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STRUCTURAL RELIABILITY – FUZZY SETS THEORY APPROACH

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In the paper two kinds of uncertainty: randomness and imprecision are proposed to be considered in a structure description. Imprecise experts' opinions can be described using fuzzy numbers. As a result, the reliability analysis of a structure can be based on the limit state function with fuzzy parameters. As a consequence, the structural failure or survival can be treated as fuzzy events. The probabilities of these fuzzy events can be the upper and the lower estimations of the structural reliability. They can be achieved using well-known reliability methods (e.g. Hasofer-Lind index and Monte Carlo simulations). They can be used as a base for the calibration of partial safety factors in design codes.

Key words: probability theory, fuzzy sets theory, reliability of structure

1. Introduction

There are two kinds of uncertainty associated with civil engineering (Blockley, 1980) – and all engineering activities – randomness and imprecision (Gasparski, 1988).

The randomness is the unpredictability of events. The randomness is described by the probability distributions based on the observation of the event frequency. It is a task of the probability theory.

The imprecision is a lack of certainty of experts' assessments. An expert of a given domain formulates his opinion arbitrary based on his knowledge, experience and intuition, using words like "big", "small", "medium" instead of precise numbers. Modelling and processing of imprecise data is a task of the fuzzy set theory.

As it is known, according to the fuzzy set theory (Zadeh, 1965, 1978) the two-value logic is extended to the multi-value logic. As a result, the conventional notion of a set A (a crisp set) is extended to a fuzzy set $\underline{A}$ by the extension of the two-value characteristic function, Eq. (1.1), to the multi-value membership function, Eq. (1.2). The notion of a real number is extended to a fuzzy number which is defined as the fuzzy set satisfying several conditions (Piegat, 1999). Fuzzy numbers can be processed using the rule of extension (Kacprzyk, 1986)

- $c_A(x): \quad R \rightarrow \{0,1\}$

$$\forall x \in R \qquad c_A(x) = \begin{cases} 1 & \text{for} \quad x \in A \\ 0 & \text{for} \quad x \notin A \end{cases} \tag{1.1}$$

- $\chi_{\underline{A}}(x): \quad R \rightarrow < 0,1 >$

$$\forall x \in R \qquad \chi_{\underline{A}}(x) = \begin{cases} 1 & \text{if } x \text{ belongs to } \underline{A} \\ \alpha \in (0,1) & \text{if } x \text{ belongs to } \underline{A} \text{ to some degree} \\ 0 & \text{if } x \text{ does not belong to } \underline{A} \end{cases} \tag{1.2}$$

Similarly, the conventional notion of an event A (a crisp event), which can be defined as the crisp subset of the sample space X and described by the characteristic function, Eq. (1.1), is extended to a fuzzy event $\underline{A}$ described by the membership function, Eq. (1.2). The fuzzy event can occur, or not occur and also can occur *to some degree* because any element x of the sample space X can match up to a given event, or not, and also can match up to it to some degree. In other words, the fuzzy event is a fuzzy subset of the sample space X. The boundary between that event and its complement is fuzzyfied, not crisp.

Two kinds of probability of fuzzy events are defined (Kacprzyk, 1986):

- the probability according to Zadeh – a real number from the interval $< 0,1 >$ – for a continuous random variable defined as follows

$$P(\underline{A}) = \int\limits_{\underline{A}} f(x)\, dx = \int\limits_{R} \chi_{\underline{A}}(x) f(x)\, dx \tag{1.3}$$

  where $f(x)$ is the probability density function of the random variable X

- the probability according to Yager – a fuzzy subset of the interval $< 0, 1 >$ – for a continuous random variable defined as a set of probabilities of crisp events A_α

$$P(A_\alpha) = \int_{A_\alpha} f(x)\, dx \tag{1.4}$$

with the following membership function

$$\chi_{\underline{P(A)}} = \alpha \tag{1.5}$$

where A_α is α-level of a fuzzy set A defined as follows

$$\forall \alpha \in (0, 1 > \qquad A_\alpha = \{x \in R : \ \ \chi_{\underline{A}}(x) \geqslant \alpha\} \tag{1.6}$$

The probabilities mentioned above make the measure of two kinds of uncertainties: the randomness of the variable X and the imprecision (fuzziness) of the event A definition, Fig. 1.

