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MODELLING AND IDENTIFICATION OF CATENARY-PANTOGRAPH SYSTEM

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The purpose of this paper is to present a survey and analysis of the dynamic interaction between the pantograph and a catenary system. The work presented in this paper emphasizes the modelling, simulation of co-operating systems and identification done by experiments on a physical model. For analytical and numerical investigation the overhead electrification system is modelled by visco-elastically connected double Bernoulli-Euler beams. For the pantograph – a model of masses vibrating in the vertical plane at a constant velocity and having four degrees of freedom (DOF) is adopted. The construction of a simple virtual model for computer simulation and some results obtained from investigations carried out on this model are presented. The work contains some numerical examples that illustrate presented theoretical investigations. The analysis of the virtual and two physical experimental models gives the possibility and benefit of their mutual verification and comparison.

Key words: dynamics, catenary, pantograph

1. Introduction

The collection of the current from the overhead equipment is a problem of primary importance for a high speed railway system, and becomes a challenging task for a medium voltage line. There is a great interest in the development of new pantograph-catenary power transmission systems with improved dynamic performance. Therefore, the dynamic interaction between the pantograph and catenary has been studied extensively (Poetsch *et al.*, 1997).

The modelling and simulation of the dynamic behaviour of the catenary-pantograph interaction is an important part when assessing the capability of a current collection system for railway traffic. The capability of a current collection system depends on the interaction between the locomotive, pantograph and the catenary system. The interaction between the pantograph and the overhead wire forms dynamically coupled systems which affect each other through the contact force. To maximize the life span of the pantograph and overhead equipment, and to ensure that the locomotive receives good quality current, the dynamic behaviour of both the pantograph and overhead wire needs to be properly understood and optimized. Therefore, this system has been analyzed by several scientists and engineers in recent years (Drugge *et al.*, 1999). The variety of designs of current collection systems is large. Depending on the country, railway company and type of traffic, different designs of pantograph and catenary systems are used. Pantograph designs based on a symmetric or asymmetric configuration have been evolved. There are three common types of catenary systems: simple, stitched and compound catenary (Roman, 2001).

The large variation in infrastructure characteristics in different countries and railway companies, different designs of the pantograph and catenary, different types of railway vehicles running at different speeds makes it almost impossible to develop a final simulation model of the system.

The work presented in this paper emphasizes the modelling, simulation of co-operating systems and identification done by experiments on a physical model of the system. For analytical and numerical investigations, the overhead electrification system is modelled by two Bernoulli-Euler beams visco-elastically connected with each other and elastically supported through the upper elements (Kumaniecka *et al.*, 2002). For the pantograph a model vibrating in the vertical plane at constant velocity of four degrees of freedom is adopted (Grzyb and Kumaniecka, 2000). The dynamic state of the investigated system is described by a set of coupled partial equations with a complex boundary conditions.

The construction of a simple virtual model for computer simulations and some results obtained from investigations carried out on that model are presented. It is assumed that the supply contact wire gives kinematic excitation to the 4 DOF pantograph model, and that the longitudinal dry friction force is related to the normal load component. The analysis of the virtual and two physical experimental models is the most important task of the present paper because of prediction, verification and comparison, reasons.

The concept of model investigations and description of the testing facility which were designed and built at Department of Mechanical Engineering, Cracow University of Technology, were introduced. The criteria based on the dimensional analysis were adopted to construct physical models of the pantograph and catenary system (Prącik and Furmanik, 2000a). In the paper the dynamic coefficient of dry friction between the supply contact wire and locomotive pantograph is determined in the experimental identification process performed on special test stands. The number of published reports (Fujii and Manabe, 1993; Kumaniecka, 1998; Kumaniecka and Grzyb, 2000) proves that these problems are up-to-date.

2. Simulation of catenary-pantograph dynamics

With respect to the dynamic behaviour, the catenary and pantograph are vital components in current collection systems for trains. The pantograph is mounted on the roof of the train and its frame assembly raises its head assembly into forced contact with the catenary system. A catenary system consists of a contact wire suspended from a catenary wire via an arrangement of droppers. The catenary wire is linked to the supporting structure, while the contact wire is linked to the support via steady arms.

To study the dynamic interaction between the pantograph and catenary it is advantageous to use at first physical and then mathematical simulation models. The research has been curried out to understand and improve the performance of the catenary-pantograph system by mathematical modelling for many years (Bogacz, 1983). During the last forty years, many studies of the catenary, pantograph, and pantograph-catenary systems dynamics have been carried out. Many different, more and more complex analytical models of the pantograph-catenary interaction have been considered and developed. Simplified analytical models of such systems have been used in order to determine critical speeds and study the influence of principal system parameters (Bogacz, 1983; Bogacz and Szolc, 1993).

The catenary has been modelled using different approaches. One degree of freedom mass spring-damper system models with varying stiffness and mass, and string models with an elastic foundation (Kumaniecka and Nizioł, 2000), and recently two-beam models (Grzyb *et al.*, 2001; Kumaniecka *et al.*, 2002) have been used. Models with the two-dimensional representation of the catenary are most common (Kumaniecka *et al.*, 2000).

Some models for simulation of the catenary dynamical behaviour, developed by the authors of the presented paper, are illustrated in Fig. 1 and Fig. 2.

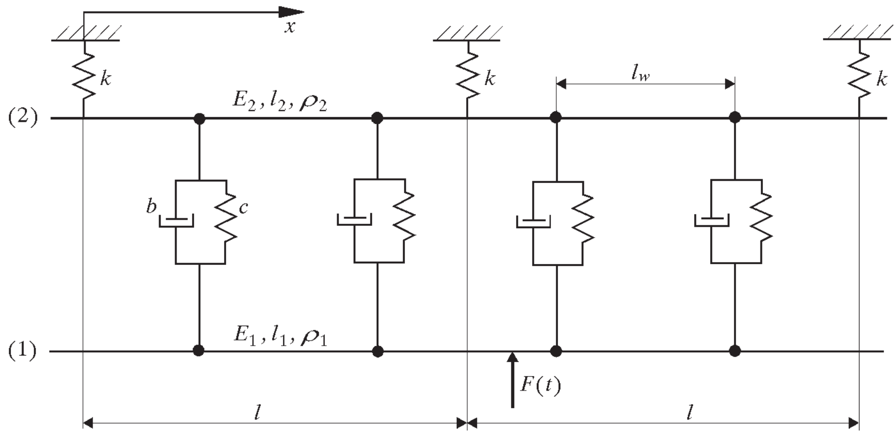

Fig. 1. The catenary model: two visco-elastically connected Bernoulli-Euler beams with the upper finite elastically supported

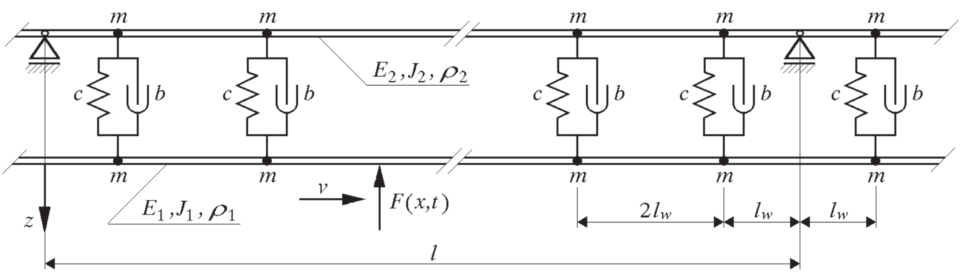

Fig. 2. The catenary model: two visco-elastically connected Bernoulli-Euler beams with concentrated masses. The upper beam rigidly supported on the foundation

In the Fig. 3, the physical model of the catenary type WBL 85 3kV, which is common in use not only in many European countries but in Asia and Africa as well, is presented. This type of pantograph will be mounted in Polish locomotives EU11 and EU43. This model refers to a real system and it has been examined by engineers from Schunk Wien G.m.b.H. (Prącik and Furmanik, 2000a).

In the present paper the system under investigation consists of two infinite visco-elastically connected Bernoulli-Euler beams, with one elastically supported on a rigid foundation and the other suspended via droppers. The model employed in the analysis is shown in Fig. 1. For the pantograph a 4 DOF model of vibrating masses in the vertical plane and moving at a constant velocity is assumed, see Fig. 3.

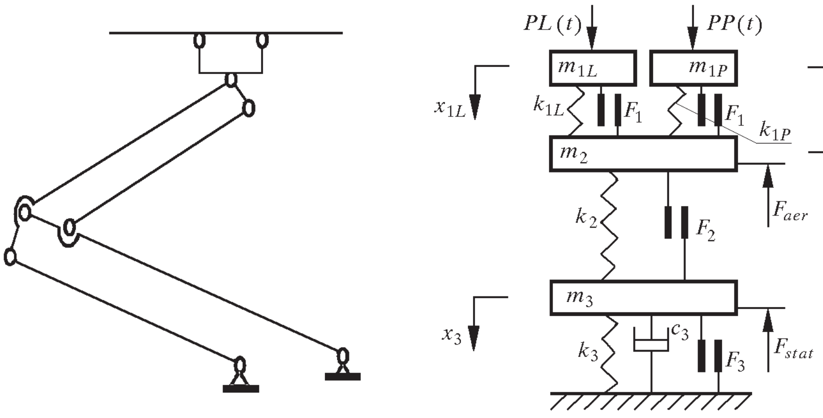

Fig. 3. Reduced physical model of a real pantograph system

The analysis of small transverse vibrations of the adopted physical model of the system was presented by Kumaniecka *et al.* (2002).

Several simplifying assumptions have been necessary to limit the scope of the problem under consideration:

- considerations are restricted to the OXY plane only,
- transverse vibrations of the beams in the vertical plane are small,
- the beams are prismatic, homogeneous, vertically loaded,
- the upper beam is multi-span and is fixed by linear springs of the stiffness k at points which are spaced by the distance l, (catenary wire linked to the supporting structure),
- the lower beam is of infinite length and linked to the upper beam via elastic elements of the stiffness c, (contact wire suspended from the catenary wire via an arrangement of droppers),
- internal damping effects in the beams are neglected,
- masses of the visco-elastic linking elements are neglected,
- the interaction between the pantograph and contact wire is limited to a single force excitation in an additive form (constant component, harmonic variable and delayed variable of the load acting from the wire onto the pantograph),
- the contact between the beam and oscillator exists all the time,
- gravitational effects are neglected.

All considerations are done in a reference system OXY. The x co-ordinate is measured along the upper beam from the point $x = 0$. The functions $w_1(x,t)$ and $w_2(x,t)$ describe the lower and upper beam transverse displacements, respectively.

For the pantograph a 4 DOF model of masses vibrating in the vertical plane and moving at a constant velocity is adopted (Prącik and Furmanik, 2000a). This constitutive a discrete non-linear model is presented in Fig. 3.

It is assumed that the supply contact wire continuously gives a kinematic excitation to the 4 DOF pantograph model, and that the longitudinal dry friction force is related to the normal load component.

3. Analysis of equations of motion of the catenary-pantograph system

The mathematical model of the catenary system given in Section 2 was obtained for small transversal vibration of the beams.

Motion of the catenary in the vertical plane is governed by equations

$$
\begin{aligned}
&E_1 J_1 \frac{\partial^4 w_1}{\partial x^4} - N_1 \frac{\partial^2 w_1}{\partial x^2} + \varrho_1 \frac{\partial^2 w_1}{\partial t^2} - p + p_F = 0\\
&E_2 J_2 \frac{\partial^4 w_2}{\partial x^4} - N_2 \frac{\partial^2 w_2}{\partial x^2} + \varrho_2 \frac{\partial^2 w_2}{\partial t^2} + p + p_p = 0
\end{aligned}
\tag{3.1}
$$

where the following notation is used

E_1, E_2	–	Young's modulus of the lower and upper beam, respectively
J_i	–	cross-sectional moment of inertia, $(i = 1, 2)$
N_i	–	tensile force in the beam
ϱ_i	–	mass density
$w_i(x,t)$	–	transverse displacements
x	–	spatial coordinate measured along the nondeformed axis of the beam
t	–	time.

The loads $p(x,t)$ acting on the beams and caused by internal forces in springs and damping elements are treated as continuous and can be expressed in the form

$$
p(x,t) = \sum_{(n)} \{c[w_2(x,t) - w_1(x,t)] + b[\dot{w}_2(x,t) - \dot{w}_1(x,t)]\}\delta(x - x_n) \tag{3.2}
$$

where

c – coefficient of the spring elasticity
b – damping coefficient
x_n – coordinates of the droppers spacing, $x_n = (2s-1)l_w$, $s \in N$
l_w – distance between the droppers
δ – Dirac's function.

The interaction force between the pantograph and contact wire p_F can be described by the term

$$p_F(x,t) = F(t)\delta(x - vt) \tag{3.3}$$

The reaction force p_p that comes from supports acting at points x_j on the upper beam is of discrete character, but can be treated as disributed and written in the form

$$p_p(x,t) = \sum_j kw_2(x,t)\delta(x - x_j) \tag{3.4}$$

where

k – coefficient of the springs elasticity
x_j – coordinates of the supports spacing.

The boundary and initial conditions adopted for numerical simulation have been based on the assumed vibration mode of the linear system (data from the identification research).

The mathematical model for a physical model of the pantograph, adopted in Section 2, was discussed in detail and presented by Prącik and Furmanik (2000b). The structure of the simulation model has been based on a formal notation of motion in the form of ordinary differential equations, and the simulation software package VisSim has been applied. Analytical approach and some results obtained from the numerical investigations carried out on the simulation model were already presented by Kumaniecka *et al.* (2002). On the basis of the given set of equations a simulating program, applying the package VisSim Analyze ver 2.0, has been built. The block scheme is presented in Fig. 4. In Fig. 5 an example of sub-blocks (up to the 2nd level depth) integrating the equation of motion of the mass m_{1P} is shown as well.

Numerical simulations have been carried out for different data sets. The calculations (results of which are presented in Fig. 4) were performed for the following parameters of the system: (construction of the pantograph WBL 85-3 kV/PKP): $m_{1L} = m_{2P} = 7.93\,\mathrm{kg}$, $m_2 = 8.73\,\mathrm{kg}$, $m_3 = 10.15\,\mathrm{kg}$, $F_1 = 2.0\,\mathrm{N}$, $F_{aer} = 0$, $F_2 = F_3 = 2.5\,\mathrm{N}$, $PP(t) = 10 + 50\sin[14\pi(t - 0.02)]$, $PL(t) = 10 + 50\sin(14\pi t)$, $c_3 = 60\,\mathrm{Ns/m}$, $F_{stat} = 110\,\mathrm{N}$.

Some results were presented in the paper by Kumaniecka *et al.* (2002).

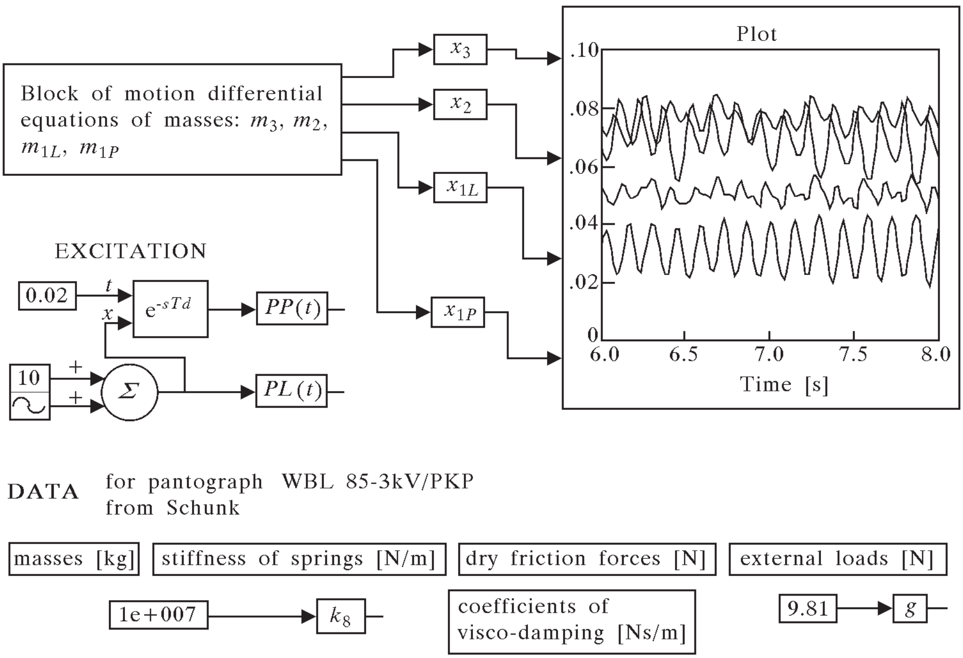

Fig. 4. Scheme of the block for simulation of motion

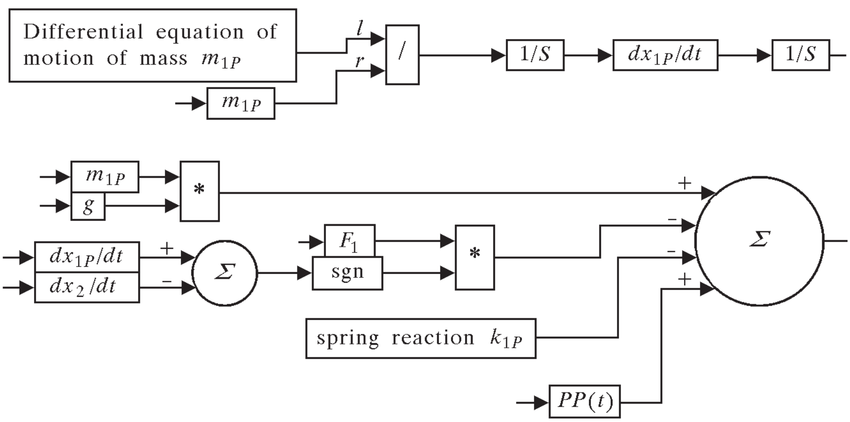

Fig. 5. Example of simulation results; displacement of a particular mass

4. Experiments on the physical model

The criteria which are determined on the base of a dimensional analysis method make it possible to construct physical models of the pantograph and catenary system (Prącik and Furmanik, 2000b). The purpose of such modelling is to preserve mechanical similarity to the real structure of the considered system. Dimensional analysis allows the estimation of the influence of chosen parameters on the investigated quantity and the determination of mechanical similarity criteria.

The testing facilities for laboratory experiments have been designed and built at the Department of Mechanical Engineering, Cracow University of Technology. The stand is situated in a long corridor (about 50 m), see Fig. 6.

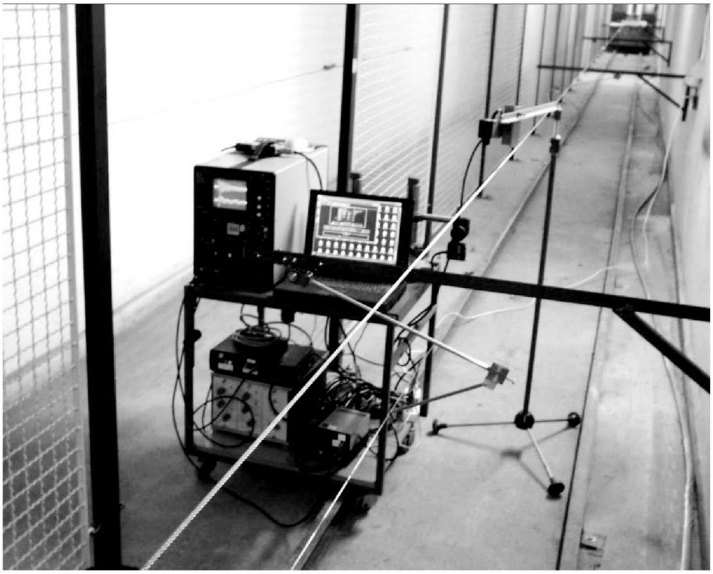

Fig. 6. Facility for testing natural frequencies and modes of vibrations of the catenary system of a physical model

The catenary interval (30 m long) is divided on sub-intervals 5 m each – where suspension elements are embedded. The suspension consists of a cantilever, at the end of which double-arm rods connected with articulated joints are mounted. The main carrying rope is stretched at the top of the whole catenary system. The supply contact wire is kept by lifting grips assembled to the edges of rocking arms of suspensions. This flexible construction is a 3 DOF system, i.e. displacements are possible in 3 directions. There are also droppers (connecting the main carrying rope with the contact wire) distributed between the suspensions. A carefully designed loading system has been adapted to make control possible and stabilize the tensile load for the carrying rope of the catenary system. An auxiliary system has been applied to achieve required levels of the supply wire tensions. An instrumentation for non-contact (laser) measurements of displacements of the vibrating supply wire and other elements of the catenary system (in the vertical and horizontal directions) has been used. The components of contact forces and characteristic vibration quantities can be measured on the moving pantograph model using strain gauges, accelerometers and other parts of the mobile instrumentation.

The pantograph model is assembled on a rail car, which is driven by a cable system with an electric engine powered and controlled via microverter (Prącik, 2001), see Fig. 7.

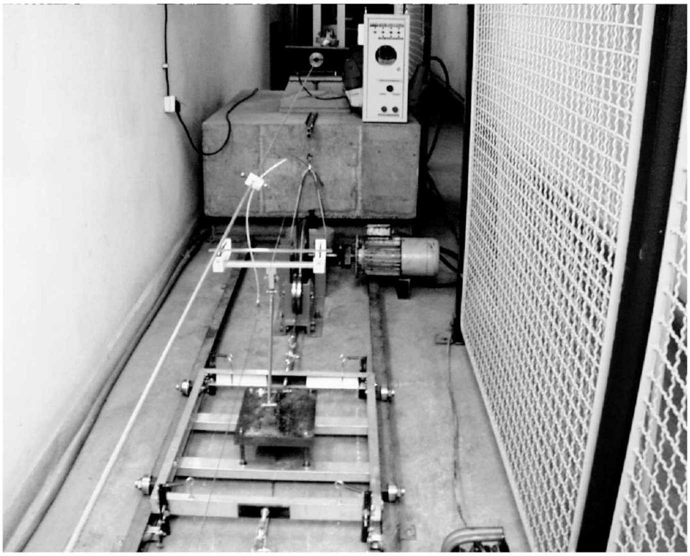

Fig. 7. A rail-car with the model of pantograph; in the background the elements of the driving system: guiding wheels, cable rewinding, electric engine can be seen

The plan of experiments included:

- modal analysis of the experimental model of the catenary system (measurements of the natural frequencies and modes of the contact wire vibration for different excitations; impulse response functions using an impact hammer),
- analysis of dynamical loads acting on parts of the pantograph model co-operating with the model of a catenary system (measurements of

forces between the slider-bar of the pantograph and the contact wire for different conditions on the stand; measurements of vibration parameters of particular pantograph elements),

- identification of friction (investigation of friction function treated as a function of load and relative slip velocity for different pairs of materials; damping measurements of chosen elements of the catenary system). The main advantage of the physical model is the possibility of taking into account real effects, for instance the fact that the supply wire forms a zig-zag pattern and that the motion can be analysed in 3D space, not only in plane,
- application of various stiffnesses of the physical catenary model (however particular results are omitted in the paper).

Some selected results of the experiments can be illustrated by diagrams in Fig. 8 - Fig. 10.

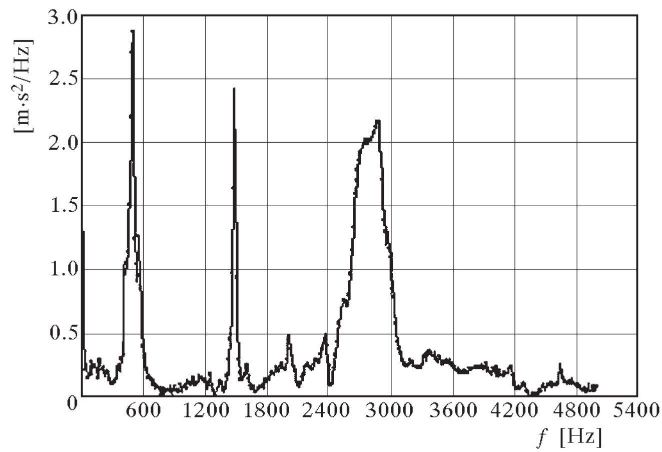

Fig. 8. Spectral density of the free horizontal vibration acceleration of the pantograph model beam (impact horizontal excitation, piezoelectric accelerometer)

The experimental investigations allow one to identify the parameters of the simulation model built accordingly to the physical model.

The results of the experimental research carried out for more general purposes, not just for the parametric identification, allow one to verify also the assumptions adopted in the simulation model. These assumptions refer, among others, to the effect of the transverse vibrations assosiated with the zig-zag motion – conditions bounded for the plane model. On the other hand, the effect of the zig-zag motion could be investigated in measurements carried out on the physical model.

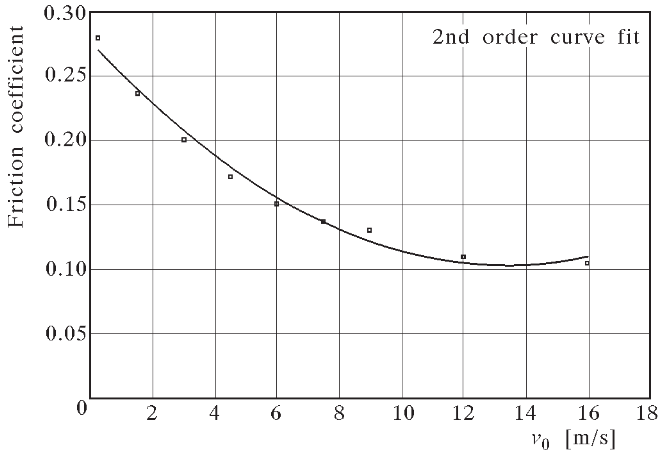

Fig. 9. Exemplary results of investigations on dynamic friction coefficient (in function to relative slip velocity)

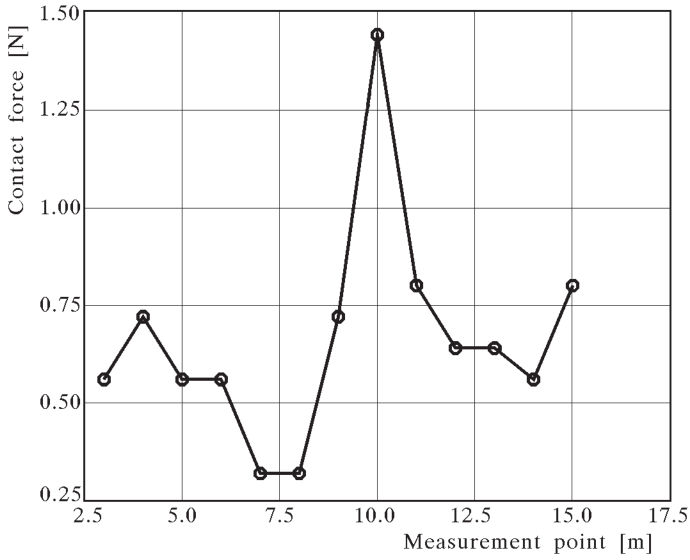

Fig. 10. Exemplary results of the contact force measurements (mean distribution along the wire)

5. Final conclusions

On the basis of theoretical investigations, numerical simulations and experiments, the following conclusions can be drawn.

- Parallel analysis of the results of investigation carried out on the physical, experimental and simulation models enable their mutual verification.
- Dimensional analysis allows the estimation of the influence of chosen parameters on the investigated quantity and determination of mechanical similarity criteria.
- During the investigations carried out on the simulation model, the convergence of numerical results by changing the integration step and the order of the applied Runge-Kutta method has been tested.
- Analysis of the simulation results indicates that when the force excitation with a dominant harmonic component acts on the pantograph, then the response of the system model is not harmonic.
- Analysis of the measurement results and calculations referring to the fexibility of the catenary linear model points out cyclic variability of the local flexibility measured along the wire length, and indicates nonlinearity when the level of initial loading changes (vertical interaction forces). This cyclic variability of the flexibility can be explained if the periodic character of the suspension structure is taken into account. The maximum of the nonlinear flexibility as a function of loading appears at the support points of the catenary wire.
- Modal analysis of the catenary wire suspension system proves the existence of a wide spectrum of free transverse vibration frequencies of the contact wire, in a range up to 40 Hz.
- The measurements of dynamic vibration and interaction force between the pantograph and contact wire were taken for different initial loads and velocity of the rail car ($\varDelta v_0 = 0$ - $2\,\mathrm{m/s}$ within time of $0.8\,\mathrm{s}$). As a result, we can state that the loss of contact between the pantograph and catenary is possible.
- The identification investigations of the friction model of cooperating materials: pantograph – contact wire, were carried out on a special experimental stand. Measurements for the relative slip velocity of 0 - 20 m/s were possible. As a result, characteristics of dynamic friction coefficient versus relative velocity were obtained for different levels of the contact force.

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Modelowanie i identyfikacja układu lina trakcyjna-pantograf

Streszczenie

Celem pracy jest analiza dynamicznej współpracy pantografu z liną trakcyjną. Rozważane są zagadnienia związane z modelowaniem i symulacją zjawisk zachodzących w układach zasilania kolejowej trakcji elektrycznej, a także identyfikacją na fizycznym modelu doświadczalnym. W pracy przyjęto model sieci trakcyjnej złożony z dwóch równolegle umieszczonych belek Bernoulliego-Eulera. Odbierak prądu, poruszający się ze stałą prędkością, modelowany jest nieliniowym cztero-masowym układem wykonującym ruch w płaszczyżnie pionowej. Przedstawiono konstrukcję modelu wirtualnego i wyniki symulacji przeprowadzonych na tym modelu. Analiza wyników eksperymentów prowadzonych na fizycznym modelu doświadczalnym i na modelach symulacyjnych pozwala na ich wzajemną weryfikację.

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