

BENDING OF A COSSERAT PLATE UNDER ITS OWN WEIGHT AND NORMAL UNIFORM LOADING

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Following the considerations published in [1] the aim of the present paper is to determine the representation of the displacement vector and the infinitesimal rotation vector describing bending of the Cosserat plate subject to its own weight and to the normal uniformly distributed loading. The presented biharmonic representation reduces the equilibrium problem of such plate to a single non-homogeneous biharmonic equation involving a plate deflection function.

1. Introduction

A generalized plane state stress (GPSS) in the micropolar plate was defined elsewhere [2] and the biharmonic representation was presented describing this problem in the plate made of Grioli-Toupin material. Generalization of this representation to the Cosserat plate was made by the Author [1]. In both cases the homogeneous problems were considered reducing the description of bending of such plates to a single biharmonic equation involving a deflection function.

In the present and next papers we will consider non-homogeneous flexure problems in plates made of a Cosserat material or a material with constrained rotations (Grioli-Toupin material). Now we will investigate the plate under its own weight and normal uniform loading acting on the faces of the plate. Representation of the displacement vector and the infinitesimal rotation vector being given is, to the best author's knowledge, unknown in the literature.

The basic equations of the Cosserat medium are given by Jemielita and Nowacki [1,3]. The summation convention is adopted. The Latin indices run over 1,2,3 and the Greek ones - over 1,2. Comma implies partial differentiation.

2. The biharmonic representation for the simple non-homogeneous problem

Let us investigate an elastic layer with thickness h subject to the bending under its own weight and normal uniform loading q acting on the faces $x^3 = z = \pm h/2$.

The solution to the differential equations

$$(1 + \kappa)(\nabla^2 u_\alpha + u''_\alpha) + \left(\frac{1}{1 - 2\nu} - \kappa\right)u^i_{,i\alpha} + 2\kappa\epsilon_{\alpha\beta}(\phi_{3,\beta} - \phi_{\beta,3}) = 0 \quad (2.1)$$

$$(1 + \kappa)(\nabla^2 u_3 + u''_3) + \left(\frac{1}{1 - 2\nu} - \kappa\right)u^i_{,i3} + 2\kappa\epsilon^{\alpha\beta}\phi_{\beta,\alpha} + \frac{1}{\mu}X_3 = 0$$

$$(\gamma + \epsilon)(\nabla^2 \phi_\alpha + \phi''_\alpha) + (\beta + \gamma - \epsilon)\phi^i_{,i\alpha} + 2\alpha[\epsilon_{\alpha\beta}(u_{3,\beta} - u_{\beta,3}) - 2\phi_\alpha] = 0 \quad (2.2)$$

$$(\gamma + \epsilon)(\nabla^2 \phi_3 + \phi''_3) + (\beta + \gamma - \epsilon)\phi^i_{,i3} + 2\alpha(\epsilon^{\alpha\beta}\phi_{\beta,\alpha} - 2\phi_3) = 0$$

with the boundary conditions on the faces

$$\begin{aligned} \sigma_{3\alpha}(x^\beta, \pm \frac{h}{2}) &= 0 \\ \sigma_{33}(x^\beta, \pm \frac{h}{2}) &= \pm \frac{q}{2} \\ \mu_{3i}(x, \pm \frac{h}{2}) &= 0 \end{aligned} \quad (2.3)$$

will be sought with the help of the semi-inverse method.

The stress tensor is denoted by (σ_{ij}) and the couple-stress tensor by (μ_{ij}) ; $(\epsilon_{\alpha\beta})$ represents the Levi-Civita permutation symbol; (u_i) stand for components of the displacement vector; (ϕ_i) represents components of the vector of infinitesimal rotations; X_3 stands for a component of body forces and $(\cdot)' = \frac{d(\cdot)}{dz}$.

