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# INFLUENCE OF THERMAL DEFORMATION ON THE STRESS STATE OF HEAT-PROOF COATINGS

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In order to study the effect of thermal deformation behaviour and magnitude on a stress level in heat-proof coatings, an investigation into the dependence of thermal expansion and shrinkage in destructing composite materials on temperature, heating rate and mode, chemical composition of gas medium and pressure as well as a specimen size and material anisotropy was carried out. Thermal stress analysis is done by means of three dimensional elasticity theory methods testified to considerable effect of conditions under which thermal deformation coefficients are obtained, on stress level and stress distribution through the coating thickness for glass-fiber-plastic cylindrical element.

### 1. Formulation of the problem

When studying coefficients of thermal linear expansion  $\alpha$  in respect to destructing composites operating under conditions of high and superhigh temperatures, of importance is to consider not only their thermal expansion but also shrinkage associated with the phase and structural transformation in the polymer matrices. It is difficult to analyse precisely all these variations, since the coking materials are multicomponent systems in which a great number of interrelated processes proceed simultaneously. Therefore the diagrams of coefficients of the thermal linear expansion  $\alpha$  for such a class of materials are of a complex (alternating and polyextremal) nature.

During single or cyclic heating of heat-proof composite materials, their physicochemical and mechanical properties undergo considerable changes, primarily due to thermal destruction of a polymeric binder. Thermal deformations in such materials are great and manifested by a complex behaviour. Thermal and shrinkage stresses arising in structural components are high enough to cause failure of the structure.

Fracture of heat-proof coating may be caused by crumbling, partial melting, errosive ablation, separating of some segments and outer surface cracking, respectively. Frequently, cracking behaviour is complicated due to the action of gas pressure generated as a result of pyrolysis of inner layers, local materials defects, etc. But more often cracks propagate along the formation lines of the cylindrical or conic coating and are caused by tensile stresses. Experimental study of thermal deformation in reinforced plastics, with maximum allowance made for service conditions and data obtained for stress-state calculations of heat-proof structural components was justified by the urgent demand for improving thermal resistance and flying life of space vehicle heat-proof coating.

### 2. Experimental solution

In the paper the methods of thermal deformation of polymer and carbonized materials are described taking into account the behaviour peculiarities under heating. The developed set-ups permit investigating the temperature strains of non-metallic destructing composites over the range of  $299 \div 3273$  K. There exists the possibility to study the effect of heating rates on chemical composition of the environment. The experimental set-ups (dilatometers) involve a high-temperature furnace, the system for specimen boating, control and measurement of its temperature, the non-contact system for thermal strain measurement in a specimen, the system for creating and measuring in the furnace for its blowing down with the inert gas as well as the system for cooling the furnace accessories.

In contrast to the thermal linear expansion coefficient for metals, the expansion and shrinkage coefficients for destructing composite materials display higher absolute values (shrinkage of some glass-carbon-fibre and carbon-metal-plastics exceeds 5%) as well as significant anisotropy, multiple extremes on curves, dependence on heating rate, specimen size, chemical composition of the medium, heat supply mode, material density, etc.

For example, Fig.1 shows thermal deformation versus heating rate for phenol-based carbon-plastics. It can be seen that heating up to temperature

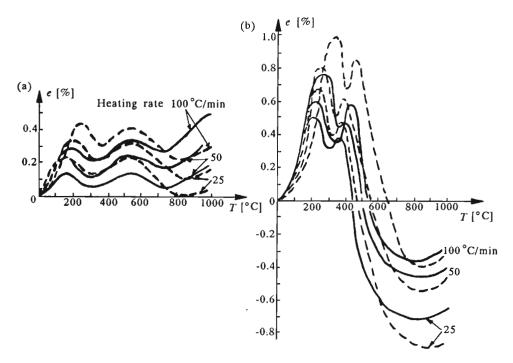


Fig. 1. Dependence of thermal deformation in carbon-metal-plastic on heating rate in neutral (solid lines) and oxidizing (broken lines) atmospheres: (a) - along the warp, (b) - along the weft

higher than 450°C results in simultaneous expansion in the warp (Fig.1a) and shrinkage in the weft (Fig.1) (cf Gracheva (1982)). In Fig.1 numbers denote a heating rate in deg/min. In neutral medium (solid lines) the intensity of deformation processes is different as compared to those in the oxidizing one (dash lines). In addition, one can see that the extremes on the expansion diagrams for specimens cut along the warp tend to increase in the neutral medium and to decrease in the oxidizing one. Thermal deformation plots for carbon-metal-plastics are identical, with similarly high anisotropy and simultaneous expansion in the warp and shrinkage in the weft and the direction of the processes in different media is analogous.