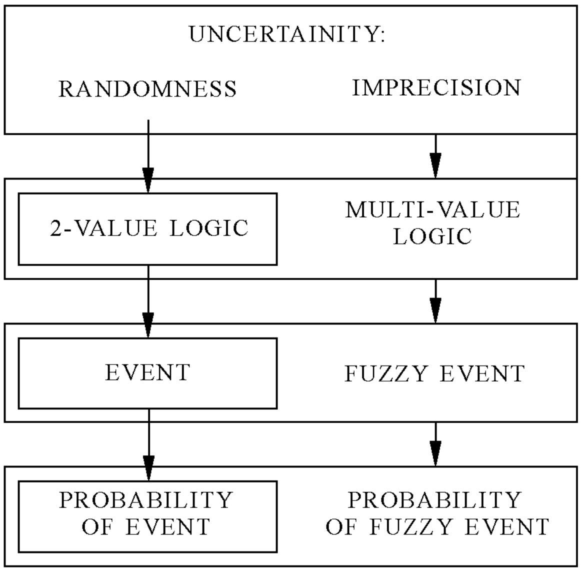

Fig. 1. Randomness and fuzziness

As it is known, according to the reliability theory, only uncertainty due to randomness is expressed (PN-ISO 2394, 2000; Nowak and Collins, 2000). The structural failure or survival is treated as an event. The probability of failure is a measure of structural reliability and is calculated as follows

$$P_f = P(g(X_1, ..., X_n) < 0) = \int_{g(x)<0} \dots \int f_{1...n}(x_1, ..., x_n)\, dx_1 ... dx_n \tag{1.7}$$

where

$\boldsymbol{X} = \{X_1, ..., X_n\}$	–	n-dimensional random variable, which represents natural randomness of loads, environmental influences, material properties, geometry etc.
$f_{1...n}(x_1, ..., x_n)$	–	joint probability density function of the n-dimensional random variable $\boldsymbol{X}$
$g = g(X_1, ..., X_n)$	–	limit state function of load capacity or serviceability, which divides the n-dimensional sample space $\boldsymbol{X}$ into the following subsets: • the area of structural failure – $g(X_1, ..., X_n) > 0$ • the area of structural survival – $g(X_1, ..., X_n) < 0$ • the limit state – $g(X_1, ..., X_n) = 0$.

That probability is used e.g. for the calibration of partial safety factors in design codes.

In the paper, two kinds of uncertainty will be modelled and taken into consideration in the limit state function (Szeliga, 2000):

- the randomness will be still represented by random variables and probability density functions
- the imprecision will be represented by fuzzy numbers and membership functions.

As a result, the structural failure or survival will be treated as fuzzy events. Partial safety factors will include "variability of fortune" and mistakes of experts' opinions (Fig. 2).

The purpose of the paper is not to prove that the structural reliability problem in fuzzy sets approach is better – but to show that it is possible.

2. Reliability of structure as a fuzzy event

Let us consider a linear limit state function. The resistance consists of two parts: the random part X_1 of normal distribution $N(\mu_1, \sigma_1)$ and the imprecise part described by a fuzzy number $\underline{a}_1 = \{\text{about } a_1^0 \text{ in } < a_1^-, a_1^+ >\}$,