The quantities μ , λ , α , γ , ϵ and β are material constants of the Cosserat medium in the Nowacki notation [1,3], where μ and λ can be viewed as Lamé's moduli, ν represents Poisson's ratio and the nondimensional constant κ is given by

$$\kappa = \frac{\alpha}{\mu} \quad (2.4)$$

Let us present the components of the displacement vector (u_i) and the components of the infinitesimal rotation vector (ϕ_i) in the form (cf [1])

$$\begin{aligned} u_\alpha(x^\beta, z) &= -zv(x^\beta)_{,\alpha} + s(z)\nabla^2 v(x^\beta)_{,\alpha} \\ u_3(x^\beta, z) &= v(x^\beta) + f(z)\nabla^2 v(x^\beta) + \frac{h}{\mu}(g(z)q + \check{g}(z)\check{q}) \end{aligned} \quad (2.5)$$

$$\begin{aligned}\phi_\alpha(x^\gamma, z) &= \epsilon_\alpha^\beta \left(v(x^\gamma)_{,\beta} + R_2(z) \nabla^2 v(x^\gamma)_{,\beta} \right) \\ \phi_3 &= 0\end{aligned}\tag{2.6}$$

where $q = \text{const.}$, $\bar{q} = X_3 h = \text{const.}$, $g(z)$ and $\bar{g}(z)$ are unknown functions.

Functions $s(z)$, $f(z)$ and $R_2(z)$ are defined by [1]

$$\begin{aligned}s(z) &= -\frac{(2-\nu)h^2}{24(1-\nu)} z \left(C_2 - \frac{4z^2}{h^2} \right) - l^2 h \frac{\sinh kz}{\sinh \frac{kh}{2}} \\ f(z) &= -\frac{h^2}{24(1-\nu)} \left[6 \left(1 - \frac{2\nu z^2}{h^2} \right) - (2-\nu)C_2 \right] \\ R_2(z) &= \frac{h^2}{24(1-\nu)} \left(3 - (2-\nu)C_2 + 12(1-\nu) \frac{z^2}{h^2} - \frac{12(1-\nu)}{kh} \frac{\cosh kz}{\sinh \frac{kh}{2}} \right)\end{aligned}\tag{2.7}$$

where

$$l^2 = \frac{\gamma + \epsilon}{4\mu} \quad k^2 = \frac{N^2}{i^2} \quad N^2 = \frac{\kappa}{1 + \kappa}\tag{2.8}$$

The constant C_2 determines the physical meaning of $v(x^\alpha)$.

We predict that the function $v(x^\alpha)$ satisfies the equation

$$\mathcal{D} \nabla^4 v = Aq + \bar{A}\bar{q}\tag{2.9}$$

where \mathcal{D} is the rigidity of the micropolar plate determined by

$$D = D\xi \quad D = \frac{\mu h^3}{6(1-\nu)} \quad \xi = 1 + 24(1-\nu) \frac{l^2}{h^2}\tag{2.10}$$

and A, \bar{A} are the unknown constants.

Substituting (2.5) and (2.6) into Eqs (2.1) and (2.2), we find the nonzero solutions to this set when the function $v(x^\alpha)$ satisfies Eq (2.9) and the functions $g(z)$ and $\bar{g}(z)$ satisfy the following simple ordinary differential equation

$$\begin{aligned}g''(z) &= -\frac{\mu(1-2\nu)A}{2h(1-2\nu)\mathcal{D}} \left[f + \frac{s'}{1-2\nu} + \kappa(f - s' - 2R_2) \right] \\ \bar{g}''(z) &= -\frac{1-2\nu}{2h^2(1-\nu)} \left\{ 1 + \frac{\mu h \bar{A}}{D} \left[f + \frac{s'}{1-2\nu} + \kappa(f - s' - 2R_2) \right] \right\}\end{aligned}\tag{2.11}$$