The effect of specimen thickness on thermal deformation for carbon-metal-plastics is shown in Fig.2. Numbers near the curves  $\varepsilon = f(T,\tau)$  denote specimen thicknesses in millimeters; solid lines represent the test at heating rate of 100 deg/min, broken lines at 25 deg/min. It is evident that a marked difference in relative thermal strain values is particularly important at high temperatures (cf Gracheva (1984)).

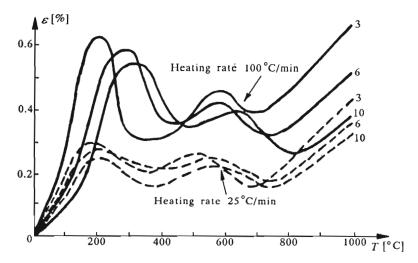


Fig. 2. Effect of specimen thickness on thermal deformation in carbon-metal-plastic at different heating rates: solid lines represent the heating rate of 100 deg/min; broken lines 25 deg/min

The mode of heat supply was found to affect strongly the thermal deformation of reinforced plastics. Fig.3 shows expansion and shrinkage diagrams for phenol-formaldehyde-resin-based glass-fiber-plastics under one-sided heating (the heating scheme and measurement points A and B are shown at the bottom of Fig.3) versus heating rate at different temperatures.

Bending load applied externally to the cooled specimen-side ensured the horizontal position of the specimen in this test.

A fourfold increase in heating rate  $(25 \div 100 \text{ deg/min})$  at, for instance, 600°C is clearly seen to result in tenfold decrease in the shrinkage magnitude. The broken line in Fig.3 represents relative values of thermal deformation for specimens from the same glass-fibre-plastics, tested under uniform heating. These results obtained for mechanically loaded specimens in one-sided heating show that a shrinkage decrease occurs simultaneously with the increase of expansion zone in the material, as compared to the dilatometric curves, with in the time and temperature range under investigation (cf Tretyachenko and Gracheva (1983)).

The above behavior is due to material physicochemical and thermophysical processes dependent on gradients, boundary conditions of thermal and mass transfer and time.

Pyrolysis in phenol- and phenol-formaldehyde-resins starts at 300÷400°C. However, shrinkage of the glass-fibre-, carbon- and carbon-metal-plastics

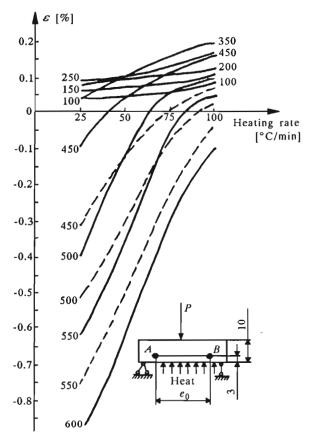


Fig. 3. Dependence of thermal deformation of glass-fibre-plastics on heating rate in one-sided heating

under investigation occurs at higher temperatures since the surface-initiated destruction process should expand through the whole specimen volume and reach a certain stage where reduction in length exceeds thermal expansion of the specimen. Isothermic zones appearing in the specimen under heating, constantly shift to the range of low temperatures, and the progress of destruction depends on temperature and time. Thus, higher heating rates were shown to decrease the material shrinkage at a given temperature, shifting the extremes on the curves  $e = f(T, \tau)$  to the range of higher temperatures. Neutral medium or larger specimen mass favors some retardation of chemical and thermophysical processes. This is illustrated by the nature of material thermal deformation diagrams.

Gravimetric analysis on carbonized carbon-plastics results provide evi-

dence for the dependence of intensity of physicochemical processes on the heating rate and the gas medium composition (Fig.4).

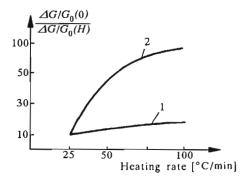


Fig. 4. Ratio of relative weight losses in oxidizing atmosphere to those in the neutral one versus heating rate

It can be seen from Fig.4 (I-relates to anisotropic carbon-plastics, II-to quasi-isotropic carbon-plastics) that the ratio of weight losse values in the above materials in the oxidizing medium and those in the neutral one greatly differ with the heating rate increase. No difference was observed however, at heating rate of 25 deg/min which is the case concerned.

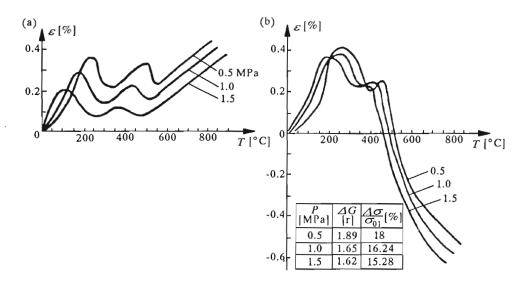


Fig. 5. Effect of pressure on thermal deformation in carbon-metal-plastics cut along
(a) - the warp, (b) - the weft

The investigation into the effect of gaseous medium on thermal deformation of heat-proof materials such as carbon-metal-plastics, proved that higher argon pressure shifts dilatograms to the lower strain and temperature values, respectively. This is in fairly good agreement with the weight analysis results on composites under conditions corresponding to those considered (Fig.5). The decrease in weight losses in the carbon-metal-plastics specimen during heating with the increase in external medium pressure is probably due to the gas "locking" effect. Gases are formed during thermal destruction of polymeric matrix which leads to material cracking.

Thermophysical properties of composite undergo noticeable changes during heating. These changes occur both in single and cyclic heating. Temperature field variation in a heat-proof cylinder during cyclic tests in high temperature gaseous flow is shown in Fig.6. With allowance made for the dependence of

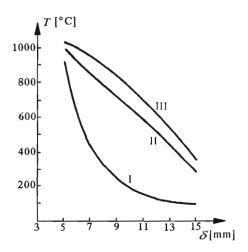


Fig. 6. Temperature distribution through the coating in high-temperature gas flow under cyclic thermal loading

thermal strain coefficients on service factors in thermal stress analysis, it is possible to provide an explanation for causes and conditions of crack initiation. By means of structural models, a laminated cylindrical sheath with phenol-formaldehyde-based glass-fibre-plastic outer coating was considered. When obtaining thermal strain coefficients the effect of heating rate, gas medium composition, anisotropy, etc. on thermal stresses in glass-fibre-laminate sheaths was evaluated from the accurate solutions to the three-dimensional elasticity theory problem for a non-uniform anisotropic body in a quasi-static setting (cf Tretyachenko et al. (1977)).

## 3. Calculation of the thermal stressed state of the construction element

The stress-deformed state of the construction element in the form of a non-homogeneous anisotropic cylinder basing on the solution of equations of the three dimensional problem of elasticity theory is considered. The method of calculation is based on the combination of equations of elasticity theory, those of heat conductivity and the method of numerical analysis. The body's material obeys the Hooke's generalized law and the Duhamel-Neuman hypothesis is taken into account. The relations of the Hooke's law are referred to the cylindrical coordinate system  $z, \theta, r$ . These equations may be written for the *i*th layer  $(r_{i+1} \leq r \leq r_i, i = 1, 2, ..., N)$  of anisotropic materials with one plane of elastic symmetry r = const in the form (cf Pankratova (1992), Grigorenko et al. (1992))

$$\begin{split} e^{i}_{z} &= a^{i}_{11}\sigma^{i}_{z} + a^{i}_{12}\sigma^{i}_{\theta} + a^{i}_{13}\sigma^{i}_{r} + a^{i}_{16}\tau^{i}_{z\theta} + \alpha^{i}_{1}T^{i} \\ e^{i}_{\theta} &= a^{i}_{12}\sigma^{i}_{z} + a^{i}_{22}\sigma^{i}_{\theta} + a^{i}_{23}\sigma^{i}_{r} + a^{i}_{26}\tau^{i}_{z\theta} + \alpha^{i}_{2}T^{i} \\ e^{i}_{r} &= a^{i}_{13}\sigma^{i}_{z} + a^{i}_{23}\sigma^{i}_{\theta} + a^{i}_{33}\sigma^{i}_{r} + a^{i}_{36}\tau^{i}_{z\theta} + \alpha^{i}_{3}T^{i} \\ e^{i}_{r\theta} &= a^{i}_{44}\tau^{i}_{r\theta} + a^{i}_{45}\tau^{i}_{rz} + \alpha^{i}_{23}T^{i} \\ e^{i}_{rz} &= a^{i}_{45}\tau^{i}_{r\theta} + a^{i}_{55}\tau^{i}_{rz} + \alpha^{i}_{13}T^{i} \\ e^{i}_{z\theta} &= a^{i}_{16}\sigma^{i}_{z} + a^{i}_{26}\sigma^{i}_{\theta} + a^{i}_{36}\sigma^{i}_{r} + a^{i}_{66}\tau^{i}_{z\theta} + \alpha^{i}_{12}T^{i} \end{split}$$