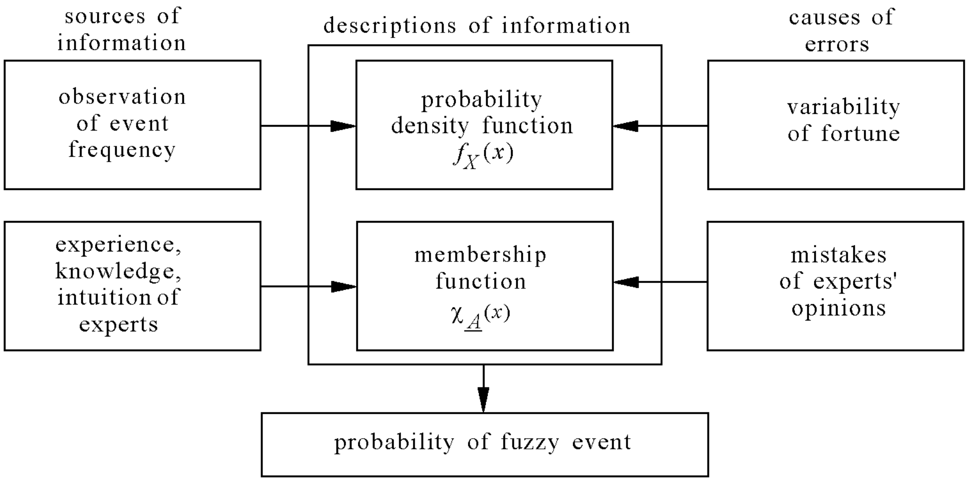

Fig. 2. Probability of a structural failure or survival as fuzzy events

$a_1^0 = 0$. The load also consists of the random part X_2 of the normal distribution $N(\mu_2, \sigma_2)$ and the imprecise part $\underline{a}_2 = \{\text{about } a_2^0 \text{ in } < a_2^-, a_2^+ >\}$, $a_2^0 = 0$

$$g(X_1, X_2) = (X_1 + a_1) - (X_2 + a_2) \qquad \begin{array}{l} a_1 \in \underline{a}_1 \\ a_2 \in \underline{a}_2 \end{array} \tag{2.1}$$

where $\underline{a}_1$, $\underline{a}_2$ – fuzzy numbers of membership functions $\chi_{\underline{a}_1}$, $\chi_{\underline{a}_2}$.

The limit state function can be expressed as follows:

$$g(Z) = Z + b \qquad b \in \underline{b} \tag{2.2}$$

where $Z = X_1 - X_2$ is the random safety margin, $\underline{b} = \underline{a}_1 - \underline{a}_2$ and $\underline{b}$ – fuzzy number of membership function $\chi_{\underline{b}}$.

$$b = \{\text{about } b^0 \text{ in } < b^-, b^+ >\} \qquad b^0 = 0 \tag{2.3}$$

If the parameter b was not fuzzy number, the following three crisp sets could be found in the axis Z: the failure area F, the limit point L and the survival area S (Fig. 3a). They could be described – by the characteristic functions – as follows

$$c_F(z) = \begin{cases} 1 & \text{for } \ Z < 0 \\ 0 & \text{for } \ Z \geqslant 0 \end{cases} \qquad c_L(z) = \begin{cases} 1 & \text{for } \ Z = 0 \\ 0 & \text{for } \ Z \neq 0 \end{cases}$$
$$c_S(z) = \begin{cases} 1 & \text{for } \ Z > 0 \\ 0 & \text{for } \ Z \leqslant 0 \end{cases} \tag{2.4}$$

Because of fuzzy numbers in the limit state function, the axis Z is divided into 3 fuzzy areas: the failure area $\underline{F}$, the limit state area $\underline{L}$ and the survival area $\underline{S}$ (Fig. 3c)

$$
\begin{aligned}
&\chi_{\underline{F}}(z) = \begin{cases} 1 & \text{for} \quad Z < b^- \\ 1 - \chi_{\underline{b}} & \text{for} \quad b^- \leqslant Z < b^0 \\ 0 & \text{for} \quad b^0 \leqslant Z \end{cases} \\
&\chi_{\underline{L}}(z) = \chi_{\underline{b}} \\
&\chi_{\underline{S}}(z) = \begin{cases} 0 & \text{for} \quad Z < b^0 \\ 1 - \chi_{\underline{b}} & \text{for} \quad b^0 \leqslant Z < b^+ \\ 1 & \text{for} \quad b^+ \leqslant Z \end{cases}
\end{aligned}
\tag{2.5}
$$

