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INFLUENCE OF BONDING LAYER ON PIEZOELECTRIC ACTUATORS OF AN AXISYMMETRICAL ANNULAR PLATE

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The purpose of this theoretical work is to present a general model of the response of an annular plate to the excitation by an annular actuator made of piezoelectric elements. The plate is clamped at the inner edge and free at the outer edge. Dynamic equations, joint conditions between sections with and without active layers as well as the boundary conditions at the two edges of the plate form a boundary value problem. The dynamic displacement response to the excitation by the applied harmonic voltage term are determined from the solution to this boundary value problem. The dynamic extensional strain on the plate surface is calculated by including the free stress conditions at the piezoelectric actuator boundaries, by considering the dynamic coupling between the actuator and the plate, and by taking into account a finite bonding layer with finite stiffness. Results from numerical simulation show influence of the bonding layer stiffness on the frequency- and space-dependent plate response.

Key words: active composite, distributed piezoactuator, annular plate, bonding layer, forced harmonic vibration

1. Introduction

In the last decade we all observe strong interest in active systems applied to, the so-called, intelligent structures, i.e. structures with highly distributed actuators, sensors, and processor networks. Such systems enable using software adjustments to modify and tune the closed-loop behavior via distributed sensors and actuators. Due to large number of actuators and sensors it is desirable that they are inexpensive, small, light-weight, and simple, and that they do not significantly modify the passive dynamical properties of the host structure. Piezoelectric materials exhibiting mechanical deformations, when an electric field is applied could be used for this purpose. They can not be modeled as point force excitations, and partial differential equations should be used to describe the response of the structure driven by them. Piezoelectric sensors and actuators have been applied successfully in the closed loop control (cf Bailey and Hubbard, 1985). The beam vibration due to the excitation by a piezoelectric actuator has been modeled by Crawley and de Luis (1987), Jie Pan et al. (1991). In particular, Crawley and de Luis presented a comprehensive static model of a piezoelectric actuator glued to a beam. This static approach was then used to predict the dynamic behavior. A dynamic model for a simply supported beam with a piezoelectric actuator glued to each of its upper and lower surfaces was developed by Jie Pan, Hansen and Snyder. In their model the actuators were assumed to be perfectly bonded. It means that the bonding layer is sufficiently thin that the shear of the layer can be neglected.

Structure vibrations that propagate from engines or another sources may be reduced by passive and active isolation, by active control, by passive and active vibration absorbers. Use of the passive vibration absorbers for the structure – borne noise/vibration control is of particular interest to this research. A shunting method has been developed for tuning the natural frequency of a piezoelectric element glued to the beam surfaces (Davis and Lesieutre, 1998). The passive vibration absorbers minimize vibration at a specific frequency related with a lightly damped structure. Large response reduction is only possible if the absorber is accurately tuned to the considered frequency. Tuning a mechanical absorber requires a change in either mass or stiffness of the device. The electromechanical properties of the piezoceramic forcing element with an external passive electrical shunt circuit are used to alter the natural frequency. An analytical distributed model of the piezoelectric vibration absorber was created to predict changes in the natural frequency due to passive electrical shunting. Capacitive shunting alters the natural frequency of the actuators. A passive vibration absorber generally acts to minimize structural vibration at a specific frequency associated with the response of lightly damped structural mode. This frequency is rarely stationary due to changing velocity. Maximum response reductions, however, are achieved only if the absorber is lightly damped and accurately tuned to the frequency of concern. The vibration of two-dimensional structures excited by a piezoelectric actuator has been modeled by Dimitriadis et al. (1991) for rectangular plates, and by Van Niekerk et al. (1995) for circular plates. Van Niekerk, Tongue and Packard presented a comprehensive static model of a circular actuator and coupled circular plate. Their static results were used to predict the dynamic behaviour of the coupled system, particularly to reduce acoustic transmissions. Piezoelectric transducers can be modeled as two-dimensional devices. That approach allows the distributed transducer shape to be included into control design process for two-dimensional structures as an additional design parameter. The essence of the approach involves replacing the piezoactuators with forces and moments distributed along the piezoelements edges. The analytical basis for damping structural vibrations with piezoelectric materials and passive electrical circuits has been developed by Hagood and von Flotov (1991). A key feature of the tunable vibration absorber developed by Davis and Lesieutre (1998) is the use of the piezoelectric ceramic elements as a part of the device stiffness. A discrete model of the piezoceramic vibration absorber was created to predict changes in natural frequency and damping due to passive electrical shunting. A comparison of passive and active damping of thermally induced vibrations of beams with piezolayers was given by Tylikowski and Hetnarski (1999). Influence of actuator shape on stabilization of plate parametric vibrations was studied by Tylikowski and Hetnarski (1998). Their approach was also used (Tylikowski, 1993) to derive parametric stabilization control especially useful in collocated sensor-actuator systems.

The goal of this research is to describe response of an annular piezoelectric plate to the excitation by annular piezoelectric actuators. The consistent distributed model based on partial differential equations of the system motion is used. The Kirchhoff annular plate of the inner radius R_1 and external radius R_2 is divided into three annular sections and the dynamic behavior of each section is analyzed separately (cf Fig.1). The analysis is confined to axially symmetric modes. The absorbers are glued to the plate in the second section $a < r < b$. The dynamic extensional strain on the plate surface is calculated by considering the dynamic coupling between the actuator and the plate, and by taking into account perfect bonding with a finite shear stiffness. The plate motion is described by partial differential equations in terms of the plate transverse displacement. The piezoelement is described by constitutive equations relating stress and electric displacement with strain and electric field. The equations are coupled with the equations of the plate motion by the surface strain term. Along the circles of connection between the sections the continuity of plate deflection, slope and curvature is considered. The condition of free normal stress for the piezoelectric element at $r = a$ and $r = b$ constraints the strain values at these two locations. The plate is excited by a harmonic voltage applied to the actuators. The displacement is harmonically varying with a constant frequency. The boundary conditions at the clamped and free edges, the joint conditions form a boundary value problem. For a single frequency excitation the equations of motion become the fourth-order homogeneous ordinary differential equations, with solutions in the form of traveling waves in the plate. Results obtained by analytical analysis and numerical simulation show influence of the bonding layer stiffness on the frequency- and space-dependent plate response.

2. Dynamics equation of axisymmetrical plate motion

Fig.1 shows a Kirchhoff's thin annular plate with identical piezoceramic elements mounted on opposite sides of the plate. The annular piezolayers are polarized perpendicularly to the plate surface, which can be used to excite or suppress the given motion of the continuous structure. The transverse piezoelectric effect is assumed axisymmetric with respect to the axis perpendicular to the plate, what implies that the transverse piezolectric constants are equal $d_{31} = d_{32}$. The piezolayers are perfectly bonded to the plate and the elastic properties of the intermediate layers are taken into account. Due to the geometry and the axisymmetric motion the plate is divided into three annular sections as shown in Fig.1, and the behavior of each part is analyzed separately.

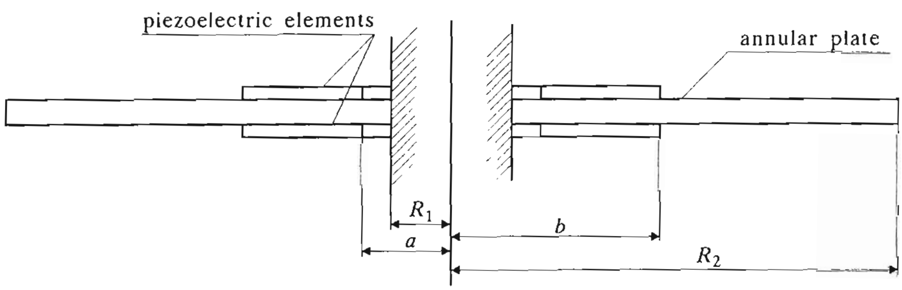

Fig. 1. Geometry of the annular plate with annular piezolayers

Equations of motion are expressed in terms of the plate transverse displacement w due to bending and in terms of the membrane in-plane displacement u of the piezoactuators. The inertia forces of the finite-thickness bonding layers are neglected and the pure one-dimensional shear in the bonding layer is assumed. The thickness of the plate, bonding layer and piezoelectric actuator is denoted by t_p, t_s, t_a, respectively.

Consider a finite element of the radial length dr in the second section. The radial stresses in the piezolayers are assumed to be uniformly distributed in the direction perpendicular to the plate due to the small thickness. For the radial actuator motion the dynamics equation has the following form

$$\Big(r\frac{\partial\sigma_r}{\partial r} + \sigma_r - \sigma_t\Big)t_a - \tau r = \rho_a t_a r\frac{\partial^2 u}{\partial t^2} \tag{2.1}$$

where

σ_r – radial stress
σ_t – circumferential stress
τ – shear stress on the interface surface
u – radial displacement
ρ_a – density of the actuator
r – radial coordinate
t – time.

We express the strains in actuators by the radial displacement

$$\epsilon_r = \frac{\partial u}{\partial r} \qquad\qquad \epsilon_t = \frac{u}{r} \tag{2.2}$$

Using Hook's law we eliminate the normal stresses and write the dynamics equation in terms of the in-plane displacement as

$$\frac{E_a t_a}{1-\nu_a^2}\Big[r\frac{\partial}{\partial r}\Big(\frac{1}{r}\frac{\partial r u}{\partial r}\Big)\Big] - \tau r = \rho_a t_a r\frac{\partial^2 u}{\partial t^2} \tag{2.3}$$

Equation of the transverse plate motion in the second section w_2 is

$$\frac{\partial T r}{\partial r} = \rho_p^* t_p r\frac{\partial^2 w_2}{\partial t^2} \tag{2.4}$$

where

T – shear force
ρ_p – modified plate density calculated according to the rule of mixture, $\rho_p^* = \rho_p + 2\rho_a t_a/t_p$
E_a – Young modulus of actuator
ν_a – Poisson ratio of actuator.

The balance of moments has the form

$$\frac{\partial M_r r}{\partial r} - M_t - Tr + \tau t_p r = 0 \tag{2.5}$$

where

M_r – radial moment
M_t – circumferential moment.

Using the Hook's law we express the moments by the plate transverse displacement in the following form

$$
\begin{aligned}
M_r &= -D\Big(\frac{\partial^2 w_2}{\partial r^2} + \frac{\nu_p}{r}\frac{\partial w_2}{\partial r}\Big)\\
M_t &= -D\Big(\frac{1}{r}\frac{\partial w_2}{\partial r} + \nu_p\frac{\partial^2 w_2}{\partial r^2}\Big)
\end{aligned}
\tag{2.6}
$$

where

$$
D = \frac{E_p t_p^3}{12(1-\nu^2)}
$$

is the plate cylindrical stiffness. Eliminating the transverse force we rewrite Eq (2.4) in the form

$$
D\nabla^2 w_2 - \frac{1}{r}\frac{\partial r t_p r}{\partial r} + \rho_p^* t_p \frac{\partial^2 w_2}{\partial t^2} = 0 \tag{2.7}
$$

where the ∇^2 operator stands for $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\Big\{r\frac{\partial}{\partial r}\frac{1}{r}\Big[\frac{\partial}{\partial r}\Big(r\frac{\partial w}{\partial r}\Big)\Big]\Big\}$.

The shear stress is expressed by the in-plane displacement of the piezoelectric layer and the plate transverse displacement

$$
\tau = \frac{G}{t_s}\Big(u + \frac{t_p}{2}\frac{\partial w_2}{\partial r}\Big) \tag{2.8}
$$

where G denotes the shear modulus of the bonding layer. Using Eqs (2.5) and (2.6) and we express the transverse force in displacements

$$
T_2 = -D\Big(\frac{\partial^3 w_2}{\partial r^3} + \frac{1}{r}\frac{\partial^2 w_2}{\partial r^2} + \frac{1}{r^2}\frac{\partial w_2}{\partial r}\Big) + \frac{G t_p}{t_s}\Big(u + \frac{t_p}{2}\frac{\partial w}{\partial r}\Big) \tag{2.9}
$$

The dynamic extensional strain on the plate surface is calculated by considering the dynamic coupling between the piezolayer and the plate.

Substituting the shear stress from Eq (2.8) the following equations are obtained for Section 2, $a < r < b$

$$
\begin{aligned}
&\rho_a t_a \frac{\partial^2 u}{\partial t^2} - \frac{E_a t_a}{1-\nu_a^2}\frac{\partial}{\partial r}\Big(\frac{1}{r}\frac{\partial r u}{\partial r}\Big) + \frac{G}{t_s}u + \frac{G t_p}{2t_s}\frac{\partial w}{\partial r} = 0\\
&D\nabla^4 w_2 + \rho_p^* t_p \frac{\partial^2 w_2}{\partial t^2} + \frac{G t_p^2}{2t_s}\nabla^2 w_2 + \frac{G t_p}{t_s}\frac{1}{r}\frac{\partial r u}{\partial r} = 0
\end{aligned}
\tag{2.10}
$$

The plate displacement equations for the plate Section 1 and 3 have the classical form

$$
\begin{aligned}
&D\nabla^4 w_1 + \rho_p t_p \frac{\partial^2 w_1}{\partial t^2} = 0 \qquad && R_1 < r < a \\
&D\nabla^4 w_3 + \rho_p t_p \frac{\partial^2 w_3}{\partial t^2} = 0 \qquad && b < r < R_2
\end{aligned}
\tag{2.11}
$$

Eqs $(2.10)_2$ and (2.11) are the fourth-order linear homogeneous partial differential equations and Eq $(2.10)_1$ is the second-order one. Thus, we have to determine fourteen constants $C_1, ..., C_{14}$ from boundary conditions. Similarly, as for piezoactuators bonded to beams (Crawley and de Luis, 1987), the piezoelectric strains do not appear explicitly in Eqs (2.10) and (2.11), but enter into the solution through boundary conditions. Solving Eq $(2.10)_2$ we can obtain the formula for calculating the in-plane displacement of the piezoactuator.

3. Boundary and joint conditions

The boundary conditions at $r = R_1$ and $r = R_2$ of the plate correspond to the clamped and free edge, respectively

$$
\begin{aligned}
&w_1(R_1) = 0 \qquad \frac{\partial w_1(R_1)}{\partial r} = 0 \\
&M_r(R_2) = -D\Big(\frac{\partial^2 w_3}{\partial r^2} + \frac{\nu_p}{r}\frac{\partial w_3}{\partial r}\Big)\Big|_{r=R_2} = 0 \\
&T(R_2) = -D\frac{\partial}{\partial r}\Big(\frac{\partial^2 w_3}{\partial r^2} + \frac{1}{r}\frac{\partial w_3}{\partial r}\Big)\Big|_{r=R_2} = 0
\end{aligned}
\tag{3.1}
$$

At the circular lines (joints) between Sections 1 and 2 and between Sections 2 and 3 the continuity of the plate deflection, slope, radial moment and transverse force have to be satisfied

$$
\begin{aligned}
&w_1(a) = w_2(a) \qquad && \frac{\partial w_1(a)}{\partial r} = \frac{\partial w_2(a)}{\partial r} \\
&w_2(b) = w_3(b) \qquad && \frac{\partial w_2(b)}{\partial r} = \frac{\partial w_3(b)}{\partial r} \\
&T_{r1}(a) = T_{r2}(a) \qquad && T_{r2}(b) = T_{r3}(b)
\end{aligned}
\tag{3.2}
$$

It should be noted that the transverse force T_2 in the second section depends not only on the transverse displacement w_2 but also on the shear stress and is calculated from Eq (2.5). The free stress condition for the piezoelectric annular element at $r = a$ and $r = b$ constraints the strains at these circles

$$\sigma_r = -\frac{E_a}{1-\nu_a^2}\Big[\frac{\partial u}{\partial r} - \Lambda_r + \nu_a\Big(\frac{u}{r} - \Lambda_t\Big)\Big] \tag{3.3}$$

where Λ_r and Λ_t denote the piezoelectric strains $\Lambda = d_{31}\mathcal{V}/t_a$ in the radial and circumferential direction, respectively. For the axisymmetric pieezoelectric effect the stress-free boundary conditions are nonhomogeneus and have the following form

$$\begin{aligned} \Big(\frac{\partial u}{\partial r} + \frac{\nu_a}{r}u\Big)\Big|_{r=a} &= \Lambda(1+\nu_a) \\ \Big(\frac{\partial u}{\partial r} + \frac{\nu_a}{r}u\Big)\Big|_{r=b} &= \Lambda(1+\nu_a) \end{aligned} \tag{3.4}$$

where the in-plane radial displacement of the actuator is calculated from Eq $(2.10)_2$. Eqs (2.10) and (2.11) with boundary conditions (3.1), joint conditions (3.2), and the stress free conditions (3.4) form a boundary value problem.

4. Analytical solution

The steady state is analyzed for a harmonic voltage with a single frequency excitation ω and amplitude $\mathcal{V}$ and the piezoelectric strain of the form

$$\Lambda = \frac{d_{31}\mathcal{V}}{t_a}\exp i\omega t \tag{4.1}$$

The steady state responses of Eqs (2.10) and (2.11) are sought as harmonics with the same angular velocity

$$\begin{bmatrix} w_1(r,t) \\ w_2(r,t) \\ w_3(r,t) \\ u(r,t) \end{bmatrix} = \exp i\omega t \begin{bmatrix} W_1(r) \\ W_2(r) \\ W_3(r) \\ U(r) \end{bmatrix} \tag{4.2}$$

where W_1, W_2, W_3, U are spatial terms of the solutions. Functions W_1, W_3 are the solutions to the classical Bessel differential equations

$$\Big(\nabla^4 - \kappa^4\Big)W = 0 \tag{4.3}$$

where the wavenumber κ is calculated from the following formula

$$\kappa^4 = \frac{\rho_p t_p \omega^2}{D}$$

$$W_1 = C_1 J_0(\kappa r) + C_2 Y_0(\kappa r) + C_3 I_0(\kappa r) + C_4 K_0(\kappa r) \tag{4.4}$$

$$W_3 = C_{11} J_0(\kappa r) + C_{12} Y_0(\kappa r) + C_{13} I_0(\kappa r) + C_{14} K_0(\kappa r)$$

where the zeroth order Bessel functions of the first and the second kind are denoted by J_0, Y_0 and I_0, K_0, respectively.

Eliminating the actuator in-plane displacement u from Eqs (2.10) we obtain the sixth order differential equation with respect to the transverse displacement w_2

$$(\nabla^2 + \alpha^2)(\nabla^2 - \beta^2)(\nabla^2 - \gamma^2)W = 0 \tag{4.5}$$

The wavenumbers α^2, β^2, γ^2 are calculated from the cubic algebraic equation

$$x^3 + \xi x^2 + \eta x + \zeta = 0 \tag{4.6}$$

where the coefficients are given by

$$\xi = e - f \qquad \eta = -\kappa^4 - \Big(\frac{1-\nu_a^2}{E_a t_a}\frac{G}{t_s} + e\Big)f$$

$$\zeta = -e\kappa^4 \qquad e = \frac{\omega^2 \rho_a t_a - \frac{G}{t_s}}{E_a t_a}(1-\nu_a^2)$$

$$f = \frac{G t_p^2}{2 t_s D}$$

The spatial term of the transverse displacement in the second section is

$$\begin{aligned} W_2 = {} & C_5 J_0(\alpha r) + C_6 Y_0(\alpha r) + C_7 I_0(\beta r) + C_8 K_0(\beta r) + \\ & + C_9 I_0(\gamma r) + C_{10} K_0(\gamma r) \end{aligned} \tag{4.7}$$

The fourteen unknown constants in Eqs $(4.4)_{2,3}$ and (4.7) are determined by making use of the boundary conditions at the plate edges and the joint conditions. For a given excitation frequency ω the constants $\boldsymbol{C} = [C_1, C_2, ..., C_{14}]^\top$ can be calculated from nonhomogeneous linear algebraic equations.

5. Results

Numerical calculations based on the formulas presented in the previous sections have been carried out for a wide range of the angular frequency and for $\Lambda = 0.00014\,\mathrm{m}$ and $e_{3r} = e_{3t}$. To include the damping effect complex Young's modulus E_p with the retardation time $\lambda = 5 \times 10^{-6}\,\mathrm{s}$ has been used. The parameters of the plate and piezoelectric elements used in the calculations are listed in Table 1.

Table 1. Material parameters used in calculations

Material	Plate-Steel	Actuator-PZTG-1195
density [kg/m^3]	7800	7275
modulus [N/m^2]	21.6×10^{10}	63×10^{9}
thickness [m]	0.002	0.0002
piezoelectric const. [m/V]	–	1.9×10^{-10}
inner radius [m]	0.02	0.03
outer radius [m]	0.15	0.07

As there is no data available relating the mechanical properties (G, t_s) of the bonding layer the calculations have been done for the following values of the G/t_s ratio: 10^9, 10^{11}, 5×10^{12}.

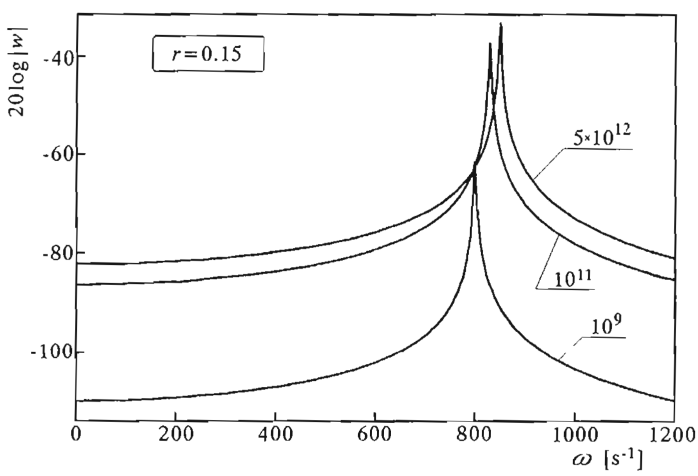

Fig. 2. Far field plate transversal response w at $r = 0.15\,\mathrm{m}$

Figure 2 shows the response of the plate outer edge in logarithmic scale (far field) to the electrical excitation of the annular piezoelectric layers.

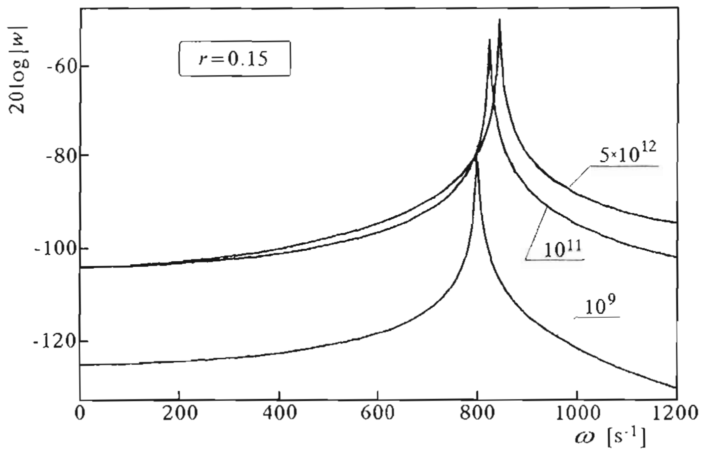

Fig. 3. Near field plate transversal response w at $r = 0.05\,\mathrm{m}$

Fig.3 demonstrates the response in the second section (near field). A dynamic distributed model has been developed on the grounds of which one can predict behavior of the piezoelectric vibration absorber. The derived dynamics equations can be reduced to the particular cases from the past studies, which were based on the assumption of static coupling between the actuator and the beam or assuming the perfect bonding in dynamical analysis. Calculations show that if the bonding layer parameter increases then the plate displacement will increase as well.

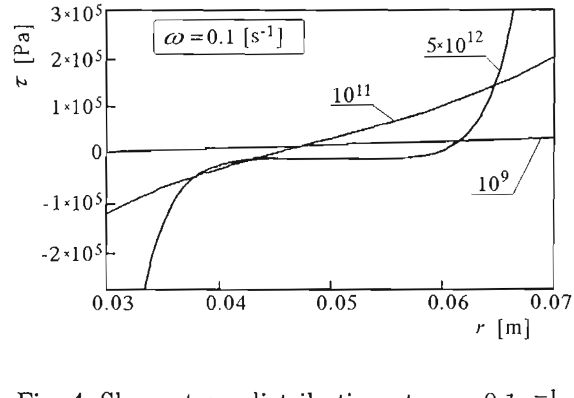

Fig. 4. Shear stress distribution at $\omega = 0.1\,\mathrm{s}^{-1}$

Fig.4 and Fig.5 show distributions of the shear stress along the second section as functions of r and the bonding layer parameter. It is seen that for

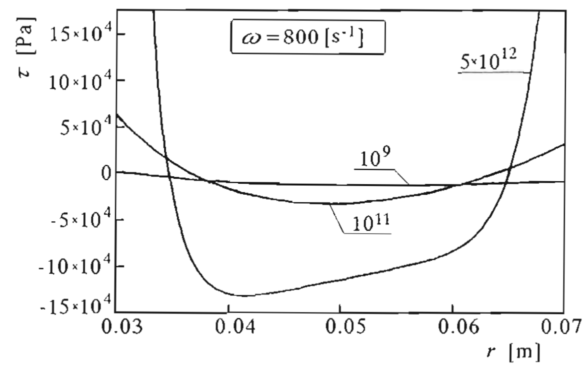

Fig. 5. Shear stress distribution at $\omega = 800\,\mathrm{s}^{-1}$

$\omega = 0.1\,\mathrm{s}^{-1}$ both the present dynamic analysis and the static one are in good agreement. The shear stresses are similar antisymmetric with respect to the center of the piezoelectric actuator. The absolute values of the dynamic shear stresses are larger for larger bonding parameters.

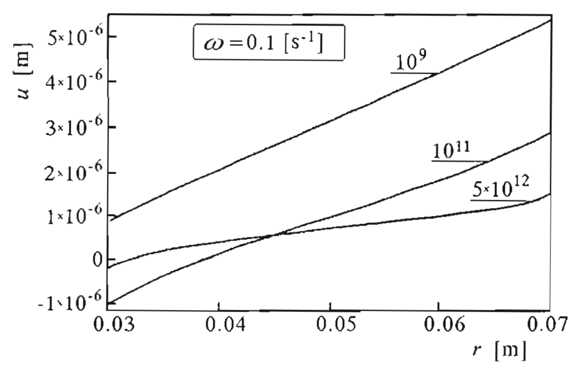

Fig. 6. In-plane actuator dispalcement at $\omega = 0.1\,\mathrm{s}^{-1}$

A comparison between the in-plane displacements u of the actuator is shown in Fig.6 and Fig.7. It is clearly seen that the static results are completely erroneous both qualitatively and quantatively.

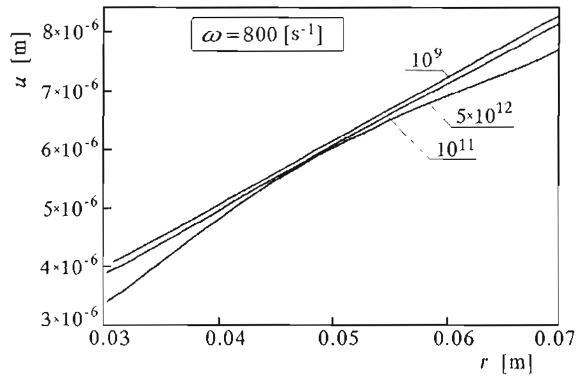

Fig. 7. In-plane actuator dispalcement at $\omega = 800\,\mathrm{s}^{-1}$

6. Conclusions

A dynamic model has been developed on the grounds of which it is possible to predict the response of the annular plate driven by the piezoelectric actuators glued to its lower and upper surface. The actuators were driven by a pair of electric fields with the same amplitute and in opposite phase. The actuators were used to excite steady-state harmonic vibrations in the plate. Spatial distributions of the plate displacements, and shear stresses for different driving frequencies have been shown. The results obtained from the analysis have been compared with particular cases from the past studies, which were based on the assumption of static coupling between the actuator and the plate or assumed the perfect bonding in dynamical analysis.

Results are in good agreement with the previous results concerning the static coupling analysis for a slow excitation $\omega = 0.1\,\mathrm{s}^{-1}$.

At the first resonance of the beam the dynamic behavior strongly depends on the bonding layer parameter G/t_s. Therefore, detailed information regarding the bonding layer parameter is of paramount importance. The omitting of the bonding layer leads to erroneous distributions of the displacements and shear stresses.

Acknowledgement

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Wpływ warstwy kleju na działanie piezoelektrycznego aktuatora płyty pierścieniowej

Streszczenie

Celem tej analitycznej pracy jest przedstawienie ogólnego modelu odpowiedzi płyty pierścieniowej na wymuszenie pochodzące od piezoelektrycznych pierścieniowych aktuatorów. Płyta jest sztywno zamocowana na brzegu wewnętrznym i swobodna na brzegu zewnętrznym. Warunki współpracy aktuatora z płytą uwzględniają właściwości sprężyste bezmasowej warstwy kleju poddawanej ścinaniu. Problem brzegowy opisano równaniami dynamiki płyty i piezoaktuatora, warunkami brzegowymi oraz warunkami zgodności na brzegach pierścieniowych obszarów rozgraniczających swobodne i poddane działaniu aktuatora części płyty. Ten kołowosymetryczny problem brzegowy rozwiązano dla harmonicznego wymuszenia napięciowego elementów piezoelektrycznych. Wyniki pokazują wpływ obecności warstwy kleju o skończonej sztywności na ścinanie, na charakterystyki częstotliwościowe przemieszczeń poprzecznych płyty, naprężeń stycznych w warstwie kleju i przemieszczeń radialnych piezoaktuatora.

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