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# COLMATAGE ACCOMPANYING THE FLOW OF GASIFIED LIQUID THROUGH POROUS MEDIA

Alfred Trzaska

Krystyna Sobowska

*Faculty of Mining, University of Mining and Metallurgy*
*e-mail: broda@uci.agh.edu.pl*

The paper deals with a flow of liquid containing dissolved gas through a porous medium. Due to a pressure drop below the saturation pressure along the flow course, bubbles of gas can be emitted in the liquid. They are deposited in the pores of the medium, decreasing its porosity and permeability. Thus, the phenomenon of colmatage occurs.
This process is described by Henry's equations (2.1), the equations of balance transport (2.2), motion (2.4), with initial boundary conditions (2.5) taken into consideration. Basing on these equations there are obtained: function of the position and time of the medium porosity $\varepsilon(x,t)$, function of the pressure distribution $h(x,t)$, and time-dependent discharge of flow $q(t)$. Diagrams of these functions are shown in Fig.1 ÷ Fig.6.

*Key words:* colmatage, filtration, porous media, suspension

## 1. Introduction

In this paper a flow of liquid containing a dissolved gas through a porous medium has been considered. During this process a pressure drop along the flow course has been observed. When additionally conditioned, this drop can result in bubbling of the gas in the liquid. These bubbles emitted from the liquid are deposited in the pores of medium, which causes a decrease in the porosity of the medium and, consequently, reduces its permeability. Such a process is known as the colmatage. Those who are interested in this problem are refered to the papers given References.

## 2. Formulation of the problem

The phenomena discussed affect the discharge of flow and the pressure distribution in the deposit. In this paper the flow at the constant pressure difference is considered. Let us assume that a liquid containing a gas dissolved of the concentration $n_0$ is forced into a one-dimensional homogeneous medium of the length $L$ under the constant pressure $h_0$, and flows out of it under the pressure $h_L$.

We assume that $h_0$ is greater than the saturation pressure $h_k$ under which the gas starts bubbling. Thus, the gas starts bubbling at a certain point in the flow course, i.e. when the pressure drops to the critical value $h_k$.

Let us introduce the following denotations

| | | |
|---|---|---|
| $C_1(x,t)$ | – | gas mass dissolved in the unit volume of liquid |
| $C_2(x,t)$ | – | volume of gas bubbles deposited in the medium pores unit |
| $\varepsilon(x,t)$ | – | function of the porosity of the medium |
| $h(x,t)$ | – | function representation the pressure distribution in the deposit |
| $q(t)$ | – | discharge of flow. |

Certain dependences among thus determined functions should be formulated, i.e., according to Henry's law, we have

$$C_1(x,t) = \begin{cases} C_0 & \text{when} \quad h(x,t) > h_k \\ \dfrac{C_0}{h_k} h(x,t) & \text{when} \quad h(x,t) \leqslant h_k \end{cases} \tag{2.1}$$

We assume that the total amount of the gas emitted remains in the medium and it is not dissolved again in the liquid.

In the model under consideration the changes in gas volume caused by the pressure change are not taken into account. Thus, we assume that the flow proceeds due to insignificant pressure changes.

Thus, we assume that the volume of gas bubbles separated from the liquid is proportional to their mass, and we obtain the balance-transport equation in the following form

$$\frac{\partial C_2(x,t)}{\partial t} + \alpha_1 q(t) \frac{\partial C_1(x,t)}{\partial x} = 0 \tag{2.2}$$

where $\alpha_1$ is a certain proportionality factor.

Let us notice, that if the initial deposit porosity is denoted by $\varepsilon_0$ than

$$\varepsilon(x,t) = \varepsilon_0 - C_2(x,t) \tag{2.3}$$

The equation of motion (cf Trzaska, 1986; Trzaska and Sobowska, 1992) is another formula describing the process in question

$$\frac{\partial h(x,t)}{\partial x} = -\frac{aq(t)}{[\varepsilon(x,t)]^3} \tag{2.4}$$

where $a$ is a certain constant.

The assumptions previously discussed give the initial boundary conditions

$$\begin{array}{ll} C_1(x,t) = C_0 & \varepsilon(x,0) = \varepsilon_0 \\ h(0,t) = h_0 & h(L,t) = h_L \end{array} \tag{2.5}$$

Let us notice that $h_L < h_k < h_0$.