Hence we obtain the following functions g and $\overset{*}{g}$ defined by

$$g(z) = \frac{\mu h^3 A}{2(1-\nu)\mathcal{D}} \left[\frac{z^2}{24(1-\nu)h^2} \left(3(1-2\nu) + \nu(2-\nu)C_2 - 2(1-\nu^2)\frac{z^2}{h^2} \right) + 2(1-\nu) \frac{l^2 \cosh kz}{kh^3 \sinh \frac{kh}{2}} - \frac{C_3}{3} \right] \quad (2.12)$$

$$\overset{*}{g}(z) = \frac{\mu h^3 A}{2(1-\nu)\mathcal{D}} \left[\frac{z^2}{24(1-\nu)h^2} \left(3(1-2\nu) + \nu(2-\nu)C_2 - 2(1-\nu^2)\frac{z^2}{h^2} \right) + 2(1-\nu) \frac{l^2 \sinh kz}{kh^3 \sinh \frac{kh}{2}} \right] - \frac{(1-2\nu)z^2}{4(1-\nu)h^2} - \overset{*}{C}_3 \frac{1}{\xi} \quad (2.13)$$

The integration constants C_3 and $\overset{*}{C}_3$ depend on the method of determination of the function $v(x^\alpha)$.

The constitutive relationships of an isotropic homogeneous and centrosymmetric medium in the Nowacki notation have the form [1,3]

$$\sigma_{ij} = (\mu + \alpha)\gamma_{ij} + (\mu - \alpha)\gamma_{ji} + \lambda\gamma^k{}_k\delta_{ij} \quad (2.14)$$

$$\mu_{ij} = (\gamma + \varepsilon)K_{ij} + (\gamma - \varepsilon)K_{ji} + \beta K^k{}_k\delta_{ij}$$

where (γ_{ij}) and (K_{ij}) are components of the deformation tensors defined by

$$\gamma_{ij} = u_{j,i} - \epsilon^k{}_{ij}\phi_k \quad K_{ij} = \phi_{j,i} \quad (2.15)$$

The boundary conditions $(2.3)_1$ and $(2.3)_3$ are identically satisfied, while the boundary condition $(2.3)_2$ will be satisfied when the constants A and $\overset{*}{A}$ are

$$A = \overset{*}{A} = 1 \quad (2.16)$$

Finally, using formulae $(2.5) \div (2.7)$ and $(2.12) \div (2.16)$, we obtain the following formulae defining the displacements, rotations and stresses, respectively

$$u_\alpha(x^\beta, z) = - \left\{ z v(x^\beta)_{,\alpha} + \left[\frac{(2-\nu)h^2}{24(1-\nu)} z \left(C_2 - \frac{4z^2}{h^2} \right) + l^2 h \frac{\sinh kz}{\sinh \frac{kh}{2}} \right] \nabla^2 v(x^\beta)_{,\alpha} \right\} \quad (2.17)$$

$$u_3(x^\beta, z) = v(x^\beta) - \frac{h^2}{24(1-\nu)} \left[6 \left(1 - \frac{2\nu z^2}{h^2} \right) - (2-\nu)C_2 \right] \nabla^2 v(x^\beta) + \frac{h}{\xi} \left[\frac{6l^2(1-\nu) \cosh kz}{kh^3 \sinh \frac{kh}{2}} + \frac{z^2}{8(1-\nu)h^2} \left(3(1-2\nu) + \nu(2-\nu)C_2 - \right. \right.$$

$$-2(1-\nu^2)\frac{z^2}{h^2}) - C_3] \frac{q}{\mu} + \frac{h}{\xi} \left[\frac{6l^2(1-\nu) \cosh kz}{kh^3 \sinh \frac{kh}{2}} - \frac{(1-2\nu)z^2}{4(1-\nu)h^2} \xi + \right. \\ \left. + \frac{z^2}{8(1-\nu)h^2} (3(1-2\nu) + \nu(2-\nu)C_2 - 2(1-\nu^2)\frac{z^2}{h^2}) - \bar{C}_3 \right] \frac{\bar{q}}{\mu}$$