It should be understood as follows: not all values of the random margin of safety Z are completely sufficient $\chi_{\underline{S}}(z) = 1$ or completely insufficient $\chi_{\underline{F}}(z) = 1$. Some values of Z in the fuzzy survival area close to the limit point $(0 < \chi_{\underline{S}}(z) < 1, 0 < \chi_{\underline{L}}(z) < 1)$ are sufficient to some degree and some values of Z in the failure area close to the limit point $(0 < \chi_{\underline{L}}(z) < 1, 0 < \chi_{\underline{F}}(z) < 1)$ are insufficient to some degree.

The structural survival or failure can be treated as fuzzy events, when the variable Z hits the following areas:

- the fuzzy failure area $\underline{F}$
- the fuzzy not survival area $\underline{-S}$ – the complement of the survival area $\underline{S}$ (Fig. 3e)

$$\chi_{\underline{-S}}(z) = 1 - \chi_{\underline{S}}(z) \tag{2.6}$$

The measure of structural reliability, taking into account both types of uncertainty – the randomness and the imprecision, can consist of the probabilities of these events:

- the probabilities according to Zadeh

$$
\begin{aligned}
P_{\underline{F}} &= P(Z \in \underline{F}) = \int\limits_{-\infty}^{+\infty} \chi_{\underline{F}}(z) f_Z(z) \, dz \\
P_{\underline{-S}} &= P(Z \in \underline{-S}) = \int\limits_{-\infty}^{+\infty} \chi_{\underline{-S}}(z) f_Z(z) \, dz
\end{aligned}
\tag{2.7}
$$