## 3. Solution of the problem

Let the point at which the pressure takes the critical value $h_k$ at the moment $t$ be denoted by $x = f(t)$. Thush

$$h\Big(f(t),t\Big) = h_k \tag{3.1}$$

Then, let the inverse of $f$ be denoted by $t = g(x)$ and represent the time at which the pressure takes the value $h_k$ at an established point $x$. So, we have

$$h\Big(x,g(x)\Big) = h_k \tag{3.2}$$

See Fig.6.

Eq (2.1), after taking Eqs (2.2), (2.3) and (3.1) into consideration, can be rewritten as

$$\frac{\partial \varepsilon(x,t)}{\partial t} = \begin{cases} 0 & \text{for} \quad 0 \leqslant x < f(t) \\ \alpha q \dfrac{\partial h(x,t)}{\partial x} & \text{for} \quad f(t) \leqslant x \leqslant L \end{cases}$$

where $\alpha = \alpha_1 C_0/h_k$. Introducing Eq (2.4)

$$\frac{\partial \varepsilon(x,t)}{\partial t} = \begin{cases} 0 & \text{for} \quad 0 \leqslant x < f(t) \\ -\dfrac{\alpha a q^2(t)}{[\varepsilon(x,t)]^3} & \text{for} \quad f(t) \leqslant x \leqslant L \end{cases} \tag{3.3}$$

Let us notice that for $f(t) \leqslant x \leqslant L$ the following equation holds

$$\varepsilon(x,t) = \sqrt[4]{\varepsilon_0^4 - 4\alpha a \int\limits_0^t q^2(t)\ dt}$$

For $x = f(t)$ we have

$$\varepsilon\Big(f(t),t\Big) = \varepsilon\Big(x,g(x)\Big) = \sqrt[4]{\varepsilon_0^4 - 4\alpha a \int\limits_0^{g(x)} q^2(t)\ dt}$$

It results from Eq (3.3) that for $0 \leqslant x < f(t)$, $\varepsilon(x,t)$ is a function of the variable $x$ only.

To retain the continuity of function $\varepsilon(x,t)$, the equality must be satisfied in this area

$$\varepsilon(x,t) = \sqrt[4]{\varepsilon_0^4 - 4\alpha a \int\limits_0^{g(x)} q^2(t)\ dt}$$

To sum up, we obtain the following formula

$$\varepsilon(x,t) = \begin{cases} \sqrt[4]{\varepsilon_0^4 - 4\alpha a \int\limits_0^{g(x)} q^2(t)\ dt} & \text{for} \quad 0 \leqslant x < f(t) \\ \sqrt[4]{\varepsilon_0^4 - 4\alpha a \int\limits_0^{t} q^2(t)\ dt} & \text{for} \quad f(t) \leqslant x \leqslant L \end{cases} \tag{3.4}$$

Integrating Eq (2.4) and introducing Eq (3.4) and taking tho conditions $(2.5)_3$ and (3.1) into account, the following formulae are obtained

$$h(x,t) = \begin{cases} h_0 - \int\limits_0^x \dfrac{aq(t)\ dx}{\sqrt[4]{\Big(\varepsilon_0^4 - 4\alpha a \int\limits_0^{g(x)} q^2(t)\ dt\Big)^3}} & \text{for} \quad 0 \leqslant x < f(t) \\ h_k - \dfrac{aq(t)[x - f(t)]}{\sqrt[4]{\Big(\varepsilon_0^4 - 4\alpha a \int\limits_0^{t} q^2(t)\ dt\Big)^3}} & \text{for} \quad f(t) \leqslant x \leqslant L \end{cases} \tag{3.5}$$

Now, the unknown functions $f(t)$, $g(x)$ and $q(t)$ appearing in the above formulae will be determined. The following system of equations will be used

$$\begin{aligned} h_0 - \int\limits_0^{f(t)} \frac{aq(t)\,dx}{\sqrt[4]{\Big(\varepsilon_0^4 - 4\alpha a \int\limits_0^{g(x)} q^2(t)\,dt\Big)^3}} &= h_k \\ h_k - \frac{aq(t)[L - f(t)]}{\sqrt[4]{\Big(\varepsilon_0^4 - 4\alpha a \int\limits_0^{t} q^2(t)\,dt\Big)^3}} &= h_L \end{aligned} \tag{3.6}$$

