MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
4, 34, 1996

# SIDE CURVATURE EFFECT ON THE STRESS INTENSITY FACTOR $K_I$ OF FINITE WIDTH CRACKED ELEMENTS SUBJECTED TO TENSION AND BENDING

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The present paper deals with the influence of side curvature on the stress intensity factor $K_I$ for a central crack, two symmetric edge cracks in a finite width plate subjected to tension and a single inner or outer edge crack in a curved bar under tension and bending. Various side curvatures and crack lengths have been analysed numerically using the boundary element method (BEM). The values of $K_I$ being calculated are compared to those obtained using the weight functions applicable to long rectangular plates with the same crack locations and regarding the real stress distributions of uncracked elements with curvilinear boundaries. It has been proven that the stress distribution along the potential crack path, due to side curvature and external loading conditions, has a dominant effect on the stress intensity factor values.

*Key words:* crack, stress intensity factor, weight function method

## 1. Introduction

The weight function method proposed by Bueckner (1970), (1973) and Rice (1972), is one of the most efective approaches to estimating stress intensity factors $K$ for various shapes and loading conditions of the cracked structure. The principle of superposition enables one to put together any linear-elastic non-uniform stress fields resulting from external loading, temperature gradients, residual stresses, etc., making this approach very useful and precise.

Some improvements of the weight function method have been suggested by Molski (1992) and (1994a), where a unitary weight function has been defined. Its unified description and a special integrating procedure make the unitary weight function method an effective tool for rapid $K$ calculations, convenient for determination of the safety margins and durability of a cracked structure under both static and variable loading conditions, especially in computer-aided calculations (cf Molski (1995)).

Since weight functions depend on the shape, crack location and loading mode of the structure, they have to be determined separately for each geometry of the body, to assure accurate further calculations of stress intensity factors $K$ for any distribution of stresses released along the crack faces.

On the other hand, the number of known weight function solutions available in the literature is strongly limited and unable to cover all real shapes of cracked structural elements. The question arises: how strongly some modifications in the shape of element side, compared to the geometries with known weight function solutions, affect $K$ values and what are the main influencing factors?

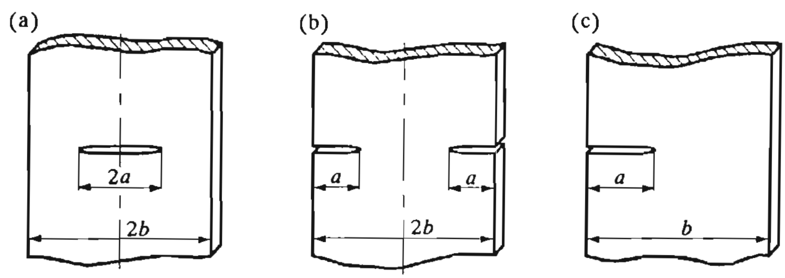


Fig. 1. Infinite strip with: (a) – symmetric central crack, (b) – two opposite edge cracks, (c) – single edge crack

Thus, the aim of the present study is to investigate the effect of side shapes (represented by various curvature radii), on the stress intensity factor $K_I$ for a symmetrical central crack, two equal and oppossite edge cracks in a symmetrical plate under tension, and for a single, inner and outer edge crack in a curved bar subjected to tension and bending. These new shapes being analysed here, are recognised as basic strips shown in Fig.1, with known weight functions, available in the literature (cf Molski (1992), (1994a); Tada et al. (1973)).

The present analysis is limited only to the Mode I and $K_I$, case which is the most important and frequently used in engineering problems.

## 2. Determination of $K_I$ values

### 2.1. Numerical approach

Four different cracked elements, shown in Fig.2 and Fig.3, have been modelled using the boundary element method (BEM) (cf Portela and Aliabadi (1993)). In both cases of symmetrical sheet under tension, shown in Fig.2, side shapes are represented by the $R/b$ ratios, describing various relative side curvatures. Three different $R/b$ values equal to 2, 1 and 0.501, respectively, have been assumed, while minimal and maximal cross-sectional lenghts of the uncracked element remain unchanged and equal to $2b$ and $4b$, respectively. The uniform tensile load $\sigma$ has been applied symmetrically to both ends of the body, sufficiently far from the cracked area. The crack length to net width ratio $a/b$ was changed with the step 0.1 within the range $0 < a/b \leq 0.7$ and for each crack length the stress intensity factor $K_I$ has been obtained using the BEM.