where $f_Z(z)$ is the probability density function of the random variable Z

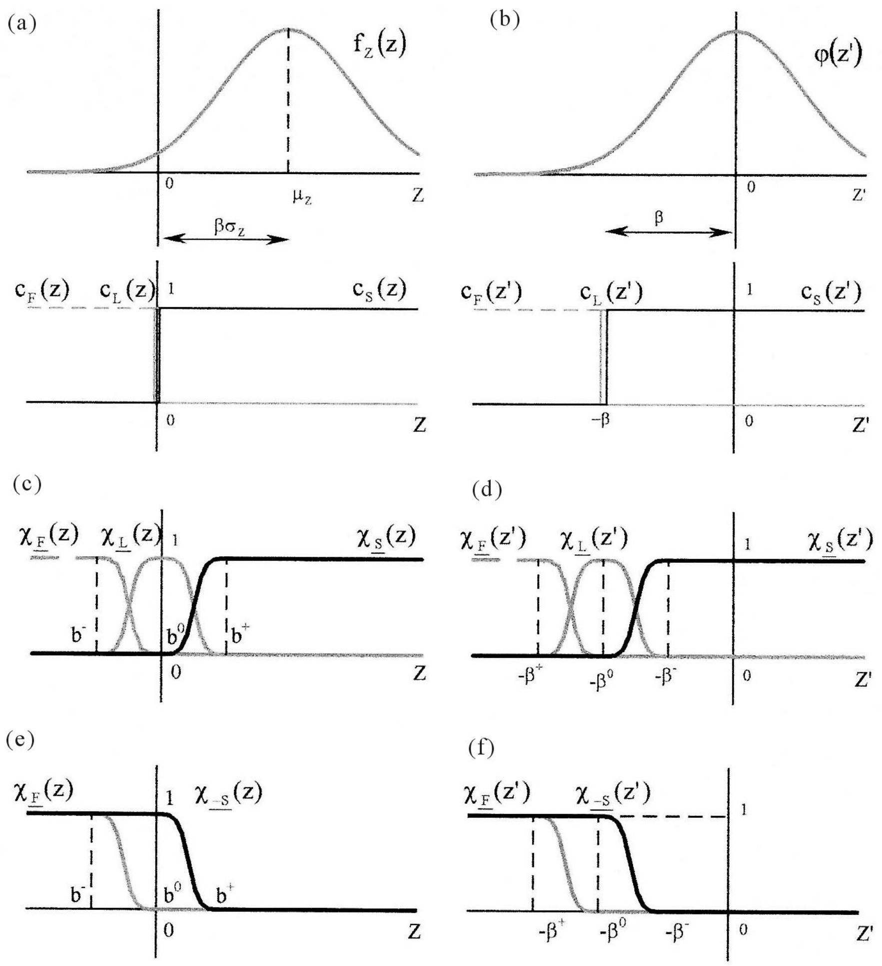

Fig. 3. The axis of the random variable Z of the normal distribution $f_Z(z)$ and the axis of the standardised random variable Z' of the standardised normal distribution $\varphi(z')$ divided into crisp and fuzzy areas of the failure, survival and limit state

- the fuzzy probabilities according to Yager with member functions $\chi_{\underline{P_F}}$ and $\chi_{\underline{P_{-S}}}$ as shown in Fig. 4.

Because

$$\underline{F} \subset F \subset \underline{-S} \tag{2.8}$$

in other words

$$\chi_{\underline{F}}(z) \leqslant C_F(z) \leqslant \chi_{\underline{-S}}(z) \tag{2.9}$$

so

$$P_{\underline{F}} \leqslant P_f \leqslant P_{\underline{-S}} \tag{2.10}$$

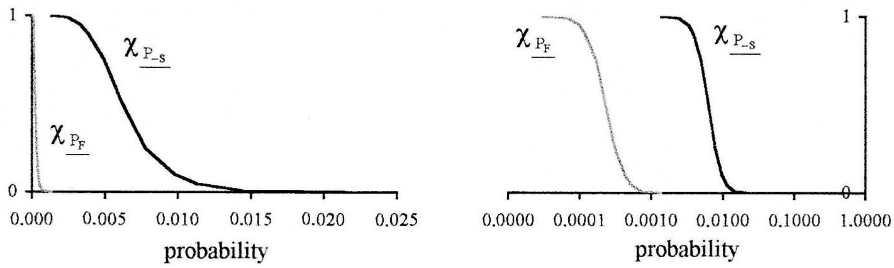

Fig. 4. Examples of the fuzzy probability according to Yager

The probabilities $P_{\underline{F}}$ and $\underline{P_{-S}}$ are the lower and the upper estimations of the failure probability P_f. That "estimation" not "calculation" is a consequence of the fuzzy numbers introduced to the limit state function.

3. Reliability index as a fuzzy number

In order to determine the probabilities mentioned above, the Cornell reliability index can be used. After standardisation of the variable Z

$$Z' = \frac{Z - \mu_Z}{\sigma_Z} \tag{3.1}$$

the limit state function takes the following form

$$g(Z') = Z' + \beta \qquad\qquad \beta \in \underline{\beta} \tag{3.2}$$

where $\underline{\beta}$ is the fuzzy number of membership function $\chi_{\underline{\beta}}$ and

$$\begin{gathered}\underline{\beta} = \{\text{about } \beta^0 \text{ in } <\beta^-, \beta^+>\} \\ \beta^0 = \frac{\mu_Z + b^0}{\sigma_Z} = 0 \qquad\qquad \beta^- = \frac{\mu_Z + b^-}{\sigma_Z} \qquad\qquad \beta^+ = \frac{\mu_Z + b^+}{\sigma_Z}\end{gathered} \tag{3.3}$$

Thus, one Cornell reliability index (Fig. 3b) is replaced by a fuzzy set of such indexes (Fig. 3d). Each of them represents the limit state to some degree $\chi_{\underline{\beta}}$.

The probabilities according to Zadeh of the fuzzy events $\underline{F}$ and $\underline{-S}$ can be calculated as follows (see Fig. 3f)

$$
\begin{aligned}
P_{\underline{F}} = P(Z' \in \underline{F}) &= \int\limits_{-\infty}^{+\infty} \chi_{\underline{F}}(z')\varphi(z')\, dz' = \Phi(-\beta^{+}) + \int\limits_{-\beta^{+}}^{-\beta^{0}} \chi_{\underline{F}}(z')\varphi(z')\, dz' \\
P_{\underline{-S}} = P(Z' \in \underline{-S}) &= \int\limits_{-\infty}^{+\infty} \chi_{\underline{-S}}(z')\varphi(z')\, dz' = \Phi(-\beta^{0}) + \int\limits_{-\beta^{0}}^{-\beta^{-}} \chi_{\underline{-S}}(z')\varphi(z')\, dz'
\end{aligned}
\tag{3.4}
$$

The fuzzy probabilities according to Yager can be calculated as follows

$$
\begin{aligned}
P_{F\alpha} = P(Z' \in F_{\alpha}) &= \int\limits_{-\infty}^{+\infty} c_{F\alpha}(z')\varphi(z')\, dz' = \Phi(-\beta^{+}) + \int\limits_{-\beta^{+}}^{-\beta(\alpha)} \varphi(z')\, dz' \\
P_{-S\alpha} = P(Z' \in -S_{\alpha}) &= \int\limits_{-\infty}^{+\infty} c_{-S\alpha}(z')\varphi(z')\, dz' = \Phi(-\beta^{0}) + \int\limits_{-\beta^{0}}^{-\beta(\alpha)} \varphi(z')\, dz'
\end{aligned}
\tag{3.5}
$$

and

$$
\chi_{\underline{P_F}} = \alpha \qquad \chi_{\underline{P_{-S}}} = \alpha \tag{3.6}
$$

The following *equivalent reliability indexes* can be also defined

$$
\beta_{\underline{F}} = -\Phi^{-1}(P_{\underline{F}}) \qquad \beta_{\underline{-S}} = -\Phi^{-1}(P_{\underline{-S}}) \tag{3.7}
$$

They determine two crisp areas in the axis Z': $(-\infty, -\beta_{\underline{F}} >$ and $(-\infty, -\beta_{\underline{-S}} >$ so that the random safety margin Z hits them with the probabilities $P_{\underline{F}}$ and $P_{\underline{-S}}$, respectively.

Because these areas satisfy

$$
(-\infty, -\beta_{\underline{F}} > \subset F \subset (-\infty, -\beta_{\underline{-S}} > \qquad \text{so} \qquad \beta_{\underline{F}} \geqslant \beta \geqslant \beta_{\underline{-S}} \tag{3.8}
$$

The *equivalent indexes* $\beta_{\underline{F}}$ and $\beta_{\underline{-S}}$ are the upper and the lower estimations of the reliability index β. That "estimation" not "calculation" is a consequence of the fuzzy numbers introduced to the limit state function.

Let us consider an n-dimensional limit state function with fuzzy parameters. Let us take the following assumptions:

- the random variables X_i have normal distributions $N(\mu_i, \sigma_i)$ and are uncoreleted

- the limit state function is linear and one of its parameter, not multiplied by the random variable X_i, is a fuzzy number described by the membership function $\chi_{\underline{a_{n+1}}}$

$$g(\boldsymbol{X}) = a_1X_1 + ... + a_nX_n + a_{n+1} \qquad a_{n+1} \in \underline{a_{n+1}} \tag{3.9}$$

where $\underline{a_{n+1}}$ is the fuzzy number of membership function $\chi_{\underline{a_{n+1}}}$.

The limit state function determines the fuzzy sets of limit states surfaces parallel to each other (Fig. 5a). Each of these surfaces describes the limit state more or less precisely. It represents the limit state to some degree $\chi_{\underline{g(\boldsymbol{x})=0}}$. According to the extension rule, each of these surfaces represents the limit state to the same degree as each number of fuzzy set $\underline{a_{n+1}}$ represents the parameter a_{n+1}

$$\chi_{\underline{g(\boldsymbol{x})=0}} = \chi_{\underline{a_{n+1}}} \tag{3.10}$$

The X space is divided into 3 fuzzy areas: the failure $\underline{F}$, limit states $\underline{L}$ and survival $\underline{S}$ area

$$\begin{aligned}
&\chi_{\underline{F}}(\boldsymbol{x}) = \begin{cases} 1 & \text{for } \ \chi_{\underline{g(\boldsymbol{x})=0}} = 0 \qquad g(\boldsymbol{X}) < 0 \\ 1 - \chi_{\underline{g(\boldsymbol{x})=0}} & \text{for } \ \chi_{\underline{g(\boldsymbol{x})=0}} \neq 0 \\ 0 & \text{for } \ \chi_{\underline{g(\boldsymbol{x})=0}} = 0 \qquad g(\boldsymbol{X}) > 0 \end{cases} \\
&\chi_{\underline{L}}(\boldsymbol{x}) = \chi_{\underline{g(\boldsymbol{x})=0}} \\
&\chi_{\underline{S}}(\boldsymbol{x}) = \begin{cases} 1 & \text{for } \ \chi_{\underline{g(\boldsymbol{x})=0}} = 0 \qquad g(\boldsymbol{X}) > 0 \\ 1 - \chi_{\underline{g(\boldsymbol{x})=0}} & \text{for } \ \chi_{\underline{g(\boldsymbol{x})=0}} \neq 0 \\ 0 & \text{for } \ \chi_{\underline{g(\boldsymbol{x})=0}} = 0 \qquad g(\boldsymbol{X}) < 0 \end{cases}
\end{aligned} \tag{3.11}$$

Now, the measure of reliability cosists of the probabilities of hitting by the random variable $\boldsymbol{X}$ the following areas:

- the fuzzy area of failure $\underline{F}$
- the fuzzy area of no survival $\underline{-S}$ (Fig. 5a)

$$\chi_{\underline{-S}}(\boldsymbol{x}) = 1 - \chi_{\underline{S}}(\boldsymbol{x}) \tag{3.12}$$

The probabilities according to Zadeh are the following

$$\begin{aligned}
P_{\underline{F}} &= P(\boldsymbol{X} \in \underline{F}) = \int_{-\infty}^{+\infty} ... \int_{-\infty}^{+\infty} \chi_{\underline{F}}(\boldsymbol{x}) f_{1...n}(\boldsymbol{x}) \, d\boldsymbol{x} \\
P_{\underline{-S}} &= P(\boldsymbol{X} \in \underline{-S}) = \int_{-\infty}^{+\infty} ... \int_{-\infty}^{+\infty} \chi_{\underline{-S}}(\boldsymbol{x}) f_{1...n}(\boldsymbol{x}) \, d\boldsymbol{x}
\end{aligned} \tag{3.13}$$

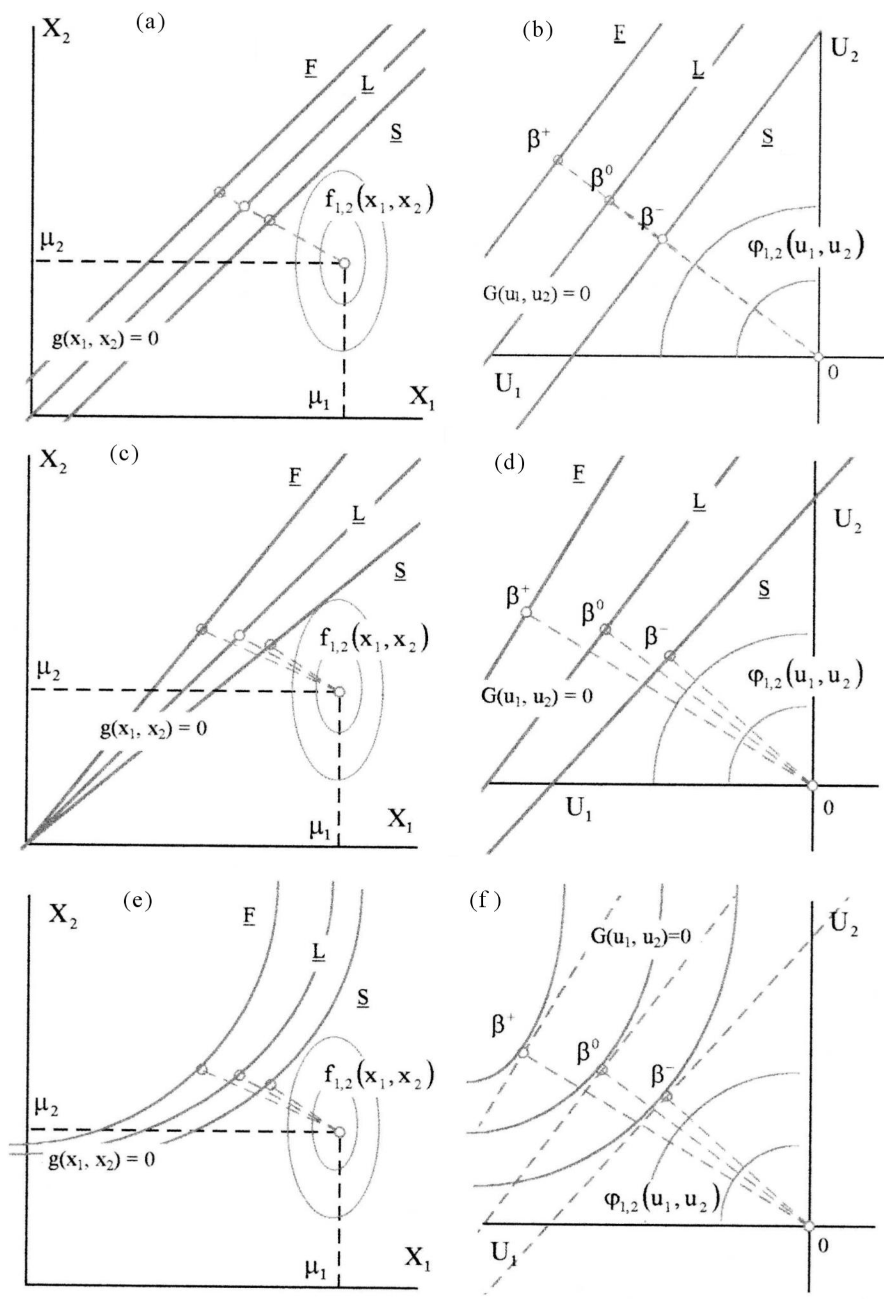

Fig. 5. Fuzzy areas of the failure $\underline{F}$, limit state $\underline{L}$ and survival $\underline{S}$ in the variables X_1, X_2 and standard variables U_1, U_2 coordinate systems

They can be calculated by transformation of the problem of n random variables to the problem of one variable. After standardisation of the variable $\boldsymbol{X}$, the equation of limit state takes the following form

$$G(\boldsymbol{u}) = \alpha_1 u_1 + ... + \alpha_n u_n + \beta = 0 \qquad \beta \in \underline{\beta} \tag{3.14}$$

where

$$\begin{aligned} \alpha_i &= \frac{a_i \sigma_i}{\sqrt{\sum\limits_{i=1}^{n} (a_i \sigma_i)^2}} \qquad && i = 1, ..., n \\ \beta &= \frac{\sum\limits_{i=1}^{n} a_i \mu_i + a_{n+1}}{\sqrt{\sum\limits_{i=1}^{n} (a_i \sigma_i)^2}} \qquad && a_{n+1} \in \underline{a_{n+1}} \end{aligned} \tag{3.15}$$

and $\underline{\beta}$, $\underline{a_{n+1}}$ – fuzzy numbers of membership functions $\chi_{\underline{\beta}}$, $\chi_{\underline{a_{n+1}}}$, respectively.

Equations (3.14) and (3.15) describe the fuzzy set of the limit state surