MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
4, 33, 1995

# ON CERTAIN METHODS OF CONSIDERING DRY FRICTION IN DYNAMIC ANALYSIS OF PLANAR OPEN KINEMATIC CHAINS

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The paper presents two different ways in which dry friction is taken into account in dynamic analysis of multibody systems considered as planar open kinematic chains with rotary joints (kinematic pairs). In both cases there are different methods of calculating reactions in kinematic pairs. The first method, based on constraint equations, is used mainly in dynamic analysis of systems with flexible links when the rigid finite element method is applied. Another method is used for analysis of chains with rigid links when the equations of motion are formulated using transformation matrices in accordance with the Denavit-Hartenberg notation. In this case unknown quantities are calculated from equations of dynamic equilibrium of a particular link. Both methods enable us to take into account the complex model of dry friction in pairs, i.e., consideration of not only kinetic friction (Coulomb friction) but also stiction phases. Numerical results obtained for both methods, in the case of analysis of the chain simplified with rigid links, are compared and good correlation between them proves the correctness of the methods used.

## 1. Introduction

Planar open kinematic chains built of $n$ links connected by rotary joints (Fig.1) are considered in this paper.

The links are independently driven by drive moments $M_i^D$ (where $i = 1,...,n$).

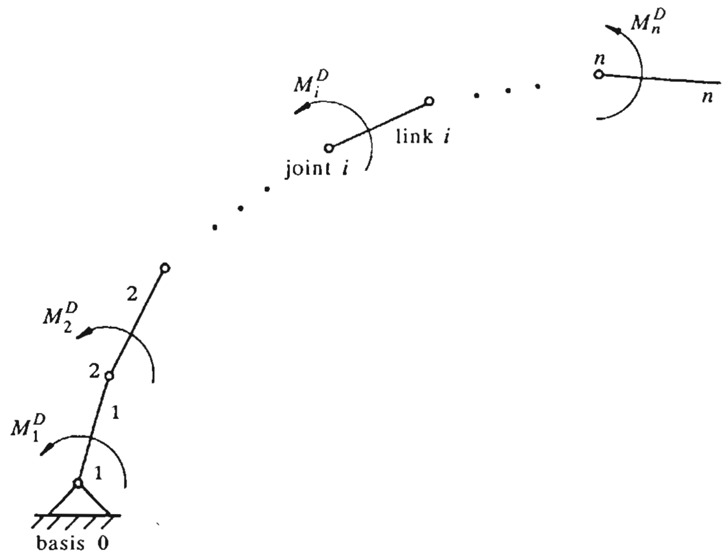


Fig. 1. Example of a planar open kinematic chain

## 2. Review of literature concerned with dynamic analysis of multibody systems with dry friction in kinematic pairs

The expression "multibody system" means any mechanical system composed of rigid bodies or particles connected by springs, dampers and kinematic pairs. These systems can form both open and closed kinematic chains of planar or spatial configuration. The review presented below concerns only publications which deal with direct problems of dynamics of multibody systems with dry friction in pairs. This means problems of analysis of motion of systems forced by known drive moments (forces). Thus, the review does not include papers dealing with inverse dynamics problems, i.e., when the motion of the system is known and drive moments (forces) necessary for this motion have to be calculated.

First publications discussed in this review concern analysis of multibody system in the form of planar linkage mechanism. Bagci (1975) presents one of the first extensive studies devoted to the subject. Equations of dynamic equilibrium of each link released from constraints are formulated and Coulomb friction and liquid friction in kinematic pairs are taken into account. These equations enable him to calculate reaction forces and drive link motion and in consequence the motion of the whole mechanism. Selected crank-androcker and slider-crank mechanisms are analysed. The procedure presented gives three times as many equations of motion as the number of moving links. Moreover, the coefficients are formulated in complicated forms even for simple mechanisms. In order to avoid these problems simplified models of so-called linkage mechanisms with varying transmission ratio are presented by Maczyński (1980). It is assumed there that only external links have masses whereas internal links are massless. The method described enables both kinematic and static friction to be taken into account. The author gives several examples of real mechanisms in which the simplification is acceptable.

An interesting method of dynamic analysis of planar linkage mechanisms is presented by Suwaj and Góral (1987). Having assumed that a mechanism has one degree of freedom, its motion can be analysed as motion of one link with reduced mass, and all external loads and friction forces are reduced to this link, so the motion of the mechanism is represented by one nonlinear differential equation with reduced parameters. Friction is considered by subtraction of the dissipation term at each step of integration. In order to calculate reduced parameters and reduce forces together with friction forces (moments), so-called transfer functions are used. Reaction forces in pairs, which are necessary to calculate the dissipation term, are calculated using iterative procedure from conditions of kinetostatic equilibrium of links. The method is applied to analysis of a slider-crank mechanism. A somewhat similar procedure for linkage mechanisms is presented by Benedict and Tesar (1971). The notion of so-called "influence coefficients" of velocities, accelerations, forces and inertia is introduced. The possibility of the use of these coefficients in dynamic analysis of planar linkage mechanisms with one or two drive links is discussed.

Wojciech (1984), Harlecki and Wojciech (1992) analyse planar linkage mechanisms with flexible links and dry friction in rotary and sliding joints. Not only the Coulomb friction is discussed but the large probability of the occurrence of stiction phases, in the case of flexible systems, is taken into account. Flexible links are modelled using the rigid finite element method. Reaction forces and unknown moments of stiction are calculated from additional constraint equations. The number of these equations varies and depends on the type of friction occurring in each pair.

Over the last few years papers have started being published which deal with dynamic analysis of spatial multibody systems. Among them, papers by Schiehlen (1983) and (1984), Schiehlen and Schramm (1982) and (1983) have a particular place. The motion of systems is described using the Newton-Euler formalism and reaction forces necessary for definition of moments of kinetic friction are calculated from transformed equations of motion in which the acceleration vector has been eliminated by premultiplication of these equations by a special distribution matrix.

Papers by Haug et al. (1986) and Wu et al. (1986a,b) are also quoted very often. The authors describe a method of dynamic analysis in which the equations of motion of systems are defined using both Lagrange and Newton-Euler formalisms. In these systems Coulomb friction and stiction are taken into account and also the impact phenomenon. Unknown quantities are calculated by formulating constraint equations. The authors present models of rotary and prismatic kinematic pairs for both planar and spatial systems and give algorithms for calculating kinetic friction moments (forces) in these pairs. The algorithms worked out are used for dynamic analysis of a simple slider-crank mechanism when the slider is connected with an additional mass by a spring.

In paper by Schwertassek and Türk (1986) the theoretical principles of a certain method of dynamic analysis of multibody systems are presented. These systems can be treated as both open and closed structures with dry friction in pairs. A lot of attention is devoted to formulation of constraint equations.

Muir and Neuman (1988) describe an effective dynamic analysis of open and closed chains with the Coulomb and static friction. According to the authors this method enables both inverse and forward dynamics problems to be solved, yet no details about the second problem are given.

Frączek (1993a) also considers the dynamics of spatial multibody systems with Coulomb and static friction in any kind of kinematic pairs. The author states that his method enables both open and closed chains to be considered. He discusses the formulation of constraint equations for different kinds of kinematic pairs. Next paper by Frączek (1993b) describes the computer program DAMS for dynamic analysis of such systems. He verifies the program using simple mechanical systems without friction.

A special place among work devoted to multibody dynamics is taken up by papers which deal with manipulators modelled as spatial open kinematic chains. Thus, in papers by Gogoussis and Donath (1988) and (1990) there is an interesting description of calculating reaction forces in kinematic pairs. Interactions between links can be described by forces and moments acting in joints between them. These quantities called joint forces and moments can be calculated from equations of the dynamic equilibrium of each link. The equations have to be formulated starting from the last free link. If joint forces and moments are known the moments of Coulomb friction can be calculated. In dynamic analysis friction in drives is also taken into account.

An other method is presented by Klosowicz (1990). The author derives the equations of motion of the spatial manipulator with rotary joints using the Newton-Euler formalism. Reaction forces in pairs are defined by premultiplication of the equations of motion by the inverse of an incidence matrix (cf Wittenburg (1977)). The equations of motion are solved using an iterative procedure in order to consider moments of kinetic friction.

An interesting method of taking into account dry friction in dynamics of spatial manipulators is presented by Wojciech [26]. The author presents a detailed algorithm for integrating equations of motion which are formulated using the Lagrange formalism. The essence of the method is described later on in this paper. The author gives results of numerical calculations for the model of a drilling vehicle which is treated as a manipulator with seven degrees of freedom.

Papers by Szwedowicz (1991) and Ostachowicz et al. (1989) are also devoted to dynamic analysis of a spatial manipulator with rotary and sliding pairs. The method of calculating reaction forces in pairs, using transformation matrices according to the Denavit-Hartenberg notation, is given by Szwedowicz and Szwedowicz (1989).

## 3. Analysis of chains with flexible links

In the case when bending flexibility of links is taken into account the links can be modelled in the form of a system of rigid finite elements (RFE) connected by elastic elements (EE). The main idea of the method is described in detail by Wojciech (1984), Harlecki and Wojciech (1992). Fig.2a presents a system of $n_i$ of RFE and $n_i - 1$ of EE which model the link $i$ of a kinematic chain. This model is a part of the model of the whole chain and is called subsystem $i$. Thus, the model of the whole chain can be treated as a collection of $n$ subsystems. Each RFE has one degree of freedom in relative motion (rotation). The motion of the subsystem can be described by generalized coordinates which are elements of the following vector

$$\xi_i = [x_i, y_i, \theta_{i1}, ..., \theta_{im}, ..., \theta_{in_i}]$$

where

$x_i, y_i$ – coordinates of point $0_i$ in the inertial coordinate system $0xy$

$\theta_{im}$ – angles of inclination of axes of each RFE to the axis $0x$ $(i = 1, ..., n;\ m = 1, ..., n_i)$.

The components of reaction forces $F_i$, $F_{i+1}$ as well as reaction moments $M_i$, $M_{i+1}$ are indicated in Fig.2a. In the case of the last free link $n$ (Fig.2b) the following should be assumed: $F_{n+1} = 0$ and $M_{n+1} = 0$.

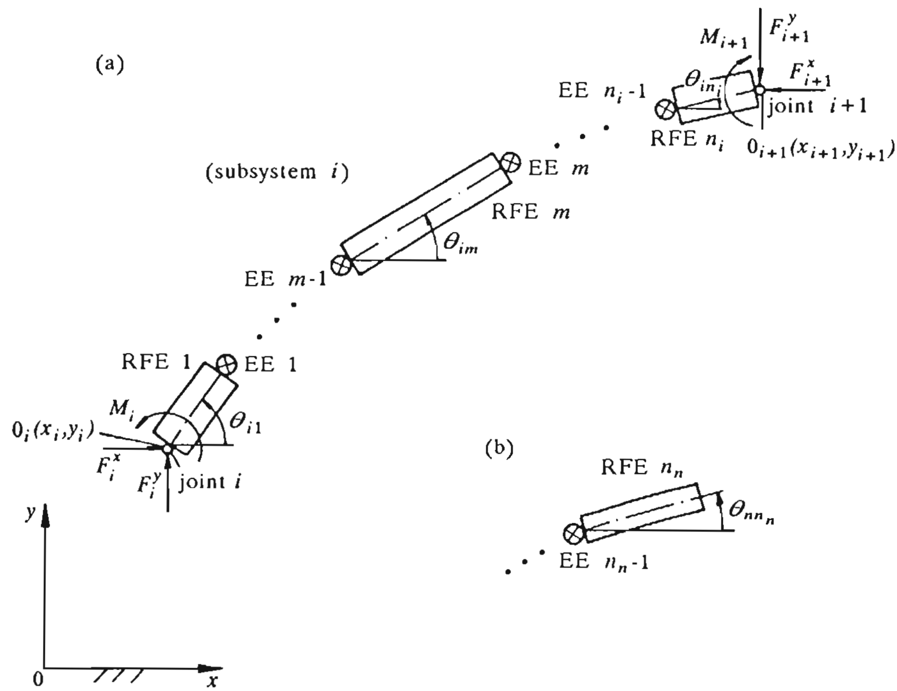


Fig. 2. (a) – Model of the $i$th link of the chain; (b) – End of the last link $n$

There can be two kinds of input of relative motion input for the $i$ kinematic pair:

- Force input

  This occurs when a link $s-1$ acts on the link $s$, where $1 \leq s \leq n$, with the known drive moment $M_s^D$ and there is relative motion in the pair $s$. Moreover, if the dry friction occurs in this pair, then the real moment acting in the pair $s$ is

$$M_s = M_s^D - M_s^F \tag{3.1}$$

  The moment of kinetic friction $M_s^F$ is defined as

$$M_s^F = \operatorname{sign}\dot{\theta}_s \mu_s r_s F_s \tag{3.2}$$

  where

  $\mu_s$ – coefficient of kinetic friction for the pair $s$
  $r_s$ – radius of the pin (sleeve) of the pair considered

$F_s$ – normal reaction force, $F_s = \sqrt{(F_s^x)^2 + (F_s^y)^2}$
$\dot{\theta}_s$ – relative velocity in this pair, $\dot{\theta}_s = \dot{\theta}_{s1} - \dot{\theta}_{s-1n_{s-1}}$.

Naturally, the moment of the same value $M_s$, opposite in direction, also acts on the link $s-1$.

- Kinematic input

This occurs when the relative velocity in a pair $p$, where $1 \le p \le n$, is the known function of time $\psi(t)$. In this case the moment $M_p^*$ from the $p-1$ link, which is necessary to assure the dynamic equilibrium of the link $p$, is unknown. It can happen that the link $p-1$ acts on the link $p$ with the drive moment $M_p^D$ but in the pair connecting both links the relative motion will be interrupted because of frictional drag. This situation is treated as a special case of kinematic input. Transmission from the stiction phase to the kinetic friction phase will follow when the condition is fulfilled

$$|M_p^D - M_p^*| > \bar{M}_p^F \tag{3.3}$$

The moment of stiction is given by the formula

$$\bar{M}_p^F = \bar{\mu}_p r_p F_p \tag{3.4}$$

where $\bar{\mu}_p$ is the coefficient of stiction in the pair $p$.

The equations of motion of the system considered can be derived using the Lagrange formalism of type II. The equations form a set of nonlinear ordinary differential equations of the second order which can be written in a matrix form

$$\mathsf{A}_i\ddot{\boldsymbol{\xi}}_i + \mathsf{B}_i\dot{\boldsymbol{\xi}}_i + \mathsf{C}_i\boldsymbol{\xi}_i + \mathsf{D}_i\boldsymbol{P}_i - \boldsymbol{T}_i^c = \boldsymbol{G}_i + \boldsymbol{T}_i \qquad i = 1,...,n \tag{3.5}$$

How to define matrices $\mathsf{A}_i$, $\mathsf{B}_i$, $\mathsf{C}_i$, $\mathsf{D}_i$ and vectors of gravity forces $\boldsymbol{G}_i$ is described by Wojciech (1984), Harlecki and Wojciech (1992).

The vector of reaction forces in kinematic pairs $i$ and $i+1$ has the form

$$\boldsymbol{P}_i = [F_i^x, F_i^y, F_{i+1}^x, F_{i+1}^y]^\top$$

The components of the friction moments $\boldsymbol{T}_i^c$ and $\boldsymbol{T}_i$ are defined as follows

$$\boldsymbol{T}_i^c = [0, 0, \delta_i M_i^*, 0, ..., 0, -\delta_{i+1} M_{i+1}^*]^\top$$
$$\boldsymbol{T}_i = [0, 0, \rho_i M_i, 0, ..., 0, -\rho_{i+1} M_{i+1}]^\top$$

and

$$\delta_i = \begin{cases} 1 & \text{when there is a kinematic input in the pair } i \\ 0 & \text{when there is a force input in the pair } i \end{cases}$$

$$\rho_i = \begin{cases} 1 & \text{when there is a force input in the pair } i \\ 0 & \text{when there is a kinematic input in the pair } i \end{cases}$$

As can be seen, that the components of the vector $\boldsymbol{T}_i^c$ which are not equal to zero appear only when there is a kinematic input in either $i$ or $i+1$ kinematic pair. The corresponding components of the vector $\boldsymbol{T}_i$ are then zero.

The equations of motion of the whole system are formulated as a composition of $n$ equations of all subsystems (cf Wojciech (1984); Harlecki and Wojciech (1992))

$$\mathsf{A}\ddot{\xi} + \mathsf{B}\dot{\xi} + \mathsf{C}\xi + \mathsf{D}P - T^c = G + T \tag{3.6}$$

In order to calculate reaction forces $F_i$ $2n$ equations of constraints have to be formulated in the following form

$$\begin{aligned} x_i &= x_{i-1} + \sum_{m=1}^{n_{i-1}} l_{i-1,m} \cos\theta_{i-1,m} \\ y_i &= y_{i-1} + \sum_{m=1}^{n_{i-1}} l_{i-1,m} \sin\theta_{i-1,m} \\ i &= 1,...,n \qquad\qquad x_0 = y_0 = 0 \qquad\qquad n_0 = 0 \end{aligned} \tag{3.7}$$

Having differentiated these equations twice they can be written in the form

$$\mathsf{A}_R^c\ddot{\xi} + \mathsf{B}_R^c\dot{\xi} = \mathbf{0} \tag{3.8}$$

In order to calculate unknown components of the vector $T^c$ a varying number of constraint equations has to be formulated in the general form

$$\begin{aligned} \theta_{p1} - \theta_{p-1n_{p-1}} &= \psi(t) \\ n_0 = 0 \qquad\qquad & \theta_{00} = 0 \end{aligned} \tag{3.9}$$

The number of equations (3.9) equals the number of kinematic pairs in which, at the moment considered, there is a kinematic input.

Having differentiated twice these equations with respect to time they can be written in the following matrix form

$$\mathsf{A}_M^c\ddot{\xi} = \psi \tag{3.10}$$

The equations of motion together with the constraint equations (3.8) and (3.10) are considered as a set of equations

$$\mathsf{A}^*\ddot{\xi} + \mathsf{B}^*\dot{\xi} + \mathsf{C}^*\xi + \mathsf{D}^*P - T^{c^*} = G^* + T^* \tag{3.11}$$

It should be underlined that these equations can be used for solving both direct and inverse dynamic problems when we search for the components of the vector $T^c$. In the first case phases of kinetic friction as well as stiction can be considered. The signal for possible occurrence of the stiction phase in the pair $s$ is the change of the sign of the relative velocity of links $s$ and $s-1$. However, motion stops only when the following condition is fulfilled at the same time

$$|M_s^D - M_s^*| \le \bar{M}_s^F \tag{3.12}$$

If not, the relative motion at once changes its direction. As has been said, the stiction phase changes to the kinetic friction phase when the condition (3.3) is fulfilled.

Eq (3.11) has been solved numerically and calculations have been carried out for a two-link chain when its links are driven by drive moments $M_1^D$ and $M_2^D$ (Fig.3).

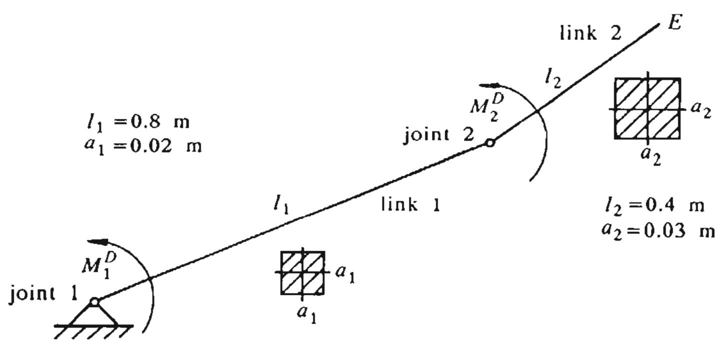


Fig. 3. Kinematic chain considered

Link 1 is treated as a beam with bending flexibility (Fig.4). It is modelled using the rigid and elastic elements (subsystem 1). Link 2 is treated as a rigid body (subsystem 2).

For calculations link 1 is divided into $n_1 = 5$ rigid finite elements. Because of concurrence of the results it is not necessary to use a larger number of rigid elements. The selected results of calculations are presented in diagrams. They are for the case when link 2 is driven by the drive moment and joint 1 (and so

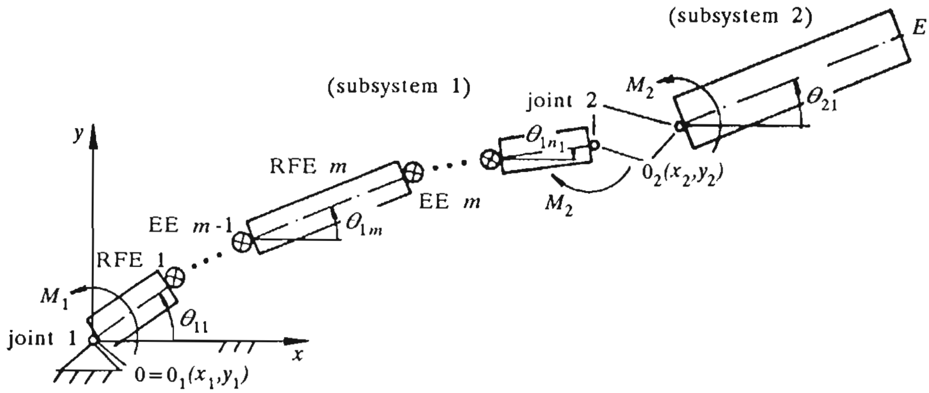


Fig. 4. Model of the analysed kinematic chain

the RFE 1) is motionless. One factor examined was a quantitative influence of friction on vibration of the end $E$ of the chain (Fig.5). This problem can be interesting, for example in analysis of accuracy of robot positioning (the chain considered can be treated as a model of the robot manipulator). For further calculations equality of kinetic and static friction coefficients is assumed $\mu_k = \bar{\mu}_k = 0.05$ $(k = 1, 2)$.

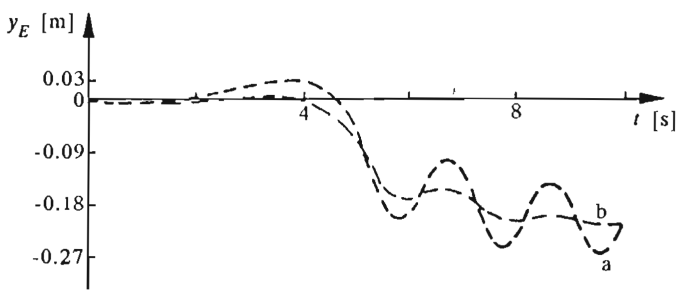


Fig. 5. Influence of friction on displacements of the end of the chain with a flexible link: (a) – lack of friction in joint 2; (b) – friction in this joint

Fig.6 presents the course of the relative velocity $\dot{\theta}_{21} - \dot{\theta}_{15}$ of link 2 and the last rigid element. The velocity in the case (b), after an increase which is caused by the drive moments, decreases under the influence of friction. Phases of stiction and kinetic friction in joint 2 are shown.

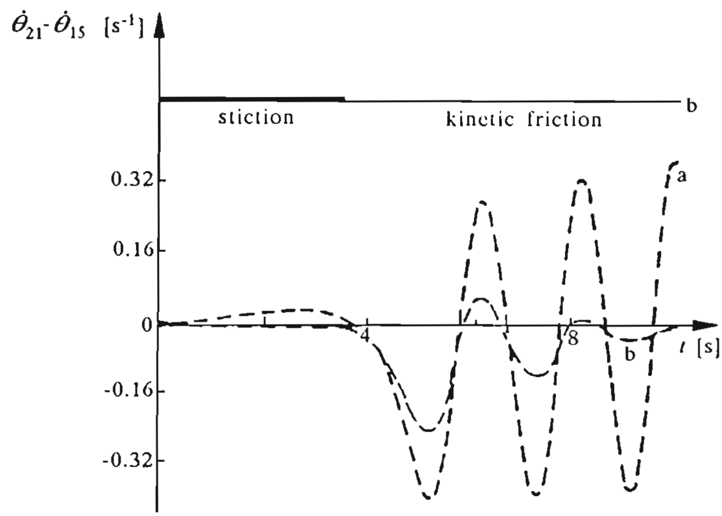


Fig. 6. Course of the relative velocity $\dot{\theta}_{21} - \dot{\theta}_{15}$: (a) – lack of friction in joint 2; (b) – friction in this joint

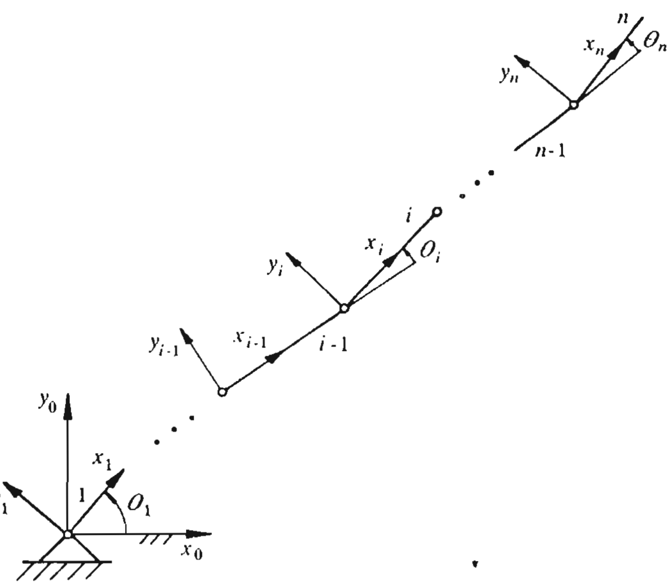


Fig. 7. Coordinate system for planar chains with rotary joint

## 4. Verification of the method presented

The formulae presented above become more simple in the case of rigid link chains. In this case subsystems consist only of one rigid element without elastic elements. Then the dynamic analysis can be also carried out using the Lagrange formalism together with Denavit-Hartenberg (cf Craig (1993)) notation for transformation of coordinate systems. This notation in principle refers to spatial systems. In the case of planar chains with rotary joints only the coordinate systems according to the Denavit-Hartenberg notation are defined as in Fig.7. Then the angles $\theta_i$ between the axes $x_{i-1}$ and $x_i$ are generalized coordinates.

If the vector $\boldsymbol{r}_i = [x_i, y_i]^\top$, defining the position of a chosen point in the coordinate system $x_i y_i$ connected with the link $i$, is known, then the vector $\boldsymbol{r}_{i-1}$ defining the position of this point in the $x_{i-1}y_{i-1}$ coordinate system connected with the link $i-1$ is defined as

$$\boldsymbol{r}_{i-1} = \mathsf{A}_i \boldsymbol{r}_i \qquad\qquad i = 1, ..., n \tag{4.1}$$

where $\mathsf{A}_i$ is the transformation matrix from $x_i y_i$ to $x_{i-1}y_{i-1}$ coordinate systems.

When transformation matrices $\mathsf{A}_i, ..., \mathsf{A}_1$ are known, the position of the point with respect to the inertial system $x_0 y_0$ can be calculated according to the formula

$$\boldsymbol{r}_0 = \mathsf{B}_i \boldsymbol{r}_i \tag{4.2}$$

where $\mathsf{B}_i = \mathsf{A}_1\mathsf{A}_2...\mathsf{A}_i$ is the transformation matrix from the coordinate system $x_i y_i$ to the inertial coordinate system.

Potential and kinetic energies of the chain as well as the equations of motion are formulated according to the procedure presented by Jurevič et al. (1984). Having differentiated matrices $\mathsf{B}_i$, the kinetic energy of the whole chain can be expressed as

$$T = \frac{1}{2}\sum_{i=1}^{n} \mathrm{tr}(\dot{\mathsf{B}}_i \mathsf{H}_i \dot{\mathsf{B}}_i^\top) \tag{4.3}$$

where

$\mathrm{tr}(\cdot)$ – trace of the matrix $(\cdot)$

$\mathsf{H}_i$ – inertial matrix of the link $i$.

Potential energy can be calculated according to the formula which contains the scalar product

$$V = g(\boldsymbol{a} \cdot \boldsymbol{b}) \tag{4.4}$$

where

$$\boldsymbol{a} = [0,1,0] \qquad\qquad \boldsymbol{b} = \sum_{i=1}^{n} m_i \mathsf{B}_i \boldsymbol{r}_{c_i}$$

and

$g$ – acceleration due to gravity

$m_i$ – mass of the link $i$

$\boldsymbol{r}_{c_i}$ – position vector of the center of mass of the link $i$ in the system $x_i y_i$.