where  $\sigma_r^i$ ,  $\sigma_z^i$ ,  $\sigma_\theta^i$ ,  $\tau_{z\theta}^i$ ,  $\tau_{rz}^i$ ,  $\tau_{r\theta}^i$  are stress components. The elastic characteristics  $a_{mn}^i$ , the coefficients of linear temperature extension  $\alpha_1^i, \alpha_2^i, \alpha_3^i$  in directions  $z, \theta, r$ , respectively, and the coefficients of shear  $\alpha_{12}^i, \alpha_{13}^i, \alpha_{23}^i$  are functions of the r coordinate. The coefficients of linear extension may be set in the form (cf Grigorenko et al. (1992))

$$a_{11}^{i} = \alpha_{1}^{i} \cos^{2} \varphi + \alpha_{2}^{i} \sin^{2} \varphi$$

$$\alpha_{22}^{i} = \alpha_{1}^{i} \sin^{2} \varphi + \alpha_{2}^{i} \cos^{2} \varphi$$

$$\alpha_{12}^{i} = 2(\alpha_{1}^{i} - \alpha_{2}^{i}) \sin \varphi \cos \varphi$$

$$\alpha_{33}^{i} = \alpha_{3}^{i} \qquad \alpha_{13}^{i} = \alpha_{23}^{i} = 0$$

$$(3.2)$$

where  $\varphi$  is the angle of inclination to the axis z.

It should be pointed out that the last condition takes into account the arbitrary change in elastic properties of materials through the body thickness. For solution, the problem of the theory of elasticity ought to satisfy not only the equations of equilibrium, strain-displacement relations, the Hooke's law, but also the boundary conditions on all surfaces of the body and the conditions of conjugation of layers in the united packet. In a majority of cases there is a rigid contact when all layers of the packet are deformed without sliding and separation. Then conditions for the components of displacement vector and the stress tensor components must be carried out respectively

$$\begin{aligned} \sigma_{r}^{i} &= \sigma_{r}^{i+1} & \tau_{rz}^{i} &= \tau_{rz}^{i+1} & \tau_{r\theta}^{i} &= \tau_{r\theta}^{i+1} \\ u_{r}^{i} &= u_{r}^{i+1} & u_{z}^{i} &= u_{z}^{i+1} & u_{\theta}^{i} &= u_{\theta}^{i+1} \end{aligned}$$
 (3.3)

The temperature field for the *i*th layer of the cylinder is defined by the equation of heat conductivity which in the cylindrical coordinate system is expressed by

$$K_r^i \frac{\partial}{\partial r} \left( \frac{\partial T^i}{\partial r} \right) + r K_z^i \frac{\partial^2 T^i}{\partial z^2} + \frac{K_\theta^i}{r} \frac{\partial^2 T^i}{\partial \theta^2} = 0$$
 (3.4)

where  $K_r^i = K_r^i(r)$ ,  $K_z^i = K_z^i(r)$ ,  $K_\theta^i = K_\theta^i(r)$  – the coefficients of heat conductivity acting in the directions  $r, z, \theta$ . It is assumed that thermal continuity conditions of layers over the entire surface of contact are fulfilled

$$T^{i} = T^{i+1}$$
  $K_{r}^{i} \frac{\partial T^{i}}{\partial r} = K_{r}^{i+1} \frac{\partial T^{i+1}}{\partial r}$  (3.5)