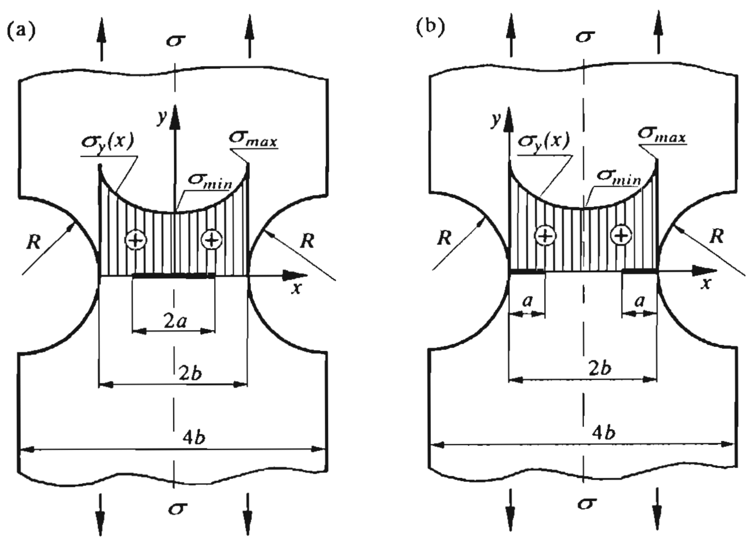


Fig. 2. Rectangular cracked sheet with two opposite blunt notches subjected to tension, with a central symmetric crack (a), and two equal edge cracks (b); $\sigma_y(x)$ for uncracked element

In both cases of the curvilinear bar shown in Fig.3, the inner and outer sides are described by constant radii $r$ and $R$, with the external radius $R$ equal to one. The following values of $r/R$ ratio have been arbitrarily chosen;

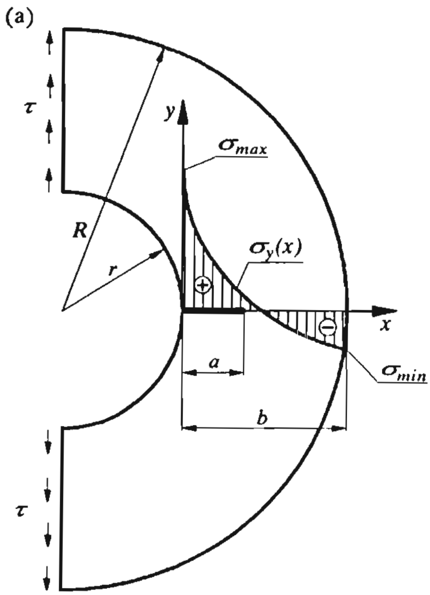


Fig. 3. Curved bar with a single inner (a) and outer (b) crack, subjected to tension and bending due to uniform stress $\tau$; $\sigma_y(x)$ for uncracked element

0.7, 0.5 and 0.3. The uniform stress $\tau$ is applied to the opposite ends, causing tensile and bending effect in the crack plane. The length of a single inner and outer edge crack was changed by 0.1 of the bar width within the range $0 < a/b \leq 0.7$. As in the previous cases, the stress intensity factor $K_{\rm I}$ has been calculated using the boundary element method incorporated in the software program "Cracker" (cf Portela and Aliabadi (1993)), with a special internal procedure capable of decomposing the stress field ahead of the crack tip into symmetrical and skew-symmetrical components, corresponding to $K_{\rm I}$ and $K_{\rm II}$, respectively. Since all the cases being analysed are symmetrical with respect to the crack plane, all values of $K_{\rm II}$ vanish.

### 2.2. Weight function approach

Application of the weight function method for determining particular values of the stress intensity factor $K$, is based on the knowledge of two functions, suitable for geometrical and loading conditions of the body under consideration.

These are:

1. The loading function $\sigma_{1j}(x)$, depending on the shape and loading conditions imposed on the body and represented by normal and tangential stresses along the potential crack path
2. The weight function $m^{(j)}(x,a)$, usually related only to the crack plane, and dependent on the geometry of cracked element; where $j = 1,2,3$ correspond to the three basic modes I, II and III.

The stress intensity factor $K_j$ is then calculated as an integral of the product of two functions: $\sigma_{1j}(x)$ and $m^{(j)}(x,a)$ over the crack length $a$, represented by the following equation

$$K_j = \int\limits_0^a \sigma_{1j}(x) m^{(j)}(x,a)\, dx \tag{2.1}$$

In the cases under investigation, three known weight functions $m^{(1)}(x,a)$, true for long rectangular cracked plates shown in Fig.1, are considered together with the normal stresses $\sigma_{11}(x) = \sigma_y(x)$ distributed along the crack plane. They have been obtained numerically from the boundary element analysis for each uncracked body subjected to uniform loads $\sigma$ and $\tau$ applied to the straight ends, as shown in Fig.2 and Fig.3. For each case, the stress values at 25 boundary points in the crack plane have been interpolated by 6th degree polynomials and applied to the "K123" program (cf Molski and Truszkowski (1995)) for calculating approximate values of the stress intensity factor $K_{\mathrm{I}}$, using known weight functions for long rectangular plates.

## 3. Results and discussion

In all elements without cracks, the stress distributions $\sigma_y(x)$ along the potential crack path varied significantly due to the changes of side curvatures. For lower $R/b$ ratios, higher stress concentration at the boundary and lower in the central part of the symmetrical sheet under tension can be observed.

For curvilinear bars shown in Fig.3, where bending effects prevail in the crack plane, lower $r/R$ values cause stronger hyperbolic effect in the stress distribution, revealed by higher values of the $\sigma_{max}/\sigma_{min}$ ratio.

Numerical results of the normalised maximal and minimal stresses together with the $\sigma_{max}/\sigma_{min}$ ratios are shown in Table 1.

**Table 1.** BEM results of maximal and minimal normal stresses in an uncracked symmetric sheet and a curvilinear bar

| Symmetric sheet under tension | | | |
|---|---|---|---|
| $R/b$ | $\sigma_{max}/(2\sigma)$ | $\sigma_{min}/(2\sigma)$ | $\sigma_{max}/\sigma_{min}$ |
| 2.0 | 1.33 | 0.869 | 1.53 |
| 1.0 | 1.62 | 0.803 | 2.02 |
| 0.501 | 2.09 | 0.753 | 2.78 |
| Curvilinear bar under tension and bending | | | |
| $r/R$ | $\sigma_{max}/\tau$ | $\sigma_{min}/\tau$ | $\sigma_{max}/\sigma_{min}$ |
| 0.7 | 20.4 | −14.3 | −1.43 |
| 0.5 | 12.88 | −6.45 | −2.00 |
| 0.3 | 10.55 | −3.17 | −3.33 |

Numerical BEM and approximate weight function results for all elements with cracks are shown graphically in Fig.4 ÷ Fig.7. For convenience, the stress intensity factors $K_I$ are transformed into correction functions $Y_I(a/b)$ defined in Fig.4 ÷ Fig.7, and drawn vs. $a/b$ ratio.

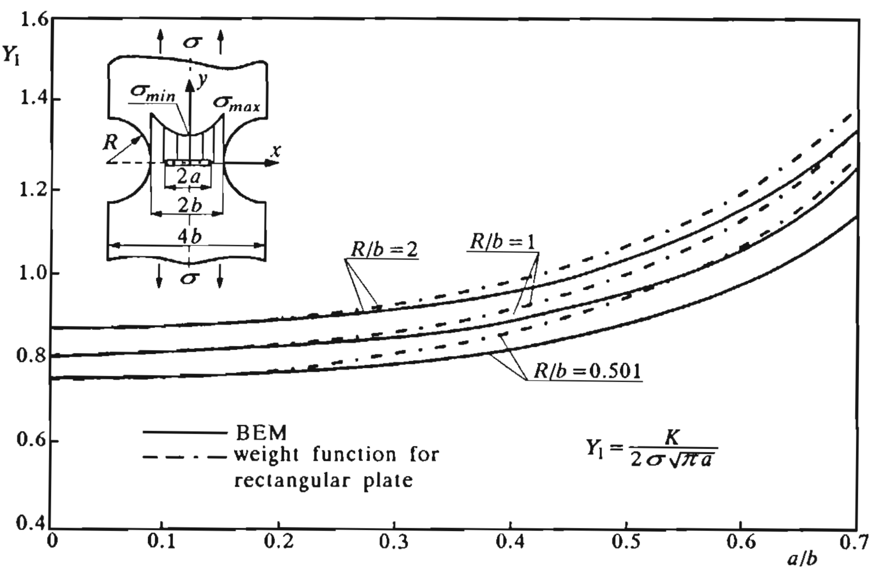


Fig. 4. Real BEM and approximate weight function correction functions $Y_I$ for a centrally cracked sheet under tension; $\sigma_y(x)$ for uncracked element

The BEM results, represented by solid lines, are expected to have maximal error lower than 0.5%, and can serve as the reference values. Dashed lines represent approximate solutions, obtained by "improper" weight functions for long rectangular plates, together with the real stresses $\sigma_y(x)$, determined numerically for uncracked bodies with curvilinear sides.

For a central, symmetrical crack in a sheet under tension (Fig.4) both solutions are close together as far as the crack length is small compared to the element width. Since the $a/b$ ratio increases, the differences between both solutions also increase, however the weight function results are always higher than the real ones. The differences depend strongly on side curvature. For the $R/b$ ratio equal to or greater than 2, the maximal deviation at $a/b = 0.7$ is about 4% and increases rapidly for lower $R/b$ values, showing that the weight function true for long, rectangular plate does not fit well to the present geometry, as far as the crack tip approaches a curvilinear side of the element.

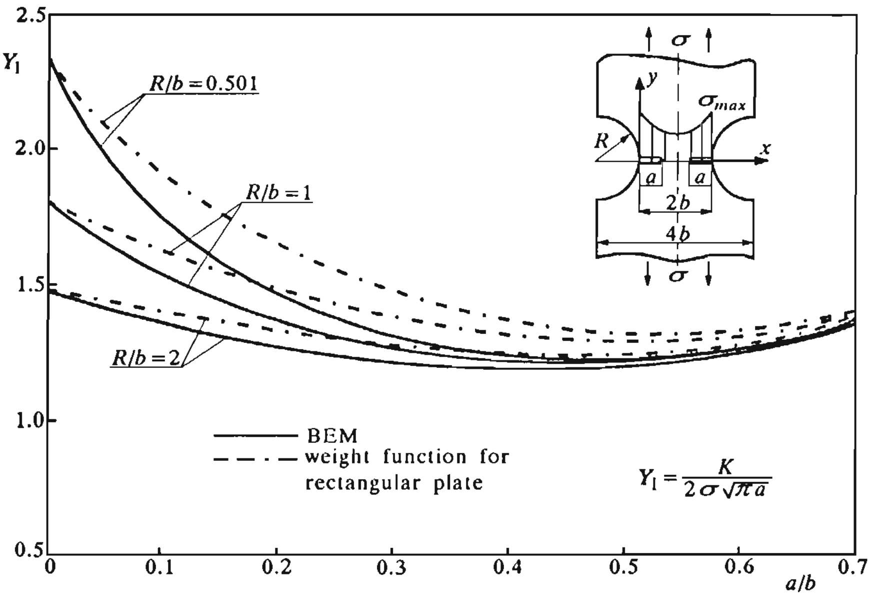


Fig. 5. Real BEM and approximate weight function correction functions $Y_I$ for a sheet under tension with two opposite edge cracks; $\sigma_y(x)$ for uncracked element

In the case of two symmetrical edge cracks shown in Fig.5, the approximate solutions also overestimate the real ones, but the deviations increase rapidly for low $a/b$ ratios, reach maximum in the range $0.25 \leq a/b \leq 0.35$ and continuously decrease. The side curvature effect is similar to that described in the previous case. For higher values of the $R/b$ ratio, the results of both solutions fit better, but for the lower ones – maximal deviation increases.

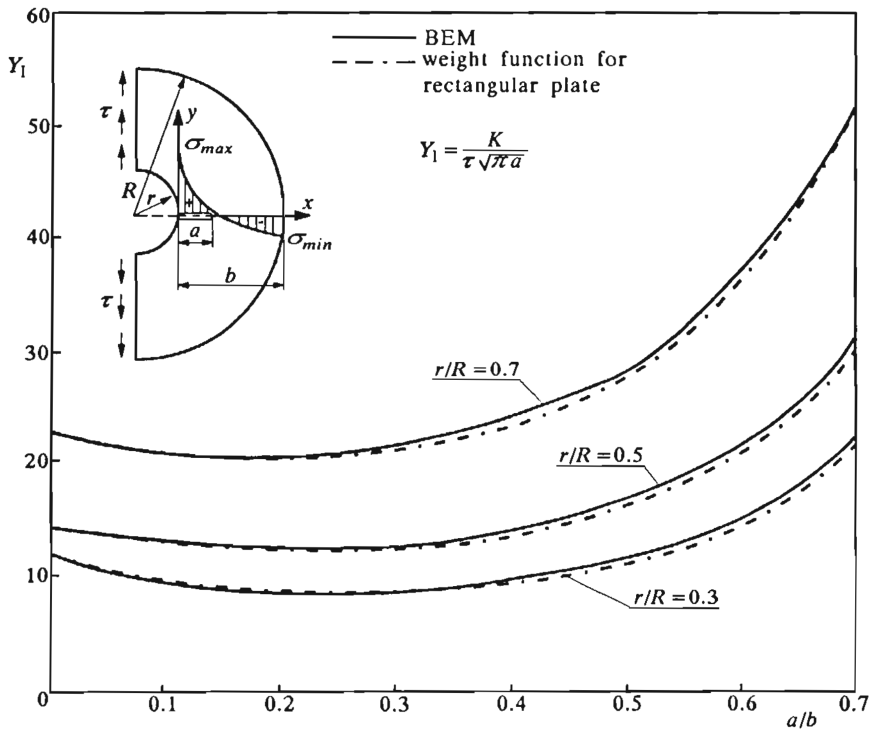


Fig. 6. Real BEM and approximate weight function correction functions $Y_I$ for a curved bar with a single inner crack; $\sigma_y(x)$ for uncracked element

Numerical BEM and approximate weight function results for a curved bar with a single inner and outer edge crack are shown in Fig.5 and Fig.6 respectively, for $R = 1$. The accuracy of the weight function solutions have appeared to be unexpectedly good in both cases, within the whole range $0.0 < a/b \leq 0.7$ being analysed, even for high curvatures i.e. small $r/R$ ratios. It is probably caused by the opposite effects of concavity at one side of the bar and the convexity at the other.

Negative values of the correction functions $Y_I(a/b)$ in Fig.7 appear due to the compressive stresses, theoretically released on the crack surface, however the real crack in such a case is entirely closed and its presence may be ignored. The negative sign, resulting from the assumed direction of the external load $\tau$, is necessary here just to make the analysis structurally consistent.

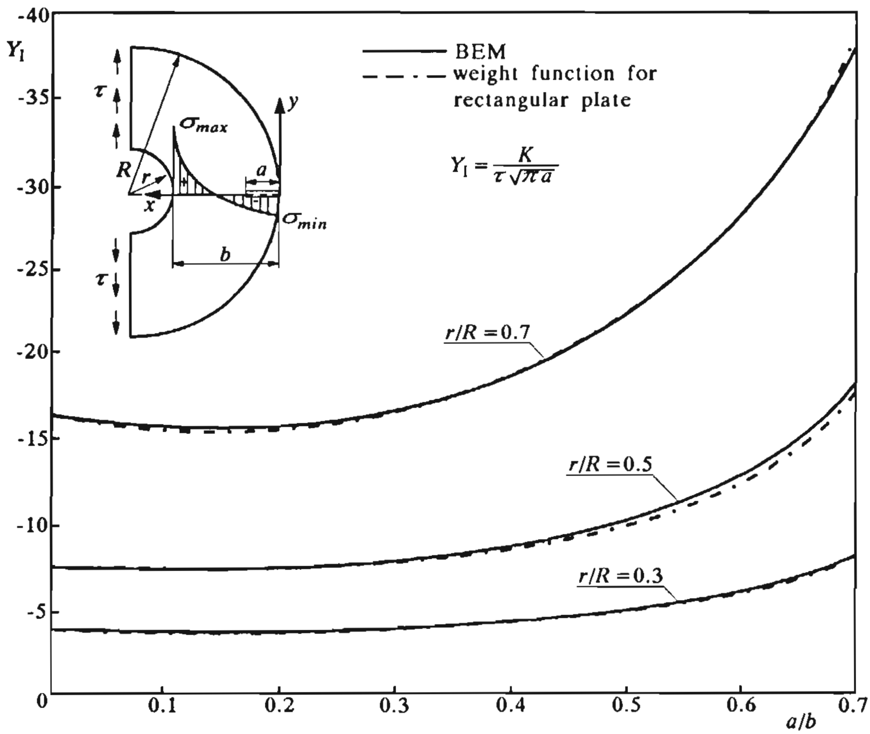


Fig. 7. Real BEM and approximate weight function correction functions $Y_I$ for a curved bar with a single outer crack; $\sigma_y(x)$ for uncracked element

## 4. Conclusions

The numerical analysis carried out using the boundary element method has shown that side curvature of any structure subjected to tension or bending affects the stress intensity factor values $K_I$ for all three cases considered above, regarding net width of the element unchanged.

$K_I$ values are strongly related to the real normal stress distributions in the crack plane and released on its faces. A smaller side radius produces higher stress concentration at the boundaries and lower stress values in the central part of the body under tension, increasing $K_I$ values of edge cracks and decreasing $K_I$ for a central crack.

The numerical BEM results of correction functions $Y_I$ compared to the approximated ones obtained using the weight function solutions true for long, rectangular plates, considered together with the real normal stress distributions, reveal good accuracy for symmetrical plates under tension with central and edge cracks, with $R/b$ ratio sufficiently high, say $R/b > 2$, giving the maximal error about 5% within the range $0 < a/b \leq 0.7$. For smaller $R/b$ ratios, the differences increase, however the weight function results always overestimate the true values. This feature is advantageous if applied to engineering problems. For a curvilinear bar under tension and bending with a single inner or outer edge crack, the results obtained using both methods are in good agreement even for a $r/R$ ratio lower than 0.5.

The present analysis has shown that in many cases, known weight function solutions true for simple geometrical forms, may be applied with good accuracy to more complicated geometrical forms, whenever real stress distributions along the potential crack path can be identified.

*Acknowledgments*

The investigation described in this paper is a part of the research project No. 7S101 040 04 sponsored by the State Committee for Scientific Research (KBN).

The boundary element results have been obtained using program "Cracker" from Wessex Institute of Technology, UK.

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## Wpływ krzywizny brzegów tarczy poddanej rozciąganiu i zginaniu na wartości współczynnika $K_{\rm I}$ dla wybranych konfiguracji szczelin

Streszczenie

Rozważono zagadnienie wpływu krzywizny bocznych brzegów płaskiego elementu o skończonej szerokości, na wartości współczynnika $K_{\rm I}$, dla szczeliny centralnej, dwóch przeciwległych szczelin brzegowych w rozciąganej, symetrycznej tarczy oraz dla przypadku pojedynczej – wewnętrznej i zewnętrznej szczeliny brzegowej w elemencie zakrzywionym. Metodą elementu brzegowego BEM obliczono wartości funkcji korekcyjnych $Y_{\rm I}$ dla różnych proporcji między długością szczeliny, szerokością tarczy a promieniem krzywizny jej brzegu. Wyniki analizy numerycznej BEM porównano z obliczeniami otrzymanymi z użyciem znanych funkcji wagowych, prawdziwych dla długiej, prostokątnej tarczy o identycznym położeniu szczelin, z uwzględnieniem rzeczywistych rozkładów naprężenia w płaszczyźnie potencjalnej szczeliny w elementach o brzegach krzywoliniowych. Stwierdzono, że w rozpatrywanych przypadkach geometrycznych, decydujący wpływ na wartości $K_{\rm I}$ ma rozkład naprężeń normalnych uwalnianych na powierzchni szczeliny, zdeterminowany krzywizną brzegów i sposobem obciążenia rzeczywistego elementu.

*Manuscript received March 7, 1996; accepted for print April 11, 1996*