$$\sigma_{\alpha\beta}(x^\gamma, z) = -\frac{2\mu}{1-\nu} \left\{ z \left((1-\nu)v_{,\alpha\beta} + \nu \nabla^2 v \delta_{\alpha\beta} \right) + \right. \\ \left. + \left[\frac{(2-\nu)h^2}{24} z \left(C_2 - \frac{4z^2}{h^2} \right) + (1-\nu)l^2 h \frac{\sinh kz}{\sinh \frac{kh}{2}} \right] \nabla^2 v(x^\gamma)_{,\alpha\beta} \right\} + \\ + \frac{\nu}{1-\nu} \frac{z}{h} \left[\frac{1}{2\xi} \left(3 - (2-\nu)C_2 + 4(1-\nu)\frac{z^2}{h^2} \right) - 1 \right] \bar{q} \delta_{\alpha\beta} + \\ + \frac{\nu z}{2(1-\nu)\xi h} \left(3 - (2-\nu)C_2 + 4(1-\nu)\frac{z^2}{h^2} \right) q \delta_{\alpha\beta} \quad (2.18)$$

$$\sigma_{\alpha 3}(x^\beta, z) = -\frac{\mu h^2}{4(1-\nu)} \left[\left(1 - \frac{4z^2}{h^2} \right) + 8(1-\nu) \frac{kl^2 \cosh kz}{h \sinh \frac{kh}{2}} \right] \nabla^2 v(x^\beta)_{,\alpha} \quad (2.19)$$

$$\sigma_{3\alpha}(x^\beta, z) = -\frac{\mu h^2}{4(1-\nu)} \left(1 - \frac{4z^2}{h^2} \right) \nabla^2 v_{,\alpha} \quad (2.20)$$

$$\sigma_{33}(x^\alpha, z) = \left\{ \frac{1}{2\xi} \left[\frac{z}{h} \left(3 - 4\frac{z^2}{h^2} \right) + \frac{24(1-\nu)l^2 \sinh kz}{h^2 \sinh \frac{kh}{2}} \right] - \frac{z}{h} \right\} \bar{q} + \\ + \frac{1}{2\xi} \left[\frac{z}{h} \left(3 - 4\frac{z^2}{h^2} \right) + \frac{24(1-\nu)l^2 \sinh kz}{h^2 \sinh \frac{kh}{2}} \right] q \quad (2.21)$$

$$\phi_\alpha(x^\gamma, z) = \epsilon_\alpha^\beta \left[v(x^\gamma)_{,\beta} + \frac{h^2}{24(1-\nu)} \left((2-\nu)C_2 - 3 - 12(1-\nu)\frac{z^2}{h^2} + \right. \right. \\ \left. \left. + \frac{12(1-\nu) \cosh kz}{kh \sinh \frac{kh}{2}} \right) \nabla^2 v(x^\gamma)_{,\alpha} \right] \quad (2.22)$$

$$\phi_3 = 0$$

$$\mu_{\alpha\beta}(x^\delta, z) = 4\mu l^2 \epsilon_\beta^\gamma \left\{ v_{,\gamma\alpha} + \left[\frac{h \cosh kz}{2k \sinh \frac{kh}{2}} - \frac{h^2}{24(1-\nu)} \left(3 + \frac{12z^2}{h^2} (1-\nu) - \right. \right. \right. \\ \left. \left. \left. - (2-\nu)C_2 \right) \right] \nabla^2 v_{,\gamma\alpha} \right\} + 4\mu l^2 \eta \epsilon_\alpha^\gamma \left\{ v_{,\gamma\beta} + \right. \\ \left. + \left[\frac{h \cosh kz}{2k \sinh \frac{kh}{2}} - \frac{h^2}{24(1-\nu)} \left(3 + \frac{12z^2}{h^2} (1-\nu) - (2-\nu)C_2 \right) \right] \nabla^2 v_{,\gamma\beta} \right\} \quad (2.23)$$

$$\mu_{33}(x^\alpha, z) = 0 \quad (2.24)$$

$$\mu_{3\alpha} = -2\mu l^2 h \epsilon_\alpha^\beta \left(2\frac{z}{h} - \frac{\sinh kz}{\sinh \frac{kh}{2}} \right) \nabla^2 v_{,\beta} \quad (2.25)$$

$$\mu_{\alpha 3} = \eta \mu_{3\alpha}$$

It can be readily seen that, if the function $v(x^\alpha)$ satisfies Eqs (2.9) and (2.16), all the differential equations of equilibrium

$$\begin{aligned} \sigma^{j\alpha}_{,j} &= 0 \\ \sigma^{j3}_{,j} + X_3 &= 0 \\ \epsilon^{ijk} \sigma_{jk} + \mu^{ji}_{,j} &= 0 \end{aligned} \quad (2.26)$$

and the boundary conditions (2.3) are satisfied.