A case is considered when the butt-ends of the cylinder z=0, z=l do not displace in their planes and are free from normal load. By taking the resolving functions for the basic ones, with the help of which we can formulate the conditions on the limiting surfaces  $r=r_0$ ,  $r=r_N$  and the interfaces of the layers  $r_i$  and fulfilling transformations the initial equations of elasticity (3.1) and the equations of heat conductivity (3.4), the resolving system of equations after division of variables for each pair values k and n for the i layer is expressed in form (cf Pankratova (1992), Grigorenko et al. (1992))

$$\begin{split} \frac{d\bar{\sigma}_{kn}^{i}}{dr} &= C_{kn}^{i}\bar{\sigma}_{kn}^{i} + \bar{f}^{i} \\ \bar{\sigma}_{kn}^{i} &= (\sigma_{r,kn}^{i}, \tau_{rz,kn}^{i}, \tau_{r\theta,kn}^{i}, u_{r,kn}^{i}, u_{z,kn}^{i}, u_{\theta,kn}^{i}, T_{kn}^{i}, T_{kn}^{'i}) \\ C_{kn}^{i} &= \|c_{mq,k}^{i}(r)\| \\ f^{i} &= (f_{1}^{i}, f_{2}^{i}, ..., f_{8}^{i}) \\ &\qquad m, q = 1, 2, ..., 8 \end{split}$$

$$(3.6)$$

Here

 $\sigma_r$  – radial stress

 $\tau_{rz}, \tau_{r\theta}$  – tangential stresses

 $u_r, u_z, u_\theta$  - radial, axial and circumferential displacements, respectively

T - temperature

T' - temperature gradient.

The matrix elements  $C_{kn}^i$  depend on the mechanical characteristics of the layer material. Integration of the Eqs (3.6) is made by means of the steady numerical method allowing to receive the solution with a high degree of precision. Selection of the basic values by which contact conditions for layer conjugation are formulated makes it possible to obtain automatically and continuously solutions for the prescribed number of layers.

On the basis of the worked out method of investigation, the influence of thermal deformation on the stress-state of heat-proof coating in the form of a hollow cylinder (Fig.7) in a quasi-static setting is studied. In agreement with the prescribed experimental initial data relating to layer thickness and mechanical and thermophysical material properties as well as the temperature field, the thickness of cylinder is broken into separate layers  $[r_i, r_{i+1}]$  (i = 1, 2, ..., 7). The specified characteristics in each of these layers are given by their own expressions (cf Grigorenko et al. (1992))

$$T = A_i + B_i r \qquad \nu = C_0^{(i)} + C_1^{(i)} T + C_2^{(i)} T^2$$

$$E = C_i + D_i T \qquad \alpha_T T = a_i + b_i T$$
(3.7)

being approximated by the initial data. Here

E - modulus of elasticity

ν – Poisson ratio

 $\alpha_T$  - coefficient of linear temperature extension.

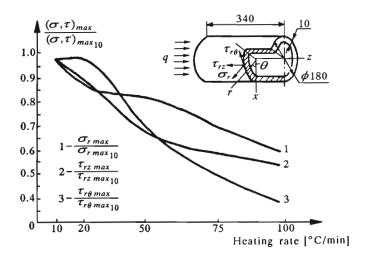


Fig. 7. Effect of heating rate on maximum relative stresses in glass-fibre cylindrical coating under one-sided heating

Investigation into the influence of heating rate V on the values of thermal stress taking into account only changes of thermal deformation coefficients  $\alpha_T$ is studied. Some results of the solution to this problem for the maximum values of the stresses  $\sigma_r, \tau_{rz}, \tau_{r\theta}$  (these values are assigned to maximum stress values with V=10 deg/min) in external (heated) layer ( $0 \le r \le 10$  mm) and distribution of the stresses  $\tau_{rz+}, \sigma_r, \tau_{rz-}$  ( $\tau_{rz+}$  - in the tension zone,  $\tau_{rz-}$  - in the compression zone) through the thickness of the layer induced by heating rate are shown in Fig.7 and Fig.8. Distribution of the tangential stresses  $\sigma_z$  at some points of the cylinder external layer for the fixed heating rate are given in the Table 1. As can be seen from Fig.7 the absolute values of maximum stresses in the external layer decrease with the increasing heating rate. The maximum stresses at the heating rate 10 deg/min are attained at the distance of 4÷5 mm from the heated surface as soon as at the heating rate 100 deg/min those have appeared in the thiner external layer  $0.8 \div 3.5 \text{ mm}$  (Fig.8). For the axis stresses  $\sigma_z$  the tension zone of action of maximum stresses is moved from 3.5 mm at V = 20 deg/min to 1 mm at V = 100 deg/min towards the heated surface. The nature of circular stresses  $\sigma_{\theta}$  distribution is analogous to that for  $\sigma_z$ . Thus, on the basis of this investigation, it is possible to conclude that with increasing the heating rate from 10÷100 deg/min, the zone of maximum thermal stresses in glass-fiber layer of the cylindrical element is moved towards the heated surface while the maximum stresses decrease by 30% in the tension zone, and by 60% in the compression zone.

