JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
42, 2, pp. 285-294, Warsaw 2004

INFLUENCE OF THE MATERIAL SENSITIVITY FACTOR ON THE STRESS RATIO FOR DIFFERENT SPECIMENS GEOMETRIES AND MATERIALS UNDER BENDING

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In the paper, the influence of mean loads on the low cycle fatigue for notched specimens of different shapes of the cross section is analysed. The specimens are characterized by a similar value of the stress concentration factor ($K_t = 2.46$ for round specimens and $K_t = 3.27$ for specimens of the rectangular cross-section). A mathematical model describing the limiting stress plane under fatigue bending is applied, taking into account the influence of mean stresses. The model has been verified by 18G2A steel and PA6 aluminium alloy specimens via experimental test results under cyclic bending for stress ratios $R = -1$, -0.5, 0. The relationship between the stress amplitude and the stress mean value taking into account the number of cycles N before failure is described by a linear model, for which good agreement of the calculations with the experimental test results is achieved.

Key words: stress ratio, numbers of cycles, bending

Notations

R	–	stress ratio
σ_a	–	amplitude of stress
σ_m	–	mean stress
N	–	number of cycles to failure
σ_y	–	yield stress
σ_u	–	ultimate stress
E	–	Young's modulus
ν	–	Poisson's ratio

1. Introduction

Additional static loadings occurring in structure elements strongly influence fatigue behaviour of these elements (Glinka *et al.*, 1995). From the known literature it appears that material reactions to the occurring additional mean loading can be different (Stephens *et al.*, 2001). Morrow (1968) proposed a widely used relation including the influence of the mean stress value and proved strong influence of this stress on the fatigue life. Application of the Morrow equation to some materials was discussed by Manson (1975). The author proved that the formula was useful for fatigue tests with mean stress values. Description of the influence of the mean loading often uses a coefficient expressing the material sensitivity to the cycle asymmetry. That coefficient is applied for formulation of a simplified Haigh graph, showing the dependence between the stress amplitude and the cycle mean value (Stephens *et al.*, 2001). Its value can be determined from tests. Serensen (1975) joined the safety coefficient related to the mean stresses with the coefficient of sensitivity to the cycle asymmetry. Bergman and Seeger (1979) introduced an additional coefficient including material sensitivity to the influence of the mean stress into the Smith-Watson-Topper energy equation. Ellyin and Kujawski (1993) modified the criterion presented by Ellyin for the critical energy of non-dilatational strain. In the paper, the coefficient of sensitivity to the cycle asymmetry was introduced. The criterion was verified for A516 steel in the form of cylindrical thin-walled specimens subjected to a combination of tension with internal and external pressure applied to the specimen walls. The coefficient of sensitivity to the cycle asymmetry was also included by Gołoś and Esthevi (1997).

In this paper, the presented criterion was verified by fatigue test results for St5 steel. Cylindrical thin-walled specimens were tested under tension with internal and external pressure acting on the specimen walls. The tests performed according to this criterion gave satisfactory results as compared with the experimental results. The coefficient of material sensitivity presented by Haigh is right for the high-cycle fatigue limit and it takes a constant value. From the results obtained by Pawliczek and Rozumek (2003) we can draw a conclusion that the material sensitivity to the cycle asymmetry depends not only on the material. It is also dependent on the number of cycles to failure for both low and high cycle ranges.

The aim of this paper is to determine the permissible fatigue loading of specimens made of 18G2A steel and PA6 aluminium alloy for different geometries of cross-sections and stress ratios.

2. A mathematical model of the limiting stress surface

In the case of a cyclic loading with participation of a mean stress, the limiting stress surface is presented in the coordinate system $\sigma_a - \sigma_m - N$. Pawliczek (2003) proposed to describe the mean stress value with the use of a function of the change in the factor of the material sensitivity to the stress ratio depending on the number of cycles N to failure. This function can be written as

$$\psi(N) = \gamma N^{\lambda} \tag{2.1}$$

where γ, λ are parameters which are determined from fatigue tests under alternating loading $(R = -1)$ and pulsating loading $(R = 0)$, N denotes the number of cycles to failure.

Values of the factor of the material sensitivity to the stress ratio $\psi(N)$ for a given N can be determined from experiments carried out according to the following formula

$$\psi(N) = \frac{2\sigma_{a_{-1}}(N) - \sigma_{a_0}(N)}{\sigma_{a_0}(N)} \tag{2.2}$$

where $\sigma_{a_{-1}}(N)$ is the stress amplitude for the fatigue life level corresponding to N cycles for loadings with the stress ratio $R = -1$ (symmetric cycles), σ_{a_0} is the maximum stress for this level and loadings with the stress ratio $R = 0$ (pulsating cycles).

Assuming a linear relationship between the stress amplitude and the mean stress $\sigma_a = \sigma_{a_{-1}}(N) - \psi(N)\sigma_m$ and using Eq. (2.1) for description of the factor of the material sensitivity to the stress ratio together with the number of cycles to failure, we obtain a relationship for calculation of the stress amplitude

$$\sigma_a = \sigma_{a_{-1}}(N) - \gamma N^{\lambda}\sigma_m \tag{2.3}$$

The Wõhler equation for symmetric loadings $(R = -1)$ takes the following form

$$\big(\sigma_{a_{-1}}(N)\big)^k N = \sigma_{af}^k N_0 \tag{2.4}$$

where σ_{af} is the fatigue limit, k – coefficient of the regression equation, N_0 – limiting number of cycles.

Substituting expression (2.4) to relationship (2.3), we obtain

$$\sigma_a(\sigma_m, N) = \Big(\frac{\sigma_{af}^k N_0}{N}\Big)^{\frac{1}{k}} - \gamma N^{\lambda}\sigma_m \tag{2.5}$$

Equation (2.5) expresses a mathematical model of the limiting stress surface determined from the fatigue test results obtained for symmetric and pulsating courses on the assumption that the influence of the mean value on the stress amplitude takes the form of a linear function.

3. Materials and the test procedure

The subject of the investigations are construction steel 18G2A and PA6 aluminium alloy. Round specimens were cut off from a drawn bar, flat specimens were cut off from a sheet, according to the rolling direction. Their chemical compositions and mechanical properties are given in Table 1.

Table 1. Characteristics of 18G2A steel and PA6 aluminium alloy

Material	Chemical compositions [%]			Mechanical properties
18G2A	0.20C	1.49Mn	0.33Si	$\sigma_y = 357$ MPa, $\sigma_u = 535$ MPa,
	0.023P	0.024S	0.01Cr	$E = 2.10 \cdot 10^5$ MPa, $\nu = 0.30$
	0.01Ni	0.035Cu		
PA6	4.15Cu	0.69Mg	0.45Si	$\sigma_y = 395$ MPa, $\sigma_u = 545$ MPa,
	0.65Mn	0.7Fe	0.2Ti	$E = 7.2 \cdot 10^4$ MPa, $\nu = 0.32$
	0.5Zn	0.1Cr		

The fatigue tests were performed at the fatigue test stand MZGS-100 (Achtelik and Jamroz, 1982) enabling realization of cyclically variable stress courses with participation of a mean stress. The tests included sinusoidally variable bending with participation of a loading mean value. The tests were done for the stress ratios $R = -1$, -0.5, 0, changing the amplitude of the moment M_a and the mean values of the moment M_m for bending. The tests were performed for round and flat notched specimens (Fig. 1). The notches were made by machining (turning and milling), and next the specimen surfaces were ground. The theoretical stress concentration factor in the round specimens $K_t = 2.46$ and flat specimens $K_t = 3.27$, were estimated with use of the model by Thum *et al.* (1960).

The regression equations for specimens made of 18G2A steel and PA6 aluminium alloy subjected to cyclic bending for various stress ratios are shown in Table 2.

From the performed tests for $R = -1$ and $R = 0$, Eq. (2.1) was determined under bending for 18G2A steel and round smooth specimens $\psi(N) = 3.124N^{-0.162}$, for round notched specimens $\psi(N) = 1.363N^{-0.105}$, for plane notched specimens $\psi(N) = 0.046N^{-0.197}$ and for PA6 aluminium alloy and plane notched specimens $\psi(N) = 0.238N^{-0.076}$. Different values of

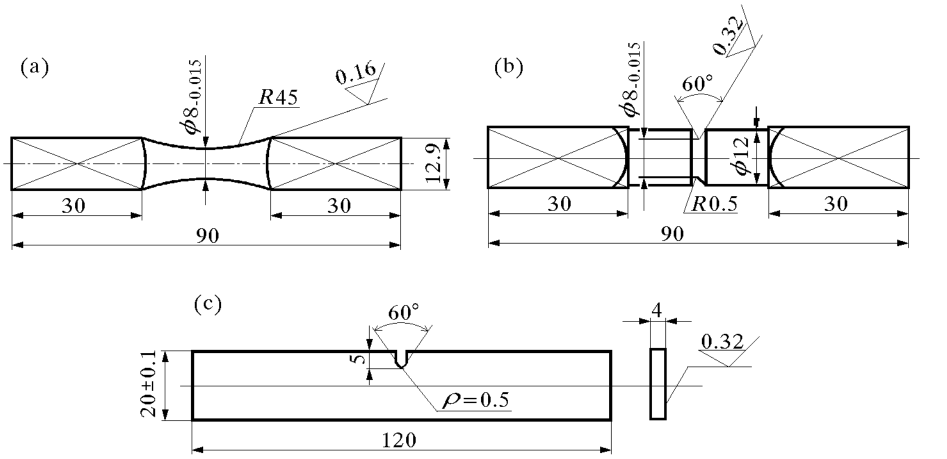

Fig. 1. Specimens subjected to tests, (a) round, (b) round notched, (c) flat notched

the coefficients of the equation $\psi(N)$ prove that the influence of the cross section geometry and the material on the fatigue process intensity is significant. The correlation coefficients r for the equations presented in Table 2 were calculated with the least square method. In all considered cases, the correlation coefficients take great values ranging from 0.97 to 0.99, which bespeaks a significant correlation of the test results.

Table 2. Parameters of the regression equation for bending in round and flat specimens

R	Round specimens – 18G2A	Round notched specimens – 18G2A
-1	$\log N = 23.93 - 7.19 \log \sigma_a$	$\log N = 19.94 - 6.02 \log \sigma_a$
-0.5	$\log N = 23.71 - 7.40 \log \sigma_a$	$\log N = 22.46 - 7.34 \log \sigma_a$
0	$\log N = 31.40 - 10.73 \log \sigma_a$	$\log N = 21.86 - 7.23 \log \sigma_a$
R	Flat notched specimens – 18G2A	Flat notched specimens – PA6
-1	$\log N = 19.96 - 6.70 \log \sigma_a$	$\log N = 17.33 - 6.05 \log \sigma_a$
-0.5	$\log N = 14.19 - 4.27 \log \sigma_a$	$\log N = 14.37 - 4.79 \log \sigma_a$
0	$\log N = 14.35 - 4.52 \log \sigma_a$	$\log N = 14.44 - 5.13 \log \sigma_a$

4. Experimental results and discussion

Figures 2 and 3 show the test results in the form of a surface of boundary stresses for the considered materials and different cross section geometries, and for smooth and notched specimens, obtained from the regression equations presented in Table 2. Moreover, Fig. 2 and Fig. 3 show theoretical contours of the stress amplitudes σ_a in the plane of the material life N and the stress ratio R, calculated from Eq. (2.5). Figures 2 and 3 also show the relative error distribution determined in an analytical way in relation to the experimental amplitude. In the case of the round smooth specimens (Fig. 2a) for $N = 10^5$ cycles, a change in the stress ratio from $R = -1$ to $R = 0$ causes a large decrease in the permissible stress amplitude (from 440 to 280 MPa). The stress amplitude decreases as R increases and is almost uniform in the whole tested range. However, for $N > 1.5 \cdot 10^6$ cycles, a change in the stress ratio from -1 to -0.5 causes large drops of the permissible stress amplitudes, and any further increase of the mean stress value ($R = -0.5$ to 0) hardly influences the stress amplitudes. Analysing the round notched specimens (Fig. 2b), we can observe that for $N = 10^5$ cycles, the change of the stress ratio from $R = -1$ to $R = 0$ takes place similarly as in the case of the specimens shown in Fig. 2a.

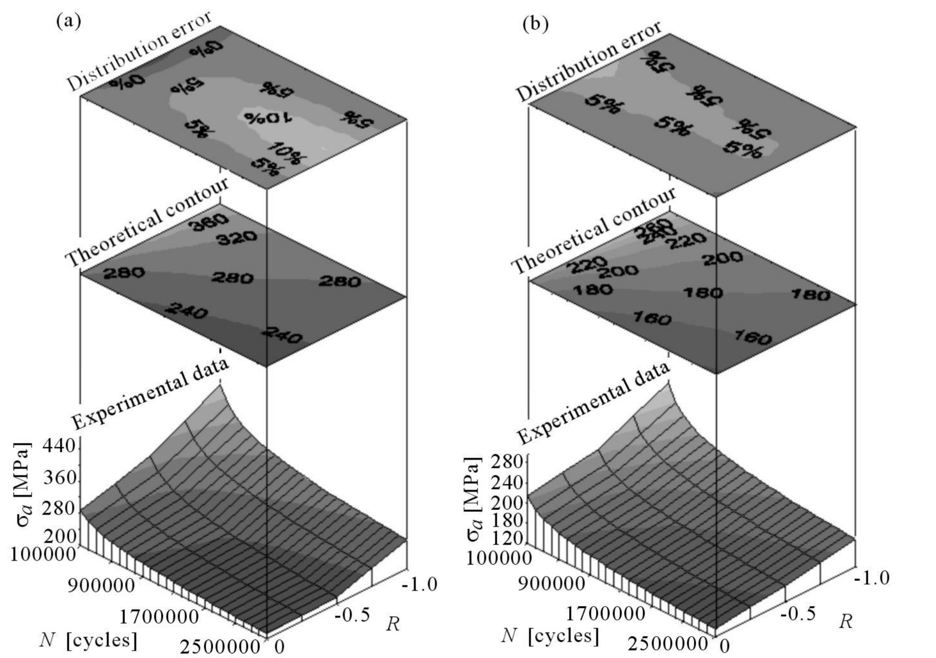

Fig. 2. The limiting stress surface under bending determined from tests, the theoretical contour and error distribution, (a) round specimens – 18G2A, (b) round notched specimens – 18G2A

Lower stress amplitudes are caused by the notch (decrease of the stress amplitude from 280 down to 215 MPa). For $N > 1.5 \cdot 10^6$ cycles, a change in the stress ratio from -1 to 0 causes uniform linear drops of the stress amplitude. Further results obtained for flat notched specimens made of 18G2A steel (Fig. 3a) had the maximum stress amplitudes of 170 MPa. It can be observed that for $N = 10^5$ cycles a change in the stress ratio from $R = -1$ to $R = -0.5$ causes a small reduction of the stress amplitude, and for R in the range of -0.5 to 0, we observe large changes of the stress amplitude from about 167 down to 115 MPa. For $N > 1.5 \cdot 10^6$ cycles, a change in the stress ratio from -1 to -0.5 and from -0.5 to 0 causes uniform linear drops of the stress amplitude.

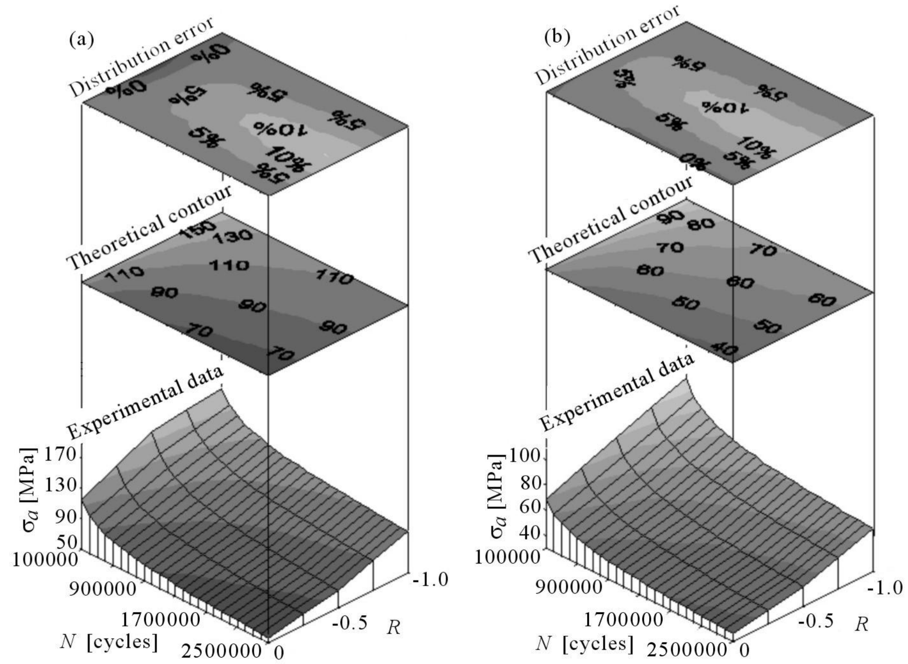

Fig. 3. The limiting stress surface under bending determined from tests, the theoretical contour and error distribution, (a) flat notched specimens – 18G2A, (b) flat notched specimens – PA6

In the case presented in Fig. 3b, we can find that for $N = 10^5$ cycles a change in the stress ratio from $R = -1$ to $R = 0$ causes linear reduction of the stress amplitude from 100 down to 67 MPa. Similar behaviour is observed for $N > 1.5 \cdot 10^6$ cycles. From the analysis of the experimental results it also appears that for flat notched specimens (Fig. 3) compared with round smooth

and notched specimens (Fig. 2) we can observe smaller differences of the stress amplitudes passing from $N = 10^5$ to $N = 2.5 \cdot 10^6$ cycles. From the fatigue test results for 18G2A steel and PA6 aluminium alloy, if the boundary stress surface is assumed according to Eq. (2.5), the equations of that surface have been determined for the considered cases:
— for round specimens (18G2A)

$$\sigma_a = \Big(\frac{N}{10^{23.93}}\Big)^{\frac{-1}{7.19}} - 3.124N^{-0.162}\sigma_m \tag{4.1}$$

— for round notched specimens (18G2A)

$$\sigma_a = \Big(\frac{N}{10^{19.93}}\Big)^{\frac{-1}{6.02}} - 1.363N^{-0.105}\sigma_m \tag{4.2}$$

— for flat notched specimens (18G2A)

$$\sigma_a = \Big(\frac{N}{10^{19.96}}\Big)^{\frac{-1}{6.70}} - 0.046N^{0.197}\sigma_m \tag{4.3}$$

— for flat notched specimens (PA6)

$$\sigma_a = \Big(\frac{N}{10^{17.33}}\Big)^{\frac{-1}{6.05}} - 0.238N^{-0.076}\sigma_m \tag{4.4}$$

From the above equations, it results that the specimen geometry and the kind of the material strongly influence the life of the tested element. The greatest changes of the regression equation coefficient m are observed for round smooth specimens. In the other cases, the coefficients m have similar values (surface equation – Eq. (2.5)). Also, the coefficient of the material sensitivity to the cycle asymmetry changed – the highest fatigue parameters γ, λ (Eq. 2.5) are observed for round smooth specimens and the least ones – for flat notched specimens (18G2A steel). The maximum relative error (Fig. 2a and Fig 3) was 10% under $N > 1.5 \cdot 10^6$ cycles, and for round notched specimens (Fig. 2b) that error was 5% for $N \approx 10^5$ cycles.

5. Conclusion

From the analysis of the obtained test results we can draw the following conclusions:

- The proposed mathematical relation, (2.5), well describes boundary stress surfaces under bending where the maximum error does not exceed 10% for both kinds of specimen geometries and materials.

- A change in the stress ratio from $R = -1$ to $R = -0.5$ for 10^5 cycles for round specimens of 18G2A steel and flat specimens of PA6 aluminium alloy causes a significant drop of the stress amplitude, but for flat specimens of 18G2A steel a certain insensitivity can be observed.

- In the range from $R = -0.5$ to 0 and $N = 10^5$ cycles for round specimens, we observe a small drop of the stress amplitude, but in the case of flat specimens greater drops of the permissible stress amplitudes are observed.

- For the fatigue life $2.5 \cdot 10^6$ cycles and both round and flat specimens similar behaviour of the material can be observed.

- Passing from the fatigue life 10^5 cycles to $2.5 \cdot 10^6$, we observe significantly smaller differences in the stress amplitude reduction in flat specimens as compared with the round ones.

Acknowledgements

This work was supported by the Commission of the European Community under the FP5, GROWTH Programme, contract No. G1MA-CT-2002-04058 (CESTI).

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Wpływ współczynnika wrażliwości materiału na asymetrię cyklu dla różnych geometrii i materiałów próbek przy zginaniu

Streszczenie

W pracy przeanalizowano wpływ obciążeń średnich przy niskocyklowym zmęczeniu dla próbek z karbem o różnych kształtach przekroju poprzecznego. Próbki cechowały się zbliżoną wartością współczynnika kształtu ($K_t = 2.46$ dla próbek okrągłych i $K_t = 3.27$ dla próbek o przekroju prostokątnym). Zastosowano model matematyczny opisujący płaszczyznę naprężeń granicznych przy cyklicznie zmiennym zginaniu z uwzględnieniem wpływu naprężeń średnich. Model zweryfikowano wynikami badań eksperymentalnych próbek ze stali 18G2A i stopu aluminium PA6 w warunkach cyklicznego zginania dla przebiegów o współczynniku asymetrii cyklu $R = -1; -0,5; 0$. Zależność amplitudy naprężeń od wartości średniej naprężeń z uwzględnieniem liczby cykli N do zniszczenia opisano linowym modelem, dla którego uzyskano dobrą zgodność obliczeń z wynikami badań eksperymentalnych.

Manuscript received November 13, 2003; accepted for print January 13, 2004