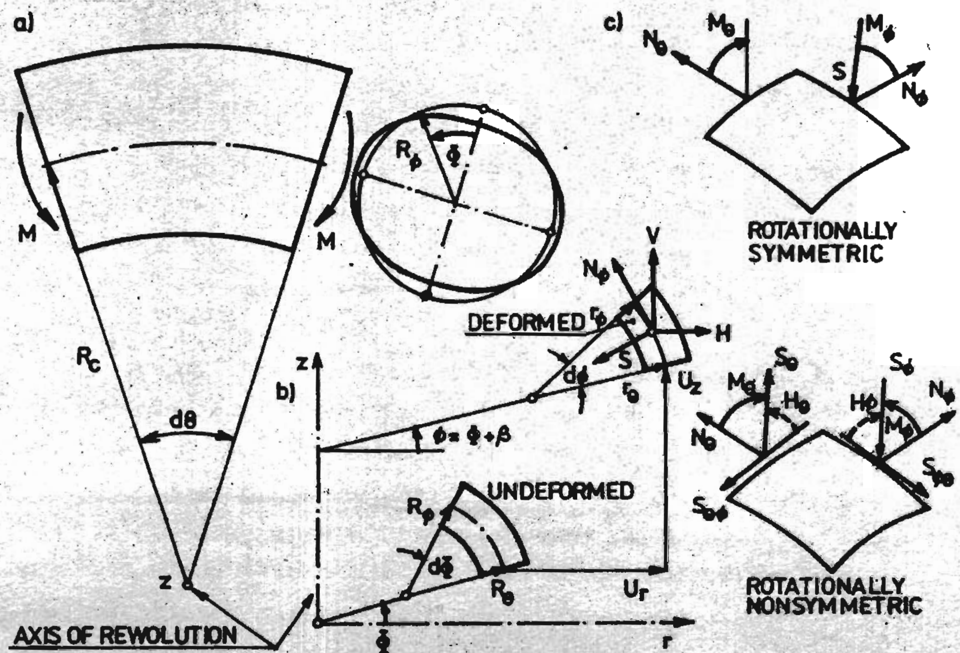

Fig. 1. Geometry of toroidal shell and convention for generalized stresses.

General finite deflection equations for the axisymmetric deformations of thin shells of revolution were derived by Reissner (1950, 1963b). A set of equilibrium, stress-strain, and compatibility equations, was reduced to two simultaneous, second-order coupled differential equations for the unknown meridional angle of slope of normal $\phi(\Phi)$ and the stress function Ψ defined in terms of the horizontal stress resultant H by $\Psi = rH$. These equations are valid for arbitrarily large deformations (finite value of the difference $\beta=\phi-\Phi$). Linearization reduces them to the Reissner and Meissner equations obtained in 1912 -1913. Retaining β^2 terms in expansions of $\sin\phi$ and $\cos\phi$ the Reissner small-finite deflection equations for axisymmetric deformation are obtained. Small deflection equations (retaining β terms), in the case of the 'rotational symmetry in broader sense', were obtained by Clark and Reissner (1951) for the elastically orthotropic or isotropic tube under bending. For thin shells ($h/b<<1$) these equations may be reduced to a simpler form, and finally, setting $b/a=0$ (tubes with the small toroidal curvature), the fundamental simplified Clark and Reissner's equations are obtained. To solve these linearized equations Clark and Reissner (1951) and Clark (1958) proposed either the trigonometric series or the asymptotic solutions. In both cases ovalization of initially curved tubes under bending is assumed to be symmetric about a tube diameter normal to the plane of the principal toroidal curvature. For particular cases the trigonometric series solution may be reduced to the Lorenz, Karl, Beskin and Karman results obtained in 1911- 1945 on the bases of a simplified energy approach. An extension of the theory of finite deflections to the case of finite strains was done by Reissner (1972, 1981). It was shown that the derived toroidal shell equations follow from the general nonlinear shell theory formulated by Simonds and Danielson (1972) and developed by Reissner (1974). The extension of the above results beyond the range of validity of the Love hypothesis was also done by Reissner (1969, 1972), who considered the additional effect of the transverse shear deformation γ, as well as the additional strain component λ due to the effect of moments turning about the normals to the middle surface of the shell.

The more accurate equations formulated within the frame of Reissner-type finite deflection theory, with $(1+\varepsilon_{\phi,\theta})$ terms retained in equilibrium equations and a variable shell thickness along the meridian allowed for, were derived by Skrzypek and Bielski (1988-1989) for thin-walled sandwich toroidal shells with arbitrary meridional section, subject to bending and external pressure

$$y_i' = f_i[y_j(\Phi),\ h(\Phi),\ \Phi], \qquad i,j=1,\ldots,6. \tag{2.2}$$

The basic set of six quasi-linear first order ordinary differential equations, for the three geometric (ϕ, U_r, U_z) and the three static (M_ϕ, N_ϕ, S) unknown functions of coordinate Φ, Fig.1, may be reduced to Reissner's equations in Cauchy's form, by setting $1+\varepsilon_{\phi,\theta}\approx1$ and by neglecting $(1/R_\theta-1/R_\phi)$ etc. terms in secondary relations. The explicit formulae were derived for circular, elliptical-symmetric and elliptical-nonsymmetric cross-sections. To solve the basic set of equations for the case of symmetric or asymmetric forms of deformation Bielski (1985-1986), and Skrzypek and Bielski (1988-1989) discussed the two types of boundary conditions: the symmetry conditions

$$B_s(X_s)=0\ , \tag{2.3}$$

or the periodicity conditions

$$B_A(X_A)=0\ . \tag{2.4}$$

The values of the vector functions B_s or B_A are determined, for a given X_s or X_A by the direct numerical integration (DNI) method. In both cases a numerical method of solving must be appropriate for the two-point boundary value problem of the type 3x3 or 5x5, respectively.

(b). Axelrad's model. The above discussed models are formulated under the fundamental assumption of rotational symmetry of deformation, in narrower or broader sense. To consider problems for which deformations do not satisfy assumption of rotational symmetry (e.g. longitudinal bending- type buckling of curved tubes, bending of elbows with end effects accounted for, bending of bellows, etc.) a general two-dimensional shell theory must be applied (partial differential equations). The 'semi-momentless' nonlinear, *flexible shell* (FS) theory was developed by Axelrad (1965, 1967, 1976, 1978, 1985), as an extension of the 'semi-membrane' theory, originated in 1932 to 1950 with Vlasov, Goldenveiser and Novozhilov for cylindrical shells. Axelrad's FS theory is based on simplifying assumptions:

i) the extensional strain in the hoop direction ε_ϕ and bending moment in longitudinal direction M_θ are ignored in the strain and equilibrium analysis;

ii) shear strain γ and torsional moments H_θ and H_ϕ are ignored;

iii) the Poisson factor terms are neglected in the Hooke relations:

$$\varepsilon_\phi=M_\theta=0;\ \gamma=H_\theta=H_\phi=0;\ \varepsilon_\theta Eh=N_\theta-\nu N_\phi\cong N_\theta;\ M_\phi/D=\kappa_\phi+\nu\kappa_\theta\cong\kappa_\phi. \qquad (2.5)$$

With these assumptions taken into account, the nonlinear equations of the FS theory may be reduced to the basic set of six partial differential equations, the equilibrium and the compatibility, for the unknowns $N_\phi, N_\theta, S_{\phi\theta}, \kappa_\phi, \kappa_\theta, \kappa_{\phi\theta}$, Fig. 1c. The transverse shear stress S_ϕ may be obtained from the additional equilibrium equation whereas $M_\phi, \varepsilon_\theta$ from the Hooke law. The linearized FS equations can be obtained through an appropriate elimination of the unknowns from the basic set of equations which finally leads, if constant thickness is assumed, to the two forth-order linearized equations. Next, the Fourier series solution is applied (Axelrad 1976, 1978) for the two basic unknowns: the meridional (hoop) curvature κ_ϕ and the longitudinal (circumferential) force N_θ. In the case if finite displacements are allowed for, the perturbation method is used. Expressing all unknowns as a series of small parameter proportional to applied loading or displacement, the problem is finally reduced to the linear solutions for each approximation.

A review of the existing computer programs for stress, buckling and stability analysis of elastic shells of revolution was done by Bushnell (1984). For Love's 'first approximation' shell theory differences in the basic kinematic relations for reference surface deformation as well as in the expressions for force and moment resultants were discussed from the point of view of the computerized formulation (BOSOR).

2.2 Nonelastic models

(a). Rigid-plastic models. A nonlinear theory of large rotationally symmetric deformation (in broader sense) for a rigid-plastic sandwich toroidal shell was formulated first by Skrzypek and Hodge (1975) and then developed by Skrzypek (1979, 1980a, 1980b), Skrzypek and Życzkowski (1983).

In the first paper the Levy-Mises theory of plastic flow and the HMH yield condition were applied to obtain the basic set of six nonlinear equations for the following unknowns: the two stress functions ω^{+},ω^{-}, the shear force S and the displacements and angle velocities $\dot{U}_r,\dot{U}_z,\dot{\phi}$. Both, the Hencky-Ilyushin small strain theory and the Nadai-Davis theory for large logarithmic strains were used by Skrzypek and Życzkowski (1983). In both cases the strain paths were compared for the in-plane bending or the combined bending with internal pressure. Deformation was assumed to retain the rotational symmetry and the axial symmetry with respect to the plane of principal curvature.

(b). Elastic-plastic models. According to Bushnell (1981) there are three basic approaches to elastic-plastic analysis of straight and curved tubes and elbows:

i) an approximate *beam type models* in which the resultant forces and moments are related to the tube axis strains and changes of curvature;

ii) a simplified *shell models* based on a one-dimensional discretization or trigonometric expansion in the hoop direction (with the end effect disregarded);

iii) a *brute force method* based on a two-dimensional discretization (with the end effects considered).

Approximate beam type models were used by Spence and Findlay (1973, 1977), Touboul et al.(1988). Calladine (1974a) applied Clark and Reissner's elastic asymptotic analysis in conjunction with the lower bound theorem of plasticity and the simplified HMH yield condition to obtain the approximate value of σ_y for which a curved tube collapses for a given value of bending moment.

Marcal (1967) recognized that axisymmetric shell theories can easily be adopted for pipe-bend analysis. Hibbitt et al. (1973) developed a finite element method (FEM) for the nonlinear analysis of pipelines with elastic-plastic and creep behavior. They introduced the special pipe-bend element into the MARC (1971) computer program (library element 17). MARC El.17 is basically a one-dimensional, axisymmetric isoparametric shell element modified through the addition of 'beam-type' deformation modes. This simplified analysis assumes that each elbow element deforms uniformly over its axial length and it does not account for the end effects provided by the straight pipe portions. Each straight pipe portion of the piping structure is modeled by one beam element (library element 14) with suitable constraint equations for joining pipe-bend sections to straight pipes (Fig.2a).

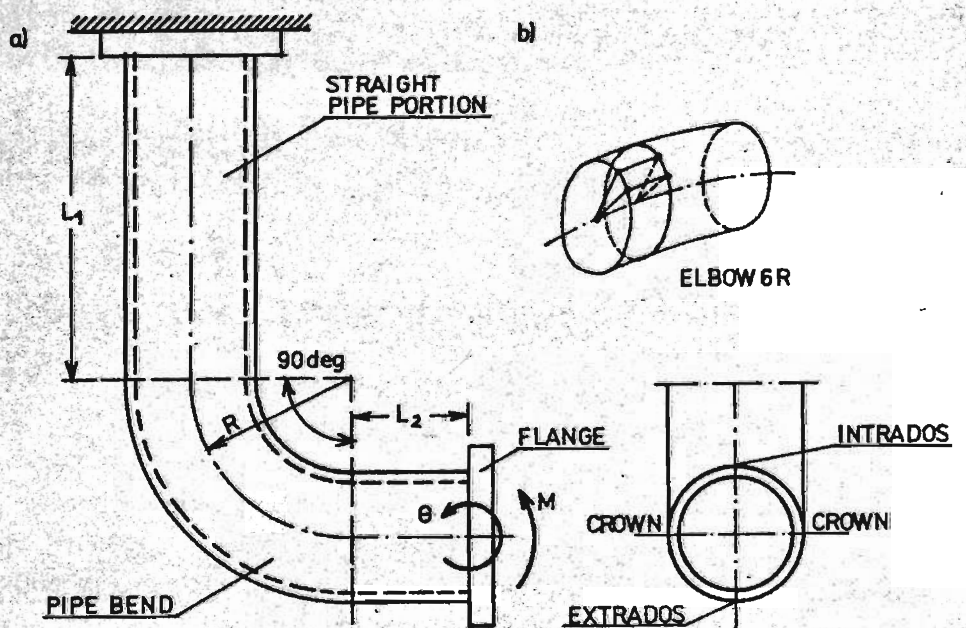

Fig.2. Example of: a) elbow structure subject to in-plane moment (Sobel and Newman 1986), b) elbow-element (Boyle and Spence 1980).

A more advanced element, called ELBOW6R, was described by Takeda et al. (1979). A piping system is decomposed here into a number of finite rings, these idealized with quadrilateral elements around the cross-section. ELBOW6R are doubly-curved element, based on a shell theory with transverse shear deformations including, consisting of four nodal points at the corners of elements and two nodes at the centres of the two end sections (Fig.2b). A one-dimensional FEM model was used by Bushnell (1981) for the elastic-plastic bending and buckling analysis of elbows.

The modified BOSOR5 program was applied to stimulate a bending problem by a problem of nonuniformly heated torus; the isotropic strain-hardening and the HMH yield criterion were assumed.

Muc (1985) and Muc and Skrzypek (1988) formulated the general theory of elastic-plastic deformation of sandwich toroidal shells based on the nonlinear geometric equations derived by Reissner and on the Prandtl-Reuss theory of plasticity and the HMH yield condition. Deformation was assumed as rotationally symmetric and, additionally, only the axially symmetric deformation of a cross-section was allowed for. The basic set of equations was reduced to the six quasi-linear equations for the unknown increments of the state functions: $\delta\phi$, δU_r, δU_z, δN_ϕ, δS, δM_ϕ. Then, the one-dimensional discretization in the hoop direction, with the symmetry condition assumed, was introduced and the Runge-Kutta IV method of direct numerical integration (DNI) with the standard iterative technique to solve the nonlinear two-point 3x3 boundary value problem was used on each step of time. Bielski and Skrzypek (1989) extended this theory to the case of a torus of arbitrary cross-section with variable thickness and asymmetric deformations of radial cross-sections considered, to end up with the 5x5 nonlinear two-point boundary value problem.

For the 'detailed' two-dimensional nonlinear analysis of elastic-plastic curved tubes and elbows the new doubly-curved shell element was developed for MARC (1979) computer program (library element 4). MARC El.4 is based on bi-cubic shape function and Koiter-Sanders shell theory. A two-dimensional FEM elastic-plastic analysis of elbows with the end effects taken into account was done by Vrillon et al. (1975). Sobel and Newman (1986) and Dhalla (1987) used MARC FEM program for both the 'simplified' one-dimensional (Element 17) or the 'detailed' two-dimensional (Element 4) analysis of the in-plane bending of elbows. The Koiter-Sanders shell theory was applied for the elastic-plastic deformation with the strain hardening stress-strain curve used.

(c). Creep models. Boyle and Spence (1980) provided an extensive state-of-art review of creep pipework analysis methods and suggested a following classification scheme:

i) Primary methods based on a one-dimensional beam-type models.

ii) Secondary methods based on a two-dimensional shell-type models, usually discretized using FEM technique (MARC, ADINA, ELBOW).

iii) Tertially methods based on two- or three-dimensional geometric model using local constitutive relations.

The effect of creep in smooth pipe-bends was examined by Spence (1969, 1973), applying the one-dimensional model with the end effect ignored. The power creep law and strain or complementary energy methods were used to obtain lower or upper bounds for creep flexibility factors. Hibbitt et al. (1973) used MARC E1.17 for the one-dimensional FEM study of a complex, spatial pipeline subject to creep under thermal cyclings.

A two-dimensional model of steady creep deformation of pipe-bends with the end constraints, based on the thin shell theory proposed by Novozhilov in 1964 and the Norton creep law , was developed by Chan and Boyle (1984, 1986). Double trigonometric series expansions were used for displacement rates and boundary conditions for the in-plane and the out-of-plane bending. Direct minimization of the total potential energy rate function with respect to the unknown coefficients in the strain field was applied.The two-dimensional FEM analysis, using MARC E14 and the exponential Blackburn creep law, was applied for the creep analysis of elbows by Sobel and Newman (1986). The simplified analysis of steady state creep of pipe bends with the end effect taken into account is due to Thomson (1980).

3. Curved tubes and pipe elbows

3.1. In-plane bending of curved tubes. Brazier (1927) was the first who considered geometric softening of curved tubes when subject to closing in-plane bending. Deformation of the initially circular cross-section was determined from the minimum strain energy. Other solutions, based on the principle of minimum potential energy, were obtained by Karl, Beskin and Huber. Reissner (1949b) and Clark and Reissner (1951) applied the theory of small-finite deflection to pure bending of curved tubes with the uniform circular or the elliptical cross-section. A concept of fully stressed torus in deformed state, optimal from the point of view of the Brazier effect, was introduced by Skrzypek (1977-1978). The stabilizing effect of internal pressure against the Brazier flattening of elastic tori and curved tubes under bending was the subject of study of Hamada and Nakatani (1977) (circular, incomplete toroidal shell; finite difference method FDM), as well as Boyle and Spence (1977) and Boyle (1981) (curved tubes with elliptic or circular cross-section; the DNI method applied to Reissner's equations for small finite deflections). Influence of the geometric effects on the plastic bending deformation of toroidal shells was also analyzed by Afendik (1968), Bilobran (1976), Rozhdestvensky and Cherny (1976).

3.2 In-plane and out-of-plane bending of pipe elbows. Pipe elbows are the most flexible members in a piping system and, hence, are forced to accommodate displacements arising from different movements, mainly, due to the thermal expansions and the seismic and dead weight loads. Both the in-plane (closing or opening) and the out-of-plane bending may appear here, however, predominantly elastic deflections are admitted. Bolt and Greenstreet (1972) determined experimentally the plastic collapse loads for the carbon steel- or stainless steelelbows subject to bending with or without internal pressure. The moment at collapse was increased, although the load at the onset of nonlinear responce was decreased due to internal pressure. Sobel (1977) applied the one-dimensional MARC pipe-bend element to the FEM analysis of 90 deg elbows subject to the in-plane closing bending (no account for 'stiffening effect' provided by the straight pipe portions). A comparison with the results predicted by the more limited scope ELBOW computer program and with Clark and Reissner asymptotic formulae was done. A displacement based FEM enhanced to account for interaction effects between elbows and rigid flanges, elbows of different curvatures and elbows joining straight pipe porsions was used by Bathe and Almeida (1982). They explored the computer program ADINA, for both the in-plane and out-of-plane bending.

Elastic or elastic-plastic bending (opening or closing) or bending combined with pressure (external or internal) of 90 or 180 deg piping elbows was analyzed by Bushnell (1981), who applied one-dimensional modified BOSOR5 FEM. A comparison with Reissner's results (elastic range) and with the test results by Bung et al. (1978) showed a very good agreement. The DNI method was applied by Skrzypek and Życzkowski (1978, 1983), Skrzypek (1978-1982), Skrzypek and Muc (1982, 1988) for the analysis of the limit curves of rigid-plastic or elastic-plastic toroidal shells subject to in-plane bending and pressure (internal or external). The DNI method was also applied by Bielski and Skrzypek (1989) to extend these results to the case of asymmetric deformations of a cross-section. Paths in stress space followed by the inner and the outer layer points on the plane of symmetry as the externally pressurized elastic-plastic curved tube was bent, exhibit a good qualitative agreement with Bushnell's (1981) results.

Dhalla (1987) used the two-dimensional MARC E14 to analyze the overall and local deformations of the 90 deg elbow joint to the two straight pipe portions. The numerical results were compared with those obtained experimentally from tests performed at the room temperature or at an elevated temperature (21°C and 513°C). At collapse the analysis overpredicts the measured deformation by as much as 30%. An extensive experimental study of the ferritic steel elbows and the austenitic steel elbows with elastic-plastic deformations accounted for was done by Hilsenkopf et al. (1988). In-plane and out-of-plane bending moments as well as the influence of internal pressure, temperature and cyclic loadings were studied. A comparison of one-dimensional and two-dimensional MARC finite elements (EL.17 and El.4) for elastic-plastic in-plane bending deformation of elbow, without or with end constraints taken into account, was performed by Sobel and Newman (1986), (Fig.2).

Furthermore, creep behavior of elbows was analyzed here using MARC El.4 and comparing the numerical results with a test. The analysis significantly overestimates the measured deformations. The El.17 (one-dimensional) occurred to be totally inadequate. The creep flexibility of pipe bends with the end constraints ('flanged bends' or 'tangent pipe bends') were analyzed by Chan and Boyle (1984, 1986). Both in-plane and out-of-plane bending were considered.

3.3 Stability of curved tubes subject to bending or/and pressure. The instability of toroidal shell (curved tube) can be considered within the frame of one of the three following symmetry restrictions, Fig.3:

a) rotational symmetry of a torus and axisymmetric deformation of a cross-section,

b) rotational symmetry of a torus but asymmetric deformation of a cross-section,

c) rotationally nonsymmetric deformation of a torus and axisymmetric or asymmetric deformation of a cross-section.

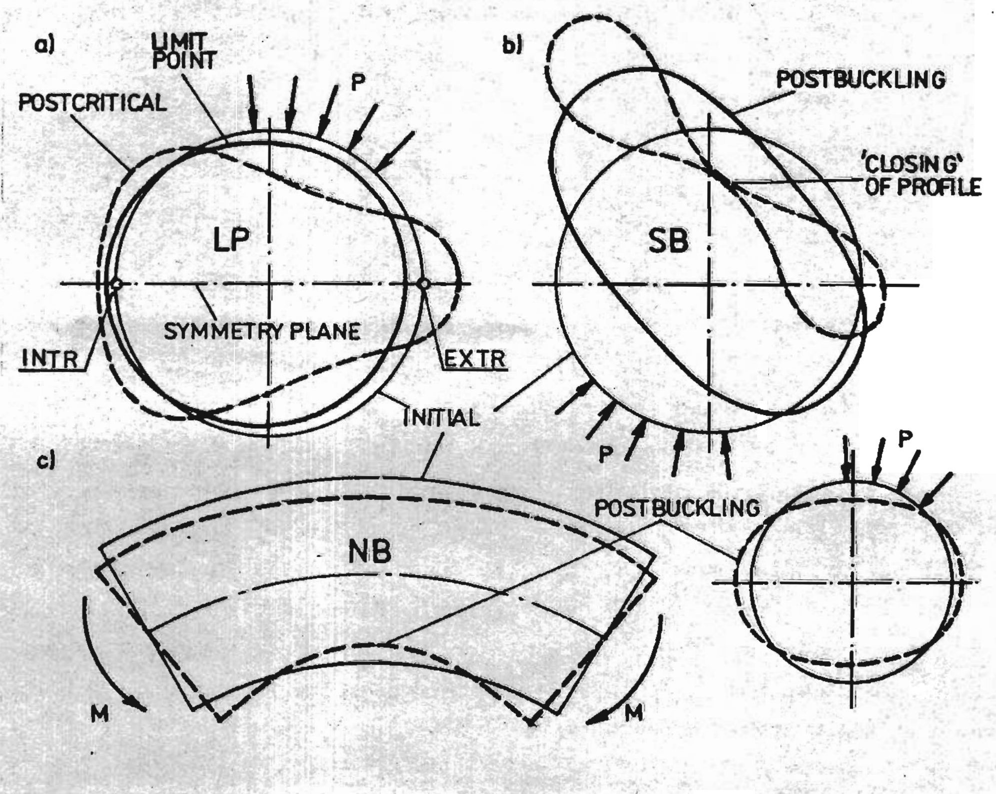

Fig. 3. Forms of instability of curved tubes with external pressure and bending: a) limit point (LP) instability, b) rotationally symmetric buckling (SB), c) rotationally nonsymmetric buckling(NB); (Bielski 1985/6, Axelrad 1976).

(a).Limit point (LP) instability of curved tubes subject to in-plane bending was first considered by Brazier. The simplified nonlinear Reissner's thin shell theory ($\lambda=0$) was used by Emmerling (1982), who examined the influence of internal pressure on the LP instability of curved tubes with circular or elliptical cross-sections, subject to bending. A related problem was considered by Bielski and Skrzypek (1982). Symmetric instability modes of the plastic toroidal shells and postcritical deformations were analyzed by Skrzypek (1978-1982), Skrzypek and Życzkowski (1978), (Brazier effect or bulging of the wall; rigid-plastic model), as well as by Muc and Skrzypek (1982), Skrzypek and Muc (1988), (pressure-curvature interaction curves; elastic-plastic model). Symmetric deformations of a plastic shell-arch with an open semicircular profile was analyzed by Skrzypek and Skoczeń (1988). The three types of boundary conditions were considered, for the shell subject to external pressure and bending, to obtain surfaces of limit states.

(b).Rotationally symmetric buckling (SB) analysis of a toroidal shell when subject to external pressure, under the additional assumption of a membrane precritical state was done by Sobel and Flügge (1967), Jordan (1973), Fedosov (1971). Fedosov noticed that, even in the case of a rotationally nonsymmetric instability mode, the critical pressure is approximately the same as for a simplified rotationally symmetric analysis. However, the analysis based on the hypothesis of a membrane precritical state considerably overestimates critical loads. To perform the meridional buckling analysis the nonlinear system of equations, corresponding to elastic or elastic-plastic models (p.2), may be used. When solving the nonlinear governing equations, e.g. by means of the Newton-Raphson algorithm, it can happen that the matrices of derivatives $[\partial B_s/\partial X_S]$ or $[\partial B_A/\partial X_A]$, are singular for symmetric or asymmetric deformations respectively. Finally, for the point of symmetric buckling the adjacent equilibrium state is asymmetric:

$$\det[\partial B_A/\partial X_A]_{\mathbf{x}_{Ao}}=0 \quad \text{and} \quad \det[\partial B_S/\partial X_S]_{\mathbf{x}_{So}}\neq 0, \tag{3.1}$$

whereas, for the limit point the adjacent equilibrium state is symmetric

$$\det[\partial B_s/\partial X_s]_{X_{so}}=0. \tag{3.2}$$

The method of detection of the singularity of nonlinear operator was applied by Gaydaychuk et al. (1978a, 1978b), Gulayev et al. (1982), Bulygin (1973, 1973-1974) to the stability analysis under external pressure of curved tubes with circular or elliptical cross-sections and constant or variable thickness. Kosheleva and Myachenkov (1971) considered stability of toroidal shells subject to concentrated forces. Bielski (1985-1986) applied the finite deflection and rotation Reissner type theory to analyze the critical and postcritical deformations of toroidal shells subject to external pressure.

Using Pontriagin's formalism Skrzypek and Bielski (1988-1989) solved the problem of optimal design of elastic toroidal shell subject to buckling under external pressure. The optimal design problem was formulated to maximize the lower of the critical pressures, p_{bif} or p_{max}, within the constraint of constant volume of material of the sandwich core where wall thickness $h(\Phi)$ was chosen as a control variable:

$$\begin{gathered} \max\min\{p_{bif}[h(\Phi)],\ p_{max}[h(\Phi)]\}\ , \\ V[h(\Phi)]/V_0=1, \quad h_{inf}\le h(\Phi)\le h_{sup}\ . \end{gathered} \tag{3.3}$$

Both unimodal and bimodal formulation were performed to obtain an increase of critical pressure by more than 30%. Bielski (1990) showed, however, that optimal shell is more sensitive to geometric imperfections (out-of-roundness) then the original shell of constant thickness. A related problem of stability and postcritical analysis of toroidal shell-arches subject to external pressure, considered as rotationally symmetric toroidal shells with open profile, was analyzed by Skoczeń (1990). A double-step parametric optimization was performed to obtain the middle surface shape and the thickness distribution, for which the critical pressure is maximized.

(c). Rotationally nonsymmetric buckling (NB) analysis was performed by Axelrad (1978). The Fourier series solution with the cos$j\Phi$ and sin$j\Phi$ functions was used for the analysis of buckling of short tubes with transverse ribs, tubes with built-in ends, toroidal shells without ribs and toroidal shell out-of-plane flexure, all subject to external pressure. A local stability concept was formulated and applied by Axelrad (1976, 1985), Emmerling (1982) and Axelrad and Emmerling (1985-1986) to consider the buckling of toroidal shell under bending. This approach is based on the observation that the nonsymmetric bifurcation buckling instability is determined by the stress state and the shape of the shell inside the zone of the initial buckle. Functions describing stress state and the shape of the shell outside the buckling zone may be extended analytically in any way with a negligible effect on the critical loads. The approximate stability condition, hence, contains only the stress and strain resultants at a point of the shell. Using this hypothesis Axelrad (1985) proposed the asymptotic stability condition in the form

$$|N_\theta| \le N_\theta^{cr}, \quad N_\theta^{cr} = Eh^2/\{r_\phi\,[3(1-\nu^2)]^{1/2}\}, \tag{3.4}$$

where r_ϕ represents the deformed normal section curvature at the 'buckling point'. In all cases under consideration the nonsymmetric buckling (NB) instability occurs before the maximum bending moment could be reached and, hence, the limit point instability (LP) is out of practical significance.

Elastic-plastic bucking and collapse considerations of large diameter subsea pipelines has recently become the aim of an increasing number of papers. Pipeline operations involve a combination of the circumferential buckling due to hydrostatic pressure and the buckling due to the longitudinal bending; Haagsma (1976), Palmer (1981), Jinsi (1982), Johns and McConnell (1984). The authors presented simple empirical buckling relationships for pipes which are subject to both hydrostatic pressure and bending, including the effect of out-of-roundness (Jinsi, Johns and McConnell). An approximate method of collapse analysis of submarine pipelines, based on simple kinematical models, was proposed by deWinter

(1981). The models were based on the observations that deformation in the collapsed cross-section of a pipe is localized in four plastic hinges. The critical points for plastic hinge formation have been also examined by Lang (1984, 1985) on the basis of the fully three-dimensional elasticity applied to a toroidal geometry. A transition curve between the two yield mechanisms (first yielding in the 'shell crowns' or in the 'intrados') was derived. This curve corresponds to the transition curves also obtained by Bielski and Skrzypek (1989) who considered the two types of collapse of elastic-plastic curved tubes: the 'instantaneous decohesion' in the shell crowns or a 'beam type' plastification.

4. Axisymmetric bellows

Structurally bellows can be considered as thin shells of revolution (toroidal bellows), which usually can be described by the two circular segments of a toroidal shell of the angle γ (convex and concave) joined by a straight segment of an annular plate or a conical shell. The majority of effort was done to C-bellows (γ=90 deg), S-bellows (γ>90 deg), Ω-bellows (one toroidal segment), in the absence of straight segments, and on U-bellows, in the presence of plate segments, Fig.4. A critical review of models used for the bellows analysis was done by Wilson (1983).

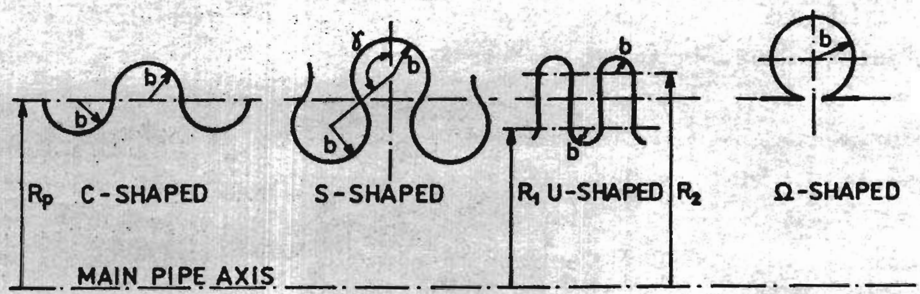

Fig.4. Representative bellows configurations: C-, S-, U- and Ω- shaped.

The models can be classified according to the two basic approaches (Chand and Garg (1981)):

a) *toroidal shell analysis* in conjunction with the annular plate and cylindrical shell theory,

b) approximate *beam type* or *plate type models*.

Clark (1950) was the first to apply the linear elastic theory of thin shells of revolution with the method of asymptotic integration of differential equations to the analysis of Ω-bellows subject to axial load as well as the corrugated pipe subject to axial load and internal pressure. Clark and Reissner's small deflection theory of thin-walled toroidal shells, with only linear β terms retained, was adopted in the sixties and early seventies by Ota and Hamada (1963) and Hamada and Takezono (1964, 1965, 1966a, 1966b, 1967a, 1967b) to the analysis of U-bellows. A more extended model, consisting of toroidal shell, annular plate, and cylindrical shell, was employed by Hamada et al. (1970) to calculate stresses and displacements in U-shaped expansion joints of pressure vessels. The accuracy of the method proposed was confirmed by comparing with the experimental results of Turner and Ford (1957). The problem of limitation of linearized approach was considered by Hamada et al (1968). A rigorous numerical analysis, based on the small-finite deflection equations derived by Reissner (1950, 1963a) (with β^2 terms retained) and on the FDM, was applied in Hamada's paper to the large deflection analysis of U-bellows and of the corrugated diaphragms. A numerical method for problems of unsymmetric bending deformation of the axisymmetric U-bellows was proposed by Hamada et al. (1971). The dependent variables were expanded in the Fourier series in the circumferential direction to reduce a partial differential equation problem to a one-dimensional problem of the meridional independent variable, next solved by the use of the FDM. Later, Hamada et al. (1976) employed the FEM to solve the thin shell equations for the U-bellows.

Recently, Singh (1988) examined the accuracy and validity of the application of the axisymmetric curved thick-shell isoparametric element for the linear elastic analysis of U-shape expansion bellows, having an arbitrary profile, subject to axial load and internal pressure. A comparison of numerical results with those from Turner and Ford experiments illustrates accuracy of the proposed method. Boyle and Spence (1984) developed the large-deflection analysis procedure, based on Reissner's finite-deflection equations, for the rotationally symmetric bellows of arbitrary section under axial loadings and internal pressure, with the toroidal arcs and the thickness variations represented by a Fourier series. The solution of a nonlinear two-point boundary value problem, was based on the DNI method and a nonlinear shooting technique. The similar method was employed by Bujar and Skrzypek (1990) for the analysis of effect of various boundary conditions at a junction between the S- or Ω-bellows unit and the main pipe. The pipe has been alternatively taken as a rigid, rigid and hinged or flexible to obtain elastic interaction curves of bellows when subject to axial load and internal pressure. An optimal design, to assure the maximal axial flexibility of S-shaped bellows, described by the two circular segments, with γ considered as the optimization variable, was done by Calladine (1974b). The extended and a more rigorous analysis was performed by Skoczeń (1990), who considered the influence of pressure and temperature on the axial flexibility of the 'optimal' bellows. A numerical approach based on the FDM was established by Hamada and Tanaka (1973) for the large-deflection problems of elastic-plastic shells of revolution. A low-cycle fatigue life was estimated for U-bellows subject to a deflection controlled cyclic loading by Hamada and Takezono (1974).

For engineering applications simple beam or plate models can be applied to predict the maximum stress levels and deflections in toroidal bellows. Theoretical results for the equivalent beam or ring plate (having identical geometric relationships with the bellows) are usually correlated with the experimental stress and deflection behavior for half convolution of bellows. Such an approach was used by Feely and Goryl (1950), who applied a beam theory for stress and deflection analysis of disc-type U-shaped, bellows subject to axial loading and pressure. A more general proposals to apply a concept of equivalent beam or annular-plate models for S- or U-bellows subject to axial loading were presented by Chand and Garg (1981). They were based on Clark's and Hamada's solutions for S- and U-bellows, obtained from the rigorous toroidal shell theories. The projected distance between the points of the maximum stress at the inner and outer convolution was assumed equal to the width of the annular plate. Finally, the constant factor was used to modify the plate stress and deflection results.

A critical review of papers dealing with the computer aided bellows models: beam model, strength of material shell model, plate model, plate and cylindrical shell model and shell model based on classical shell theory, an approximate energy method and on finite element analysis, was done by Wilson (1983). A discussion of solutions based on the energy approach, was performed by Findlay and Spence (1979). The existing Clark's, Dahl's and other's results, were used to demonstrate that the application of the theorem of minimum potential energy leads to lower bounds for flexibility factors, whereas, the analysis based on the theorem of minimum complementary energy provides upper bound flexibility factors.

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Summary

TOROIDALNE KONSTRUKCJE POWŁOKOWE - RÓWNANIA PODSTAWOWE ORAZ KOMPUTEROWA ANALIZA RUR ZAKRZYWIONYCH, KOLAN I KOMPENSATORÓW

W pracy dokonano przeglądu ponad 100 publikacji poświęconych nieliniowej analizie cienkościennych konstrukcji w kształcie powłok toroidalnych. Omówiono podstawowe teorie, modele i metody obliczeniowe oraz programy komputerowe usystematyzowano problemy analizy naprężeń, ugięć oraz stanów krytycznych związanych z utratą stateczności, bądź też z powstawaniem niedopuszczalnych nieciągłości kinematycznych. Omówiono rozwiązania inżynierskie dla kolan rurociągów i kompensatorów toroidalnych, poddanych działaniu różnych obciążeń (zginanie w płaszczyźnie lub z płaszczyzny kolan, zginanie z udziałem ciśnienia wewnętrznego, wyboczenie pod działaniem ciśnienia zewnętrznego i zginania, osiowe obciążenie kompensatorów).