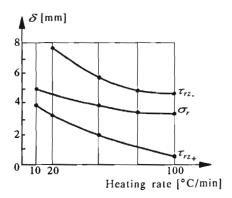


Fig. 8. Movement of maximum thermal stress front as a function of heating rate in glass-fibre cylindrical sheath under one-sided heating

**Table 1.** Distribution of the stresses  $\sigma_z/\sigma_0$  through the thickness of external layer for the fixed heating rate

r	V [deg/min]				
[mm]	10	20	50	75	100
0	0.1480E4	0.1236E4	0.1107E4	0.1071E4	0.1044E4
1	0.1182E4	0.1236E4	0.1107E4	0.1071E4	0.1044E4
2	0.9837E3	0.1236E4	0.8329E4	0.3521E3	0.2841E3
3	0.6407E3	0.9241E3	0.5581E3	0.2931E3	0.3077E2
4	0.2977E3	0.2875E3	0.2833E3	-0.2703E3	-0.2292E3
5	0.6189 E2	-0.3460E3	-0.2438E3	-0.1618E3	-0.1541E3
6	-0.5209E2	-0.2149E3	-0.1692E3	-0.1583E3	-0.1469E3
7	-0.1702E3	-0.1897E3	-0.1507E3	-0.1358E3	-0.1307E3
8	-0.1818E3	-0.1869E3	-0.1469E3	-0.1305E3	-0.1276E3
9	-0.1849E3	-0.1834E3	-0.1429E3	-0.1239E3	-0.1285E3
10	-0.1860E3	-0.1815E3	-0.1401E3	-0.1211E3	-0.1251E

#### 4. Final remarks

Thus, basing on the experimental results obtained, considering the conditions simulating the service ones, and on the result of thermal stress calculation in terms of the three-dimensional elasticity theory the possibility is shown of predicting serviceability of cylindrical heat-proof glass-fiber coatings for the actual service conditions. Such comprehensive experimental – ana-

lytical investigations reveal that the ballast weight of flying vehicles can be reduced considerably without degradation of strength and reliability of the whole structure.

#### References

- GRACHEVA L.J., 1982, Peculiarities of the Curves of Thermal Deformation of Carbon-Plastics under the High Temperatures, (in Russian), Prob. Proc., 3, 98-101
- GRACHEVA L.J., 1984, Dependence of Thermal Deformation Destructing of Carbon – and Carbonmetalplastics of Thickness of Specimen, (in Russian), Prob. Proc., 4, 61-66
- 3. GRIGORENKO J.M., VASILENKO A.T., PANKRATOVA N.D., 1992, The Problems of the Elasticity Theory of the Non-Homogeneous Bodies, (in Russian), Nauk. Dumka, Kiev
- PANKRATOVA N.D., 1992, The Calculation of Anisotropic construction Elements in Space Formulation, in XXXI Symposium "Modelling in mechanics", Gliwice, 327-334
- 5. TRETYACHENKO G.M., GRACHEVA L.J., 1983, Thermal Deformation of Non-Metallic Destructing Materials, (in Russian), Nauk. Dumka, Kiev
- TRETYACHENKO G.M., GRACHEVA L.J., GRIGORENKO J.M., VASILENKO A.T., PANKRATOVA N.D., 1977, Effect of the Dependence of Thermal Deformation on Heating Rate on the Stressed State of Heat-Rejecting Coatings, (in Russian), 3, 69-73

## Wpływ odkształcenia termicznego na poziom naprężeń w pokryciach izolacji cieplnej

#### Streszczenie

W pracy rozpatrzono wplyw odksztalcenia termicznego na poziom naprężeń w pokryciach izolacji cieplnej. Ustalono charakter i wartości odksztalceń cieplnych, powodujących destrukcję kompozytów, która zależy od temperatury, rodzaju prędkości nagrzewu, skladu chemicznego, ciśnienia środowiska gazowego, a także od wymiarów próbek i anizotropii materialu. Stanu termonaprężeń przeanalizowano przy pomocy metody opartej na rozwiązaniu równań sprężystości w trójosiowym stanie, pozwalającym uwzględniać dowolną niejednorodność materialu na grubości izolacji cieplnej w postaci elementu cylindrycznego z włókien szklanych. Uwzględniono warunki, przy których współczynniki rozszerzalności cieplnej wyznaczano doświadczalnie.