The formulae (2.17) ÷ (2.24) obtained above are given in the most general form, not encountered by the author in literature. We select the constants C_2 , C_3 and $\overset{*}{C}_3$ in such a way that the function $v(x^\alpha)$ has a simple physical meaning
- for

$$\begin{aligned} C_2 &= 3 \\ C_3 &= \frac{5(1+\nu)}{64} + \frac{6l^2(1-\nu)}{kh^3} \coth \frac{kh}{2} \\ \overset{*}{C}_3 &= \frac{5(1+\nu)}{64} - \frac{1-2\nu}{16(1-\nu)} \xi + \frac{6l^2(1-\nu)}{kh^3} \coth \frac{kh}{2} \end{aligned} \quad (2.27)$$

the function v represents a deflection of plate faces

$$\hat{w}(x^\alpha) \stackrel{\text{df}}{=} u_3(x^\alpha, \pm \frac{h}{2}) \quad (2.28)$$

- for

$$C_2 = \frac{6}{2-\nu} \quad C_3 = \overset{*}{C}_3 = \frac{6l^2(1-\nu)}{kh^3 \sinh \frac{kh}{2}} \quad (2.29)$$

w represents a deflection of the mid-plane

$$w(x^\alpha) \stackrel{\text{df}}{=} u_3(x^\alpha, 0) \quad (2.30)$$

- for

$$\begin{aligned} C_2 &= \frac{6-\nu}{2-\nu} \\ C_3 &= \frac{27-7\nu^2}{960(1-\nu)} + \frac{12l^2}{k^2 h^4} (1-\nu) \\ \overset{*}{C}_3 &= \frac{27-7\nu^2}{960(1-\nu)} - \frac{1-2\nu}{48(1-\nu)} \xi + \frac{12l^2}{k^2 h^4} (1-\nu) \end{aligned} \quad (2.31)$$

the function v is a simple average

$$\bar{v}(x^\alpha) \stackrel{\text{df}}{=} \frac{1}{h} \int_{-h/2}^{h/2} u_3(x^\alpha, z) dz$$

- for

$$\begin{aligned} C_2 &= \frac{3(10 - \nu)}{5(2 - \nu)} \\ C_3 &= \frac{3(65 - 9\nu^2)}{11200(1 - \nu)} + \frac{72l^2(1 - \nu)}{k^3h^5} \left(\coth \frac{kh}{2} - \frac{2}{k^2h^2} \right) \\ \bar{C}_3 &= \frac{3(65 - 9\nu^2)}{11200(1 - \nu)} - \frac{1 - 2\nu}{80(1 - \nu)} \xi + \frac{72l^2(1 - \nu)}{k^3h^5} \left(\coth \frac{kh}{2} - \frac{2}{k^2h^2} \right) \end{aligned} \quad (2.32)$$

is a weighted average

$$\bar{w}(x^\alpha) \stackrel{\text{df}}{=} \frac{3}{2h} \int_{-h/2}^{h/2} \left(1 - 4\frac{z^2}{h^2} \right) u_3(x^\alpha, z) dz$$

It is evident that according to the introduced deflection, we can represent displacements and stresses in different forms.

3. Limiting cases. G-T and Hooke's plate

The representation of the displacement vector in the plate made of Girelli-Toupin material is a limiting case when κ tends to infinity [1].

Assuming $\kappa \rightarrow \infty$ in formulae (2.8), we obtain

$$N^2 = 1 \quad k^2l^2 = 1 \quad (3.1)$$

Substituting the equality (3.1) into formulae (2.17) ÷ (2.25), we obtain the displacements, rotations and stresses in the plate made of G-T material. In this case $\phi = \frac{1}{2} \text{rot} u$ [1].

To obtain the displacements and stresses in the plate made of the classical Hookean material one should pass to zero with l [1,2]. On computing limits of the expressions (2.17) ÷ (2.21), we arrive at

$$u_\alpha(x^\beta, z) = - \left\{ z v(x^\beta)_{,\alpha} + \left[\frac{(2 - \nu)h^2}{24(1 - \nu)} z \left(C_2 - \frac{4z^2}{h^2} \right) \right] \nabla^2 v(x^\beta)_{,\alpha} \right\}$$

$$\begin{aligned}
 u_3(x^\beta, z) = & v(x^\beta) - \frac{h^2}{24(1-\nu)} \left[6 \left(1 - \frac{2\nu z^2}{h^2} \right) - (2-\nu)C_2 \right] \nabla^2 v(x^\beta) + \quad (3.2) \\
 & + h \left[\frac{z^2}{8(1-\nu)h^2} \left(3(1-2\nu) + \nu(2-\nu)C_2 - 2(1-\nu^2) \frac{z^2}{h^2} \right) - C_3 \right] \frac{q}{\mu} + \\
 & + h \left[\frac{z^2}{8(1-\nu)h^2} \left(3(1-2\nu) + \nu(2-\nu)C_2 - 2(1-\nu^2) \frac{z^2}{h^2} \right) - \right. \\
 & \left. - \frac{(1-2\nu)z^2}{4(1-\nu)h^2} - \overset{*}{C}_3 \right] \frac{\overset{*}{q}}{\mu}
 \end{aligned}$$

$$\begin{aligned}
 \sigma_{\alpha\beta}(x^\gamma, z) = & -\frac{2\mu}{1-\nu} \left\{ z \left((1-\nu)v_{,\alpha\beta} + \nu \nabla^2 v \delta_{\alpha\beta} \right) + \right. \\
 & + \frac{(2-\nu)h^2}{24} z \left(C_2 - \frac{4z^2}{h^2} \right) \nabla^2 v(x^\gamma)_{,\alpha\beta} \left. \right\} + \\
 & + \frac{\nu}{1-\nu} \frac{z}{h} \left[\frac{1}{2} \left(3 - (2-\nu)C_2 + 4(1-\nu) \frac{z^2}{h^2} \right) - 1 \right] \overset{*}{q} \delta_{\alpha\beta} + \\
 & + \frac{\nu z}{2(1-\nu)h} \left(3 - (2-\nu)C_2 + 4(1-\nu) \frac{z^2}{h^2} \right) q \delta_{\alpha\beta} \quad (3.3)
 \end{aligned}$$

$$\sigma_{\alpha 3}(x^\beta, z) = \sigma_{3\alpha}(x^\beta, z) = -\frac{\mu h^2}{4(1-\nu)} \left(1 - \frac{4z^2}{h^2} \right) \nabla^2 v_{,\alpha} \quad (3.4)$$

$$\sigma_{33}(x^\alpha, z) = \frac{z}{2h} \left(1 - 4 \frac{z^2}{h^2} \right) \overset{*}{q} + \frac{z}{2h} \left(3 - 4 \frac{z^2}{h^2} \right) q \quad (3.5)$$

Certain particular cases of the above formulae for the self-weight loading for $C_2 = \frac{6}{2-\nu}$ and $\overset{*}{C}_3 = 0$ were obtained earlier, among others, by Dougall, Love and Gutman [4÷6]. Simillary for the uniform loading they were given in the papers [5,7,8].

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Zginanie płyty Cosseratów ciężarem własnym i stałym obciążeniem normalnym

Streszczenie

W niniejszej pracy, korzystając z rozważań danych w pracy [1], wyznaczono przedstawienie wektora przemieszczenia i infinitezimalnego obrotu, opisujące zginanie płyty Cosseratów ciężarem własnym i stałym obciążeniem normalnym. Przedstawiona reprezentacja biharmoniczna sprowadza zagadnienie równowagi takiej płyty do rozwiązania niejednorodnego równania biharmonicznego na funkcję przedstawiającą ugięcie płyty.

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