Eq $(3.6)_1$ results from the continuity of function $h(x,t)$ at the point $x = f(t)$; while Eq $(3.6)_2$ is a consequence of the boundary condition $(2.5)_4$. Eq $(3.6)_1$ is transformed into

$$\int\limits_0^{f(t)} \frac{dx}{\sqrt[4]{\Big(\varepsilon_0^4 - 4\alpha a \int\limits_0^{g(x)} q^2(t)\,dt\Big)^3}} = \frac{h_0 - h_k}{aq(t)}$$

and both sides are differentiated with respect to $t$. The following is obtained

$$\frac{f'(t)}{\sqrt[4]{\Big(\varepsilon_0^4 - 4\alpha a \int\limits_0^{g(f(t))} q^2(t)\,dt\Big)^3}} = -\frac{(h_0 - h_k)q'(t)}{aq^2(t)}$$

Hence, after transformations and considering that $g\big(f(t)\big) \equiv t$, we have

$$\sqrt[4]{\Big(\varepsilon_0^4 - 4\alpha a \int\limits_0^{t} q^2(t)\,dt\Big)^3} = -\frac{af'(t)q^2(t)}{(h_0 - h_k)q'(t)}$$

Introducing the above formula into Eq $(3.6)_2$ the following equation is obtained

$$\frac{q'(t)}{q(t)} = \frac{h_L - h_k}{h_0 - h_k}\frac{f'(t)}{L - f(t)}$$

which after integrating over $t$ gives the following dependence

$$f(t) = L - (L - x_0)\Big(\frac{q(t)}{q_0}\Big)^{\frac{h_0 - h_k}{h_k - h_L}} \tag{3.7}$$

where $x_0 = f(0)$, $q_0 = q(0)$.

The values of $x_0$, $q_0$ can be found examining Eq (2.4) at the instant $t = 0$. Integrating this equation over $t$, with the conditions $(2.5)_{2,3}$ taken into consideration, the following is obtained

$$h(x,0) = h_0 - \frac{aq_0}{\varepsilon_0^3}x$$

This is a linear function and on the conditions $(2.5)_{3,4}$ it has to have the form

$$h(x,0) = h_0 - \frac{h_0 - h_L}{L}x \tag{3.8}$$

Comparing both formulae obtained, we calculate

$$q_0 = q(0) = -\frac{(h_0 - h_L)\varepsilon_0^3}{aL} \tag{3.9}$$

Next, taking into account the condition (3.1) in Eq (3.8), $x_0 = f(0)$ is found. Thus

$$h_k = h_0 - \frac{h_0 - h_L}{L}x_0$$

and hence

$$x_0 = \frac{(h_0 - h_k)L}{h_0 - h_L} \tag{3.10}$$

Eq (3.7) is introduced into Eq $(3.6)_2$. An the integral equation is obtained after transformations

$$\varepsilon_0^4 - 4\alpha a\int\limits_0^t q^2(t)\;dt = \Big[\frac{a(L - x_0)}{h_k - h_L}\Big]^{\frac{4}{3}} q_0^{-\frac{4}{3}\frac{h_0 - h_k}{h_k - h_L}} \Big[q(t)\Big]^{\frac{4}{3}\frac{h_0 - h_L}{h_k - h_L}}$$

It is differentiated with respect to $t$. Introducing Eq (3.10) and after certain transformations a differential equation of the following form is obtained

$$q'(t)\Big[q(t)\Big]^{\frac{4}{3}\frac{h_0 - h_L}{h_k - h_L} - 3} = -3\alpha a^{-\frac{1}{3}}L^{-\frac{4}{3}}(h_0 - h_L)^{\frac{1}{3}}(h_k - h_L)q_0^{\frac{4}{3}\frac{h_0 - h_k}{h_k - h_L}} \tag{3.11}$$

The following function is the solution of Eq (3.11) when $\frac{4}{3}\frac{h_o - h_L}{h_k - h_L} - 3 \neq -1$, i.e. $2h_0 - 3h_k + h_L \neq 0$

$$q(t) = \frac{\varepsilon_0^3(h_0 - h_L)}{aL}(1 - At)^{\frac{3}{2}\frac{h_k - h_L}{2h_0 - 3h_k + h_L}} \tag{3.12}$$

where

$$A = \frac{2\alpha(2h_0 - 3h_k + h_L)(h_0 - h_L)\varepsilon_0^3}{aL^2} \tag{3.13}$$

If $2h_0 - 3h_k + h_L = 0$, the following function is the solution of Eq (3.11)

$$q(t) = \frac{\varepsilon_0^3(h_0 - h_L)}{aL} e^{-Bt} \tag{3.14}$$

where

$$B = \frac{2\alpha(h_0 - h_L)^2\varepsilon_0^2}{aL^2} \tag{3.15}$$

The function $q(t)$ given both by Eqs (3.12) and (3.14) is a decreasing function of time. Fig.1 presents diagrams of this function for various cases.

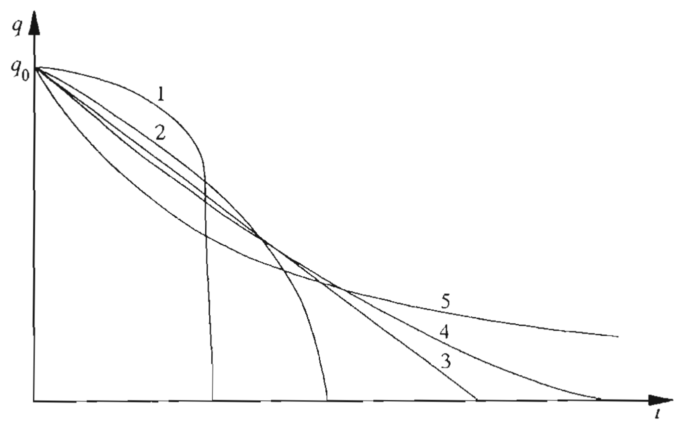


Fig. 1. Diagrams of function $q(t)$ obtained from Eq (3.12) for different values of initial saturation and terminal pressures, respecive

Introducing Eqs (3.12) or (3.14) into Eq (3.7) we get the function $f(t)$. We have

$$f(t) = L\Big[1 - \frac{h_k - h_L}{h_0 - h_L}(1 - At)^{\frac{3(h_0 - h_k)}{2(2h_0 - 3h_k + h_L)}}\Big] \tag{3.16}$$

when $2h_0 - 3h_k + h_L \neq 0$ and

$$f(t) = L\Big(1 - \frac{2}{3}e^{-\frac{1}{2}Bt}\Big) \tag{3.17}$$

when $2h_0 - 3h_k + h_L = 0$. In each case this function is increasing, i.e. $f(t) \geqslant f(0) = x_0$.

Now, the function $t = g(x)$ will be determined. This is an inverse to the function $x = f(t)$, i.e.

$$t = g(x) \Longleftrightarrow x = f(t)$$

Let $2h_0 - 3h_k + h_L \neq 0$. Now $t$ is calculated from the equation $x = f(t)$ where $f(t)$ is given by Eq (3.16). In this case for $x \geqslant x_0$ we have

$$t = g(x) = \frac{1}{A}\Big\{1 - \Big[\frac{(L-x)(h_0-h_L)}{L(h_k-h_L)}\Big]^{\frac{2(2h_0-3h_k+h_L)}{3(h_0-h_k)}}\Big\} \tag{3.18}$$

When $2h_0 - 3h_k + h_L = 0$, the function $g(x)$ has the form

$$g(x) = \frac{2}{B}\ln\frac{2L}{3(L-x)} \qquad \text{for} \quad x \geqslant \frac{1}{3}L \tag{3.19}$$

Now, it is possible to determine the functions $\varepsilon(x,t)$, $h(x,t)$. It is enough to introduce the determined formulae for the functions $q(t)$, $f(t)$, $g(x)$ into Eqs (3.4), (3.5) and made suitable calculations. When $2h_0 - 3h_k + h_L \neq 0$, then

$$\varepsilon(x,t) = \begin{cases} \varepsilon_0 & \text{for} \quad 0 \leqslant x < x_0 \\ \varepsilon_0\Big[\dfrac{(L-x)(h_0-h_L)}{L(h_k-h_L)}\Big]^{\frac{h_0-h_L}{3(h_0-h_k)}} & \text{for} \quad x_0 \leqslant x < f(t) \\ \varepsilon_0(1-At)^{\frac{h_0-h_L}{2(2h_0-3h_k+h_L)}} & \text{for} \quad f(t) \leqslant x \leqslant L \end{cases} \tag{3.20}$$

For $2h_0 - 3h_k + h_L = 0$, the function $\varepsilon(x,t)$ is expressed by

$$\varepsilon(x,t) = \begin{cases} \varepsilon_0 & \text{for} \quad 0 \leqslant x < \frac{1}{3}L \\ \frac{3}{2}\varepsilon_0\dfrac{L-x}{L} & \text{for} \quad \frac{1}{3}L \leqslant x < f(t) \\ \varepsilon_0 e^{-\frac{1}{2}Bt} & \text{for} \quad f(t) \leqslant x \leqslant L \end{cases} \tag{3.21}$$

Fig.2 and Fig.3 present the diagrams of function $\varepsilon(x,t)$ at a given point $x$ or at a determined instant $t$ for various cases.

The function $h(x,t)$ is expressed by the following formula

$$h(x,t) = \begin{cases} h_0 - C\dfrac{(h_0-h_L)x}{L} & \text{for} \quad 0 \leqslant x < x_0 \\ h_0 - C(h_0-h_k)\Big[\dfrac{L(h_k-h_L)}{(h_0-h_L)(L-x)}\Big]^{\frac{h_k-h_L}{h_0-h_k}} & \text{for} \quad x_0 \leqslant x < f(t) \\ h_L + C\dfrac{(h_0-h_L)(1-x)}{L} & \text{for} \quad f(t) \leqslant x \leqslant L \end{cases} \tag{3.22}$$

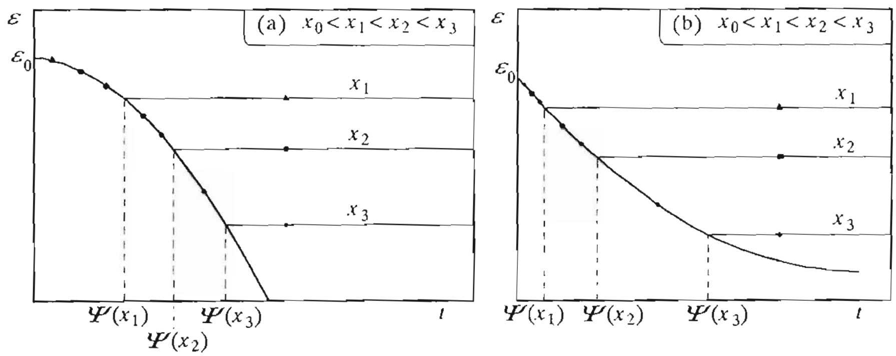


Fig. 2. Illustration of the porosity variation at the determined points $x > x_0$ when (a) $2h_0 - 3k_k + h_L > 0$, (b) $2h_0 - 3k_k + h_L < 0$

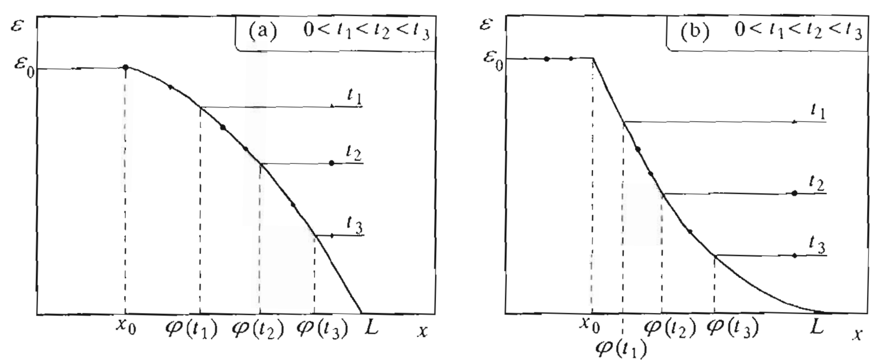


Fig. 3. Illustration of the porosity variation at the determined moments of the process duration when (a) $2h_0 - 3k_k + h_L > 0$, (b) $2h_0 - 3k_k + h_L < 0$

and

$$C = (1 - At)^{\frac{3(h_k - h_L)}{2(2h_0 - 3h_k + h_L)}}$$

when $2h_0 - 3h_k + h_L \neq 0$, and

$$h(x,t) = \begin{cases} h_0 - \dfrac{(h_0 - h_L)x}{L} \mathrm{e}^{-Bt} & \text{for} \quad 0 \leqslant x \leqslant \frac{1}{3}L \\[2ex] h_0 - \dfrac{4}{27} \dfrac{(h_0 - h_L)L^2}{(L-x)^2} \mathrm{e}^{-Bt} & \text{for} \quad \frac{1}{3}L \leqslant x < f(t) \\[2ex] \dfrac{2h_0 + h_L}{3} - \dfrac{h_0 - h_L}{L} \Big[ (x - L)\mathrm{e}^{\frac{1}{2}Bt} + \frac{2}{3}L \Big] & \text{for} \quad f(t) \leqslant x < L \end{cases} \tag{3.23}$$

when $2h_0 - 3h_k + h_L = 0$.

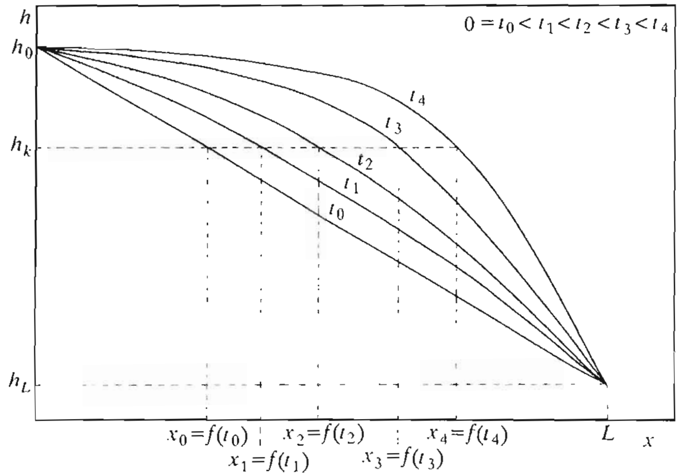


Fig. 4. Illustration of the process pointing to the moving point $\varphi(t)$ at which the pressure takes the critical value of saturation $h_k$

Fig.4 shows diagrams of the function $h(x,t)$ for various moments of the process duration.

Summing up, we want to say that the way in which the process proceeds in affected by the sign of expression $2h_0 - 3h_k + h_L$ in an essential manner.

If $2h_0 - 3h_k + h_L > 0$, then the discharge of flow $q(t)$ (Eq (3.12)) at the instant $t = 1/A$ has the zero value, i.e. the flow of a liquid through a porous medium dies out. The function $f(t)$ (Eq (3.16)) takes the value $L$ for $t = 1/A$.

If $2h_0 - 3h_k + h_L \leqslant 0$, the discharge of flow never taks the zero value, the medium is never "choked", but by virtue of Eqs (3.12), (3.14) $\lim_{t\to\infty} q(t) = 0$. Basing on Eqs (3.16) and (3.17) we have $\lim_{t\to\infty} f(t) = L$.

When $t \geqslant 1/A$ in the case of $2h_0 - 3h_k + h_L > 0$, or as $t \to \infty$ for $2h_0 - 3h_k + h_L < 0$, by virtue of Eq (3.20) the asymptotic distribution of porosity in the medium has the form

$$\varepsilon(x,t) = \begin{cases} \varepsilon_0 & \text{for} \quad 0 \leqslant x < x_0 \\ \\ \varepsilon_0 \Big[\dfrac{(L-x)(h_0-h_L)}{L(h_k-h_L)}\Big]^{\frac{h_0-h_L}{3(h_0-h_k)}} & \text{for} \quad x_0 \leqslant x < L \end{cases} \tag{3.24}$$

If $2h_0 - 3h_k + h_L = 0$, then basing on Eq (3.21) the following equation is obtained

$$\varepsilon(x,t) = \begin{cases} \varepsilon_0 & \text{when} \quad 0 \leqslant x < \frac{1}{3}L \\ \\ \frac{3}{2}\varepsilon_0 \frac{L-x}{L} & \text{when} \quad \frac{1}{3}L \leqslant x < L \end{cases} \tag{3.25}$$

From Eqs (3.22) and (3.23) the boundary distribution of pressure has the form

$$h(x) = \begin{cases} h_0 & \text{when} \quad 0 \leqslant x < L \\ h_L & \text{when} \quad x = L \end{cases} \tag{3.26}$$

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## Kolmatacja towarzysząca przepływowi zgazowanej cieczy przez ośrodki porowate

Streszczenie

Tematem prac jest przepływ cieczy zawierającej rozpuszczony gaz przez ośrodek porowaty. W wyniku spadku ciśnienia poniżej ciśnienia nasycenia na drodze przepływu może dochodzić do wytrącania się gazu w postaci pęcherzyków. Osadzają się

one w przestrzeni porowej ośrodka doprowadzając do zmniejszenia jego porowatości i przepuszczalności. Zachodzi więc zjawisko kolmatacji.

Proces ten opisano równaniami Henry'ego (2.1), bilansu transportu (2.2) i ruchu (2.4), z uwzględnieniem warunków początkowo-brzegowych (2.5). W oparciu o te równania uzyskano funkcję położenia i czasu porowatości ośrodka $\varepsilon(x,t)$, rozkładu ciśnienia $h(x,t)$, oraz wydatek przepływu w funkcji czasu $q(t)$. Wykresy tych funkcji ilustrują rysunki $1 \div 4$.

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