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PROBABILISTIC *B*-MODELS OF CUMULATIVE FATIGUE DAMAGE

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A review of phenomenological cumulative fatigue damage models, based on the theory of embedded Markoff chains (MC), is presented in the paper. There are two types of Bogdanoff-Kozin models (called *B*-models), i.e., stationary time-independent and non-stationary time-dependent ones, respectively. A new improved *B*-model, called the method of dependent probability transition matrix (dependent PTM) is analyzed, as well. A comparison between the models is made upon laboratory tests carried out by the author.

1. Introduction

Theoretical bases of the Bogdanoff-Kozin models (i.e. *B*-models), their capabilities and applications were widely presented in papers by Bogdanoff (1978a), Bogdanoff and Krieger (1978), Bogdanoff and Kozin (1980). A chapter of the monograph by Bogdanoff and Kozin (1985) was devoted to application of the models to several practical engineering problems. Models of cumulative damage are considered as the finite Markoff chains (MC). Based on this assumption, the structure of the models is very simple and therefore application of them to practical engineering problems is straightforward. A versatility is a big advantage of the *B*-model, after all, they can be applied to evaluation of damage in the case of visible crack growth and also in the case of invisible, internal change of material structure during the process of continuous fatigue damage. The only drawback of the model is that, there is no possibility of taking into consideration the influence of successive loading effect on the change in intensity of the cumulative damage process (i.e., on the fatigue life). Therefore, an additional method of time transformation should be performed, what allows us for taking into account the influence of overloading, but only from a theoretical point of view. Practically, an application of the aforementioned method is impossible, because the time function (which transforms the stationary Markoff chain into the non-stationary one and vice versa) is needed. An attempt at experimental determination of the function is equivalent to estimation of the real fatigue life with no necessity for using transformation. According to the above, the improved B-model, supplied with the nonlinear hypothesis of fatigue cumulative damage formulated by Hashin (1980), is presented in the paper. An influence of interaction of successive, loading on the fatigue life is taken into consideration in the improved model by Sobczyk (1980), (1982), (1986), Ditlevsen and Sobczyk (1986), Bogdanoff and Kozin (1982), (1984). However, the lately mentioned systems concern only the phase of crack growth and are hardly used in engineering practice due to its complexity. It should be mentioned that other methods based on the properties of Markoff chains with influence of overloading on fatigue life included exist (cf Drewniak (1989a), (1991), (1992a,b)).

2. The stationary B-model

The stationary B-model of cumulative fatigue damage is a phenomenological model based on the Markoff chains (cf Sobczyk (1973); Bolotin (1968) and Iosifescu (1988)) and on the concept of shock model (cf Esary et al. (1973)). The basic idea of this model is a duty cycle (DC), defined as a repetitive block of construction loading, which caused cumulative damage. Typical examples of the DC are: machine shaft loading, loading of a toothed wheel, which comes into meshing, loading of on undercarriage during take-off and landing of an airplane or block of loading in programmed fatigue tests. In all these cases the fatigue life can be measured using a discrete cycle of loading $x = 0, 1, 2, \dots$, whereas a cumulative damage level is defined by the set of discrete states $j = 1, 2, \dots, b$, where state b either represents "replacement is necessary" or "failure occurs". The basic assumption of B-model consists in damage accumulation in the DC depends only on its intensity and on the state of damage at the start of this DC. The damage can increase within the DC from its state at the start of that DC to the next, higher state. These assumptions are fundamental in the MC and therefore the process of damage accumulation is modelled by a stationary MC with discrete both time and state. Thus, the same constant probability transition matrix (PTM) $\mathbf{Q}$ is associated with each constant severity DC. Matrix $\mathbf{Q}$ has the following form

$$\mathbf{Q} = \begin{bmatrix} p_1 & q_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & p_2 & q_2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & p_3 & q_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & p_{b-1} & q_{b-1} \\ 0 & 0 & 0 & 0 & \vdots & 0 & 1 \end{bmatrix} \tag{2.1}$$

where p_j is the probability the jth damage state remains unchanged at one step, and in compatibility with shock model-this is also the probability of shock existence, which does not exceed a critical level, given damage at a start of the DC, whereas q_j is the probability of increase in damage rate at one step to the state $(j+1)$, or the probability that a shock exceeds a critical level, given damage at a start of the DC.

Values of p_j and q_j fulfill the following conditions

$$0 < p_j < 1 \qquad\qquad p_j + q_j = 1 \tag{2.2}$$

It is necessary to know the probability distribution (PD) of the initial damage state (for the time $x = 0$)

$$\boldsymbol{p}_0 = [\Pi_1, \Pi_2, ..., \Pi_{b-1}, 0] \tag{2.3}$$

where Π_i – the probability of compatibility of the initial damage state D_0 to the state j

$$P\{D_0 = 1\} = \Pi_j \geq 0 \tag{2.4}$$

especially in service conditions, for the determination of the course of cumulative damage process. Basing on the PD of initial damage state and the PTM, the full process of cumulative damage can be traced. An assumption that the level of initial damage corresponds to the first state, i.e.,

$$\boldsymbol{p}_0 = [1, 0, ..., 0] \tag{2.5}$$

is made. For defined probabilities p_1 and q_1, the sequence of Bernoulli trials should be considered, unless the shock for the first time exceeds the critical value, what corresponds to the geometric distribution with the parameter p_1

$$P\{T_1 = x\} = q_1 p_1^{x-1} \qquad\qquad x = 1, 2, ... \tag{2.6}$$

where T_1 denotes the time spent for the damage in state 1. After the time T_1 damage reaches the state 2. The sequence of Bernoulli trials should be considered until the shock of a value higher than the second critical level appears. In this manner, by repeating the succeeding transition states, the damage accumulation process can be traced until the state b is reached, which means the fatigue damage of an element. The time required to reach the state b (the time to failure) given starting state 1 at $x = 0$, is the random variable $W_{1,b}$

$$W_{1,b} = T_1 + ... + T_{b-1} \tag{2.7}$$

which states that each T_j has the geometric distribution

$$P\{T_j = x\} = q_j p_j^{x-1} \qquad x = 1,2 \tag{2.8}$$

The mean and the variance of $W_{1,b}$ are determined by the following formulas

$$E[W_{1,b}] = \sum_{j=1}^{b-1}(1 + r_j)$$
$$\text{var}W_{1,b} = \sum_{j=1}^{b-1} r_j(1 + r_j) \tag{2.9}$$

where $r_j = p_j/q_j$.

Evaluation of the cumulative distribution function (CDF) of $W_{1,j}$ for every $1 \leq j \leq b$ or a random value D_x of the damage state at the time x; $x = 0, 1, ...$ is sufficient for description of the cumulative fatigue damage process. The probability of damage D_x appearing in the state j at the time x is described by $p_x(j)$, namely

$$P\{D_x = j\} = p_x(j) \qquad j = 1, 2, ..., b \tag{2.10}$$

where $p_x(j)$ is the probability mass function (PMF) at the instant x over the damage states $j = 1, ..., b$; i.e., it is a component of the $(1 \times b)$ row vector $\boldsymbol{p}_x = [p_x(1), ..., p(b)]$ of damage accumulation PD in all states at the time x. This PD can be easily determined from the MC theory

$$\boldsymbol{p}_x = \boldsymbol{p}_0 \mathbf{Q}^x \qquad x = 0, 1, 2, ... \tag{2.11}$$

Using Eq (2.11), the CDF of the time W_b to absorption in the state b is only determined, namely $p_x(b)$

$$F(x) = P\{W_b \leq x\} = p_x(b) \qquad x = 0, 1, 2, ... \tag{2.12}$$

The initial state of random value W_b is not explicitly presented, like in Eq (2.7), because in this case the initial state can be described by the PD (see Eq

(2.3)). Then, the mean and the variance of time W_b are calculated using the following formulas

$$
\begin{aligned}
E[W_b] &= \sum_{x=0}^{\infty}\Big(1 - p_x(b)\Big) \\
\mathrm{var}W_b &= \sum_{x=0}^{\infty} x\Big(1 - p_x(b)\Big) + E[W_b] - (E[W_b])^2
\end{aligned}
\tag{2.13}
$$

3. The non-stationary B-model

The non-stationary model of fatigue damage cumulation can be applied not only in the case of variable severity of the DC, but also in the case when the material properties or the environmental conditions affecting the process of damage cumulation change with time. Determination of the CDF directly upon the formula $(3.1)_2$, in the same manner like for stationary processes, can not be practically applied due to the lack of procedure of the elements of matrix $\mathbf{Q}_k$ $(3.1)_1$ determination.

In non-stationary model, the cumulative damage process is described by the following formulas

$$
\begin{aligned}
\boldsymbol{p}_x &= \boldsymbol{p}_0 \prod_{k=1}^{x} \mathbf{Q}_k \qquad\qquad x = 1,2,\ldots \\
p_x(b) &= F(x)
\end{aligned}
\tag{3.1}
$$

where

- $\boldsymbol{p}_x$ – probability mass function at the time x over the damage states $1,2,\ldots,b$
- $\boldsymbol{p}_0$ – initial probability distribution over the damage state at $x = 0$
- $F(x)$ – cumulative distribution function (CDF)
- $\mathbf{Q}_j$ – probability transition matrix (PTM) generated by successive kth stress levels $Q_{j,k}$ $(k = 1,\ldots,n)$, which are included into the jth block Q_j $(j = 1,\ldots,x)$.

Cumulative distribution function $F(x)$ is a deterministic function for a certain stress level sequence $Q_{j,k}$ being a random function of x in the case when Q_j is a sequence of random selected stress levels $Q_{j,k}$. The mean value of $F(x)$ could be determined using the following formulas

$$E[\boldsymbol{p}_x] = \boldsymbol{p}_0 \prod_{j=1}^{x} E(\mathbf{Q}_j)$$
$$E[F(x)] = E[\boldsymbol{p}_x(b)] \tag{3.2}$$

An algorithm of evaluation of the CDF $F(x)$ could be formulated, taking the following steps:

1. Determination of parameters $r_{j,kl}$ and b dimension of the PTM $\mathbf{Q}_{j,k}$ for the kth level in the jth block ($k = 1, ..., n$; n – number of levels in the block)

$$\widehat{m}_{j,k} = (b_1 - 1)(1 + r_{j,k1}) + (b - b_1)(1 + r_{j,k2})$$
$$\widehat{s}_{j,k} = (b_1 - 1)(1 + r_{j,k1})r_{j,k1} + (b - b_1)(1 + r_{j,k2})r_{j,k2} \tag{3.3}$$

   where $\widehat{m}_{j,k}$ and $\widehat{s}_{j,k}$ are the estimates of mean life and variance, respectively.

   It is assumed that

$$r_{j,kl} = r_{j,k1} \qquad \text{for} \quad l = 1, ..., b_1 - 1$$
$$r_{j,kl} = r_{j,k2} \qquad \text{for} \quad l = b_1, ..., b - 1$$

2. Determination of mean values of the PTM $\mathbf{Q}_{j,k}$ elements for the kth stress level

$$E[q_{j,kl}] = \frac{1}{1 + r_{j,kl}}$$
$$E[p_{j,kl}] = 1 - E[q_{j,kl}] \tag{3.4}$$

3. Assuming of the relative frequencies $f_{j,k}$ of the occurrence of the kth level $Q_{j,k}$ in the jth block

4. Determination of the mean values of PTM $\mathbf{Q}_j$ for the jth block

$$E[\mathbf{Q}_j] = \sum_{k=1}^{n} f_{j,k} E[\mathbf{Q}_{j,k}] \tag{3.5}$$

where the elements of PTM $E[\mathbf{Q}_{j,k}]$ are defined by the following formulas

$$
\begin{aligned}
E[q_{j,l}] &= \sum_{k=1}^{n} f_{j,k} E[q_{j,kl}] \\
E[p_{j,l}] &= 1 - \sum_{k=1}^{n} E[q_{j,kl}]
\end{aligned}
\tag{3.6}
$$

5. Calculation of the mean value of CDF $F(x)$ using Eqs (3.2).

The influence of the $\mathbf{Q}_{j,k}$ order on fatigue life is taken into account in the presented model, because the order of matrix multiplication matters. Physical load interaction is not accounted for in this model, which property is a serious drawback of this method due to the error in fatigue life evaluation (in some cases of loading), which arises due to the loading interaction neglectiong and could be approximately equal to 200% (cf Drewniak (1992a)).

4. Cumulative damage model with dependent PTM

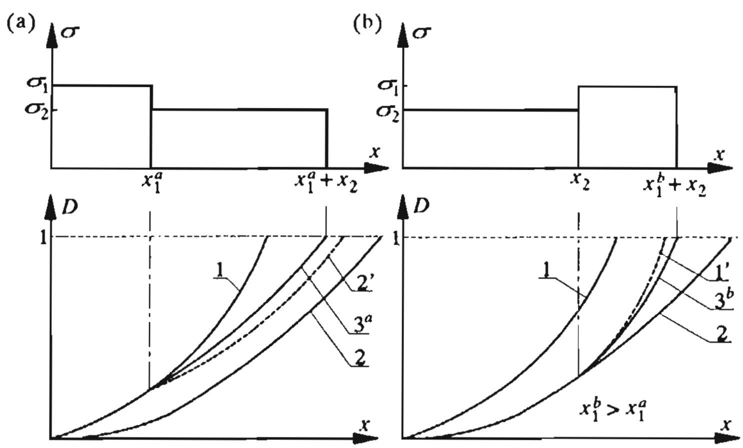

Fig. 1. Courses of the cumulative fatigue damage without and with taking into account the effect of load interaction; (a) – for "high-low" loading, (b) – for "low-high" loading

Sample courses of cumulative damage processes for two cases of loading are presented in Fig.1 (cf Drewniak (1992a)). Curves 2' and 1' represent the case, when the influence of loading sequence is not taken into account, curves 3^a and 3^b represent the opposite case. The cumulative damage process, without taking into account the influence of loading sequence on fatigue life satisfies the linear Palmgren-Miner rule. Mean value of the residual fatigue life, in the case of the two-level loading, is determined by the formula

$$x_2^{lin} = N_2\Big(1 - \frac{x_1}{N_1}\Big) \tag{4.1}$$

where N_i $(i = 1,2)$ – fatigue life under sinusoidal stress conditions (constant amplitude).

The cumulative damage process, when taking into account the effect of load interaction agrees with the nonlinear damage cumulation hypothesis (cf Hashin (1980)). The mean value of residual fatigue life in this case is defined by the following formula

$$x_2^{nonl} = N_2\Big[1 - \Big(\frac{x_1}{N_1}\Big)^{\beta}\Big] \tag{4.2}$$

where $\beta = c(\sigma_2 - Z_G)/(\sigma_1 - Z_G)$, c – constant.

A coefficient ζ of influence of the sequence of stress levels on fatigue life is described by formula

$$\zeta = \frac{x_2^{nonl}}{x_2^{lin}} \tag{4.3}$$

This coefficient will be used to determination of the dependent PTM $\mathbf{Q}_{j,kl}$. Parameters $r_{j,kl}$ $(l = 2,...,b-1)$ and b are calculated upon the above algorithm (see Eqs (3.3)), however the estimated mean value of fatigue life $\widehat{m}_x$ is substituted for $\zeta\widehat{m}_x$. Calculation formula for the CDF for two-level block of loading can be generalized for the case of multi-level loading

$$\begin{array}{ll} \boldsymbol{p}_x = \boldsymbol{p}_0\mathbf{Q}_1^x & 0 < x < x_1 \\ \boldsymbol{p}_x = \boldsymbol{p}_0\mathbf{Q}_1^{x_1}\mathbf{Q}_2^{x-x_1} & x_1 < x \le x_1 + x_2 \\ \boldsymbol{p}_x = \boldsymbol{p}_0\mathbf{Q}_1^{x_1}\mathbf{Q}_2^{x_2}\mathbf{Q}_3^{x-x_1-x_2} & x_1 + x_2 < x \le x_1 + x_2 + x_3 \\ \boldsymbol{p}_x = \boldsymbol{p}_0\mathbf{Q}_1^{x_1}\mathbf{Q}_2^{x_2}\cdots\mathbf{Q}_n^{x-x_1-...-x_{n_1}} & x > x_1 + ... + x_{n-1} \end{array} \tag{4.4}$$

where

$\mathbf{Q}_1$ – PTM for the first duty cycle (DC), therefore without taking into account the influence of previous loading

$\mathbf{Q}_i$ – dependent PTM generated by the loading Q_i $(i > 1)$, taking into account the effect of previous loading

x_i – number of cycles per level of loading i, $(i = 1,...,n-1)$

n – level number of loading.

Calculation of the CDF $F(x)$, in the case of block loading can be significantly simplified. The formula for CDF calculation, taking into account the repetition of loading blocks, can be presented by

$$\boldsymbol{p}_x = \boldsymbol{p}_0 \mathbf{Q}^x \qquad x = 1, 2, ... \tag{4.5}$$

where $\mathbf{Q}$ – mean PTM generated by the full block of loading

$$\mathbf{Q} = \sum_{j=1}^{m} \sum_{i=1}^{m} f_i f_{ji} \mathbf{Q}_{ji} \tag{4.6}$$

where

- $\mathbf{Q}_{ji}$ – dependent PTM generated by the jth level of loading taking into account the influence of previous ith loading level on fatigue life
- m – number of levels in the block of loading
- f_j – relative frequencies of occurrence of the jth level in a block
- f_{ji} – probability of existence of the jth level of loading directly after the ith level.

So simple form of Eq (4.6) is obtained due to the assumption that PTM for the first level of loading in block is the dependent PTM, which is equal to matrices for first levels in following blocks. This simplifying assumption does not affect the course of CDF $F(x)$, because the number of cycles on a selected level in the block is small in comparison to the number of cycles in the full block or in comparison to fatigue lifetime. The value of auxiliary constant c should be experimentally determined for the method of dependent PTM. It seems that the mentioned property is the only drawback of the method.

The advantages are presented in sample calculations, which were experimentally verified. Comparison between the courses of CDF $F(x)$ calculated under the model (4.4) for two-level loading (Fig.1) and the experimental CDF (ECDF) as wel as the CDF without load interaction is presented in Fig.2 and Fig.3.

The same generalized model was applied to determination of the CDF for four-level block of loading – Fig.4. The courses of CDF calculated with the aid of method of dependent PTM are close enough to the courses of ECDF and differences between mean values are less than 25 percent. Furthermore, the predicted fatigue lives are usually less than the real ones, so they are on "safety side" (in "safety region"). The most interesting conclusion can be drawn from Fig.5, Fig.6 and Fig.7, in which the courses of CDF for the non-stationary model without taking into account the effect of load interaction (Eqs

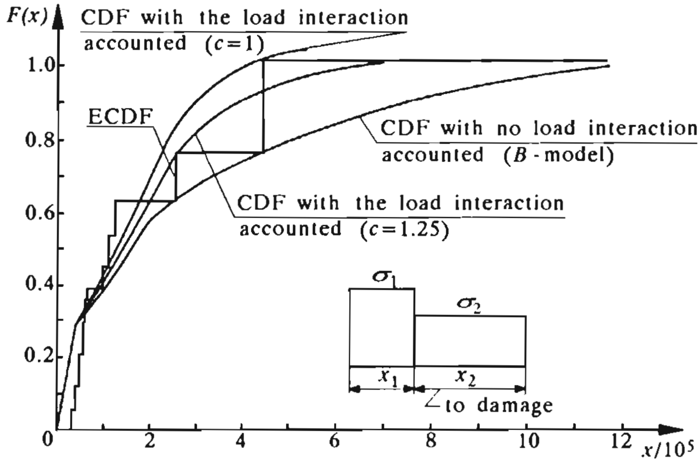

Fig. 2. Comparison of the CDFs for two-level loading $\sigma_1 = 1111\,\mathrm{MPa}$, $\sigma_2 = 841\,\mathrm{MPa}$, $x_1 = 32\,000$ cycles

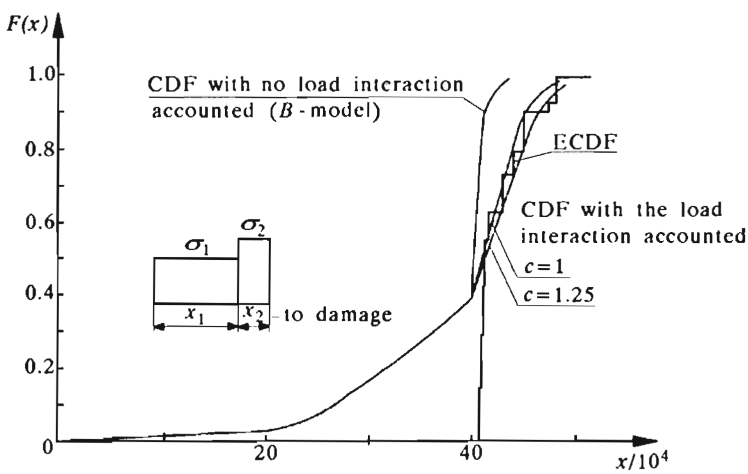

Fig. 3. Comparison of the CDFs for two-level loading $\sigma_1 = 841\,\mathrm{MPa}$, $\sigma_2 = 1111\,\mathrm{MPa}$, $x_1 = 425\,000$ cycles

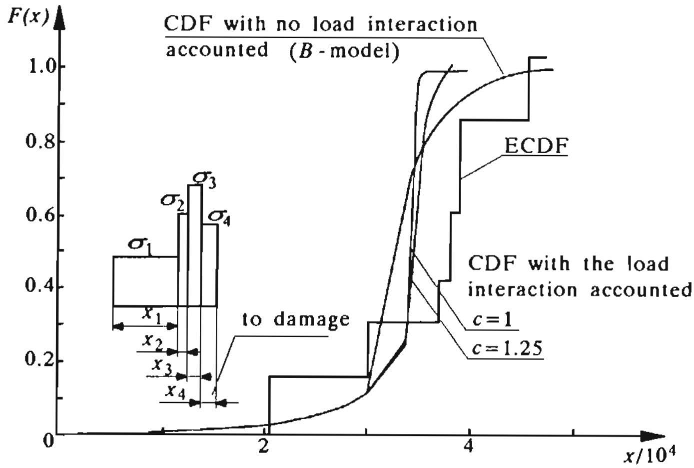

Fig. 4. Comparison of the CDFs for four-level loading $\sigma_1 = 841\,\mathrm{MPa}$, $\sigma_2 = 936\,\mathrm{MPa}$, $\sigma_3 = 1032\,\mathrm{MPa}$, $\sigma_4 = 1111\,\mathrm{MPa}$, $x_1 = 300\,000$, $x_2 = 40\,000$, $x_3 = 10\,000$ cycles

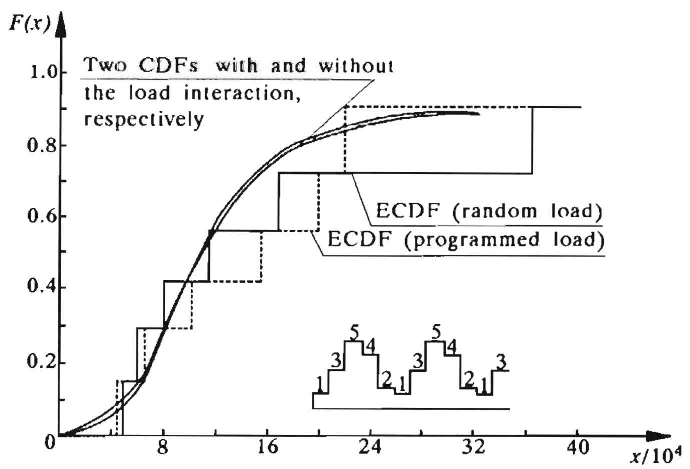

Fig. 5. Comparison of the CDFs for the programed block loading ("low-high-low"), $x_i = 1\,000$ cycles $(i = 1, ..., 5)$

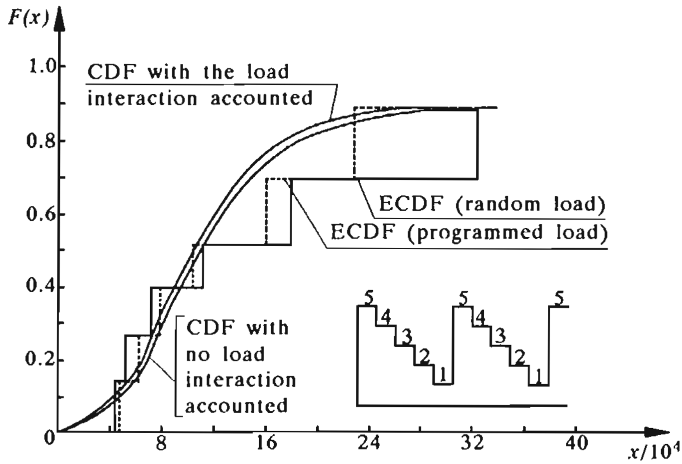

Fig. 6. Comparison of the CDFs for the programed block loading ("high-low"), $x_i = 1\ 600$ cycles $(i = 1, ..., 5)$

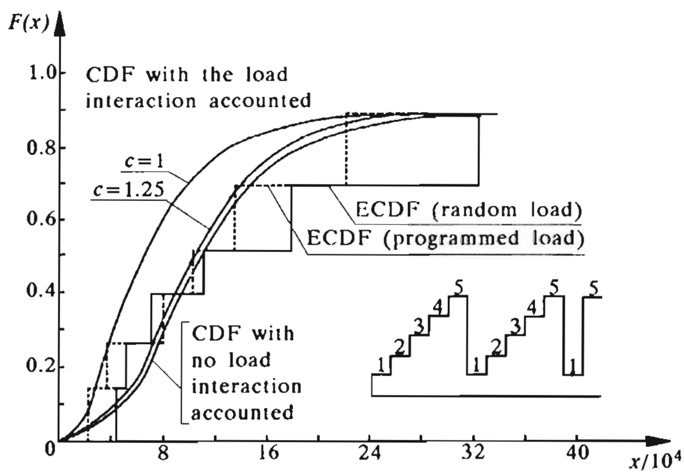

Fig. 7. Comparison of the CDFs for the programed block loading ("low-high"), $x_i = 1\ 600$ cycles $(i = 1, ..., 5)$

(3.1)) with the CDF for the model of dependent PTM (Eqs (4.5), (4.6)) with the courses of ECDF are compared. Repeated blocks of loading differentiate only by the sequence of stress levels: $\sigma_1 = 778$ MPa, $\sigma_2 = 841$ MPa, $\sigma_3 = 936$ MPa, $\sigma_4 = 1032$ MPa, $\sigma_5 = 1111$ MPa in the following way: (a) "low-high-low", (b) "high-low", (c) "low-high" (cf Drewniak (1992a)).

Differences in courses of the CDFs for the program "low-high" in Fig.7 for $c = 1$ confirm, once more, usefulness of application of the models, in which the effect of load interaction is taken into account. The courses CDFs which are close together in the case of sequence of cycles (a) and (b) confirm wide popularity and correctness of application of those schemes to fatigue investigations.

Hypotheses, without taking into account the influence of load interaction on fatigue life, can be used in construction elements fatigue life evaluation for programed loading in the case of sequence "low-high-low". So the form of stress level sequence ensures almost equivalent influence on lifetime of interaction of smaller loading with higher ones and of higher loading with smaller ones. It results from the fact that intensity of the cumulative fatigue damage process diminishes due to insensitivity to the load interaction.

The value of constant c in Eq (4.3) could be estimated by comparison between the corresponding CDF courses in Fig.2 ÷ Fig.7. Obstades in selection of a proper value of c arise from the fact, that this constant depends on the sequence of stress levels ("high-low" or "low-high") and on the number of cycles of the previous loading level. It was assumed the value of constant $c = 1$ in the examples presented in the paper, therefore the assumption of $c = 1.25$ (in some cases) dangerously overestimates the value of fatigue life.

5. Conclusions

The aim of such a detailed presentation of the stationary and non-stationary B-models was instructing future users in possibilities of application of these models to fatigue tests. Basing on the considered analysis of these models the necessity for preparation of a complementary method of fatigue life evaluation for materials, components and assemblies of machines arises in which the influence of loading sequence on fatigue life will be taken into account. The calculation model of fatigue life, suggested in the paper, called the method of dependent probability transition matrix was developed with regard to the possibility of taking into consideration the influence of loading history on fatigue life. This is achieved by multiplication of the PTM and also by taking into consideration the interaction of successive loading. This second property, consists entirely in regarding the influence of joint action of successive loading on modification of fatigue properties (i.e., retardation or acceleration of the fatigue crack). Due to positive experimental verification the considered, original model has proved to be convenient for fulfilling the assumed aim. It is necesary to add, that this model is limited to regarding only the influence of previous loading cycle. Also, due to application of the nonlinear Hashish hypothesis of fatigue damage cumulation, the presented model is valid only for high-cycle strength materials. Adaptation of the described model to determination of low-cycle fatigue life will be possible after introducting the model the nonlinear damage cumulation hypothesis working for this low-cycle range of fatigue life. The presented calculation model as a probabilistic model allows the evaluation of not only the mean value but also the distribution function of fatigue life. The moment method can be applied to its building, i.e., for determination parameters of the PMT, evidently, on condition that the mean values of fatigue lives and standard deviations corresponding to given load levels are known. Therefore, in reality, these data are possible to obtain not in every case directly from the fatigue investigation, consequently, in the next work the computer method of determination of PMT parameters based only upon fatigue curve. Other feature of the method of dependent PMT is a possibility of its application to an accelerated fatigue life investigation, i.e., in censored tests and in tests, in which the lower load levels are neglected. The proposed method of dependent PMTs can be also applied to determination of reliability of machinery. Determination of the reliability function $R(x)$, taking into account the influence of interaction of successive loads on reliability of elements (assembled into the machine set) or the set as a whole, will be the original achievement in the solution to this problem.

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Probabilistyczne B-modele kumulacji zniszczenia zmęczeniowego

Streszczenie

W pracy dokonano przeglądu modeli kumulacji uszkodzeń zmęczeniowych opartych na teorii włożonych łańcuchów Markowa. Są to dwa typy modeli Bogdanowa-Kozina, tj. stacjonarny i niestacjonarny B-model. Dokonano także analizy poprawionego B-modelu, nazwanego metodą zależnych macierzy prawdopodobieństwa przejścia. Porównania powyższych modeli dokonano na podstawie własnych badań zmęczeniowych.

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