For further consideration the following is defined

$$\begin{aligned} \mathsf{B}_i^j &= \frac{\partial \mathsf{B}_i}{\partial \theta_j} \qquad && i,j = 1,...,n \\ \mathsf{B}_i^{js} &= \frac{\partial \mathsf{B}_i^j}{\partial \theta_s} \qquad && i,j,s = 1,...,n \end{aligned} \tag{4.5}$$

The above quantities can be presented as

$$\begin{aligned} \mathsf{B}_i^j &= \begin{cases} \mathsf{A}_1...\mathsf{A}_{j-1}\mathsf{D}_j\mathsf{A}_j\mathsf{A}_{j+1}...\mathsf{A}_i & \text{when} \quad j \le i \\ \mathbf{0} & \text{when} \quad j > i \end{cases} \\ \mathsf{B}_i^{js} &= \begin{cases} \mathsf{A}_1...\mathsf{A}_{j-1}\mathsf{D}_j\mathsf{A}_j...\mathsf{A}_{s-1}\mathsf{D}_s\mathsf{A}_s...\mathsf{A}_i & \text{when} \quad j < s \le i \\ \mathsf{A}_1...\mathsf{A}_{j-1}\mathsf{D}_j^2\mathsf{A}_j...\mathsf{A}_i & \text{when} \quad j = s \le i \\ \mathbf{0} & \text{when} \quad j > i \ \text{and} \ s > i \end{cases} \end{aligned} \tag{4.6}$$

where $\mathsf{D}_j = \partial \mathsf{A}_j / \partial \theta_j$.

The relations presented enable a complicated matrix differentiation to be replaced by simple multiplication. Thus

$$\dot{\mathsf{B}}_i = \sum_{j=1}^{n} \mathsf{B}_i^j \dot{\theta}_j \tag{4.7}$$

and so the kinetic energy (4.3) can be written in the following way

$$T = \frac{1}{2} \sum_{i=1}^{n} \sum_{j,l=1}^{i} \mathrm{tr}(\dot{\mathsf{B}}_i^j \mathsf{H}_i \mathsf{B}_i^{l\top}) \dot{\theta}_j \dot{\theta}_l \tag{4.8}$$

Using the above formulation the variable number $s$ of equations describing the motion in the pairs with force input can be defined

$$\sum_{i=1}^{n} D_{si} \ddot{\theta}_i + \sum_{j=1}^{n} \sum_{i=j}^{n} D_{sji} \dot{\theta}_j \dot{\theta}_i + D_s = M_s \tag{4.9}$$

where

$$D_{si} = \sum_{l=\max\{i,s\}}^{n} \mathrm{tr}(\mathsf{B}_l^i \mathsf{H}_l \mathsf{B}_l^{s\top})$$

$$D_{sji} = \delta_{ji} \sum_{l=\max\{i,j,s\}}^{n} \mathrm{tr}(\mathsf{B}_l^{ji} \mathsf{H}_l \mathsf{B}_l^{s\top})$$

$$D_s = g(\boldsymbol{a} \cdot \boldsymbol{b}^*)$$

$$\boldsymbol{b}^* = \sum_{i=s}^{n} m_i \mathsf{B}_i^s \boldsymbol{r}_{c_i}$$

$$\delta_{ji} = \begin{cases} 1 & \text{when} \quad j = i \\ 2 & \text{when} \quad j \neq i \end{cases}$$

The number of Eqs (4.9) is equal to the number of pairs in which, at the moment considered, there is a force input. The moment $M_s$ is calculated from Eq (3.1).

In this approach normal reaction forces in the case of force input and equilibrium moments in the case of kinematic input are calculated in a different way than in the rigid finite element method. The approach proposed is based on the recursive Newton-Euler formulation (cf Luh et al. (1980)) which consists in realization of two calculation loops. The first of them enables kinematic parameters of each link and then inertial forces and moments to be calculated. In the other loop the reaction forces $F_s$ (in the case of force input) or equilibrium moments $M_p^*$ (in the case of kinematic input) are calculated. Naturally, this last case includes the ceasing of relative motion in pairs as a consequence of friction. Calculations in the first loop are realized starting from the first link, while in the second loop from the last free link $n$. The iterative algorithm, worked out by Wojciech [26] for calculating kinetic friction moments $M_s^F$ and integrating the equations of motion, has been used in this approach.

For numerical calculations, in the case of the chain with two rigid links (Fig.3), first the inverse dynamics problem has been simulated using both methods considered.

Fig.8 shows the courses of coordinates $\theta_1 = \theta_{11}$ and $\theta_2 = \theta_{21} - \theta_{11}$ which are to be realized in the time given. As can be seen, movements of links are realized one after another in the phases I and II, respectively. Both methods enable necessary drive moments $M_1^*$ and $M_2^*$ to be easily calculated.

In order to check the correctness of the results obtained, the calculated drive moments are used for the solution to direct dynamics problems. Thus,

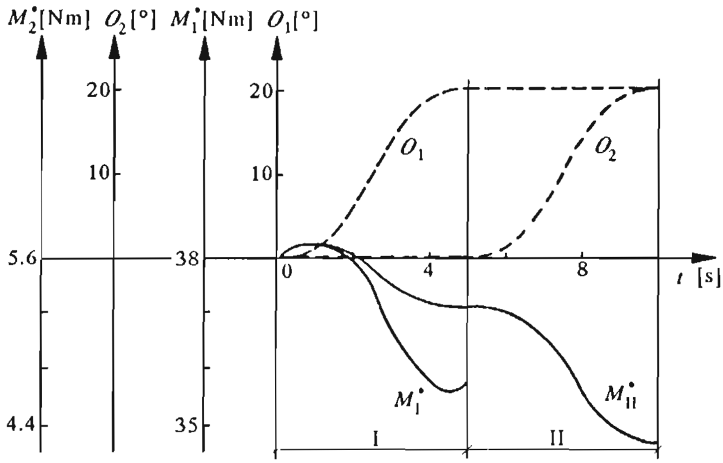


Fig. 8. Example of the inverse dynamics problem

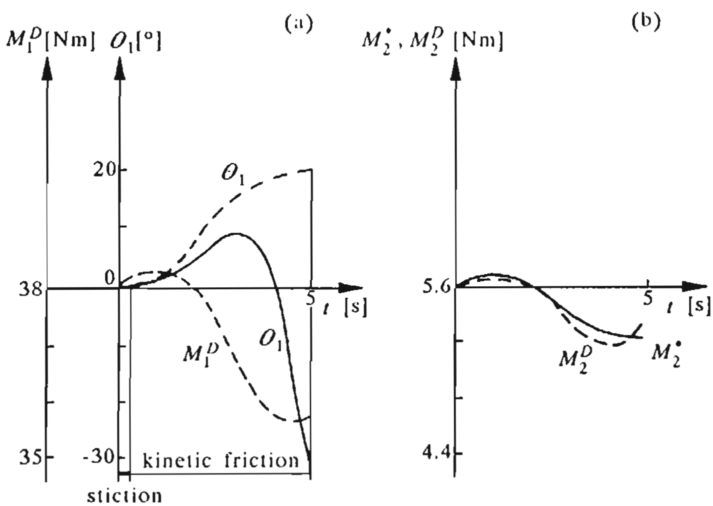


Fig. 9. (a) – Direct dynamics with friction in joints; (b) – Correction of the drive moment of the link 2

these moments act on links the movement of which is examined. When omitting friction, identical courses of coordinates $\theta_1$, $\theta_2$ are found (Fig.8), and regardless of the fact whether Eqs (3.11) or (4.9) are used. Thus both methods give the same results.

However, completely different courses are obtained when friction is taken into account.

When the drive moment $M_1^D$ is assumed to be as the first part of $M_1^*$ in Fig.8, the course of the coordinate $\theta_1$ (solid line Fig.9a) is completely different from the course required (broken line). In order to guarantee the initial lack of motion in joint 2 the moment $M_2^D$ (broken line in Fig.9b) should be applied which is different than the calculated $M_2^*$ (solid line in Fig.9b). Phases of kinetic and static friction in joint 1 are shown in Fig.9a. It appears that if the friction is not taken into account in the real systems unsuitable drive moments can be assumed in control tasks and in consequence the motion of links will differ from the expected one.

It is also important that the above diagrams have been obtained using both methods discussed. They are identical, which proves the correctness of the methods.

## 5. Conclusions

Both methods of considering dry friction in dynamic analysis of planar open kinematic chains with rigid links, using different rules, have given consistent numerical results. So the calculations carried out have been proved to be correct. Once again it is proved that friction considerably influences the motion of mechanical systems, especially when flexibility of links is taken into account. Neglect of friction can mean that wrong results are obtained, for example in control.

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## O pewnych metodach uwzględniania tarcia suchego w analizie dynamicznej otwartych łańcuchów kinematycznych o konfiguracji płaskiej

Streszczenie

W pracy przedstawiono dwa odmienne sposoby uwzględniania tarcia suchego w analizie dynamicznej układów wielomasowych, rozważanych w postaci płaskich otwartych łańcuchów kinematycznych z parami obrotowymi. W obu przypadkach zastosowano różne metody obliczania reakcji w parach kinematycznych. Pierwszy sposób, oparty na uwzględnieniu równań więzów, stosowany jest zasadniczo w przypadku analizy dynamicznej łańcuchów z członami podatnymi, przeprowadzanej przy użyciu "metody sztywnych elementów skończonych". Rozważono ponadto możliwość analizy szczególnej postaci tych łańcuchów z członami sztywnymi przy użyciu innej metody, w której do sformułowania równań ruchu wykorzystuje się macierze transformacji

układów współrzędnych związanych z poszczególnymi członami wg notacji Denavita-Hartenberga. W tym przypadku niewiadome wielkości wyznacza się, korzystając z równań określających równowagę dynamiczną poszczególych członów. Obie metody pozwalają uwzględniać złożony model tarcia suchego w parach, tzn. rozważać nie tylko stany tarcia kinetycznego (Coulomba), ale także stany tarcia statycznego. Porównano wyniki obliczeń numerycznych otrzymane w przypadku analizy uproszczonej wersji łańcucha z członami sztywnymi przy wykorzystaniu obu metod. Zgodność tych wyników stanowi potwierdzenie poprawności użytych metod.

*Manuscript received January 20, 1995; accepted for print April 27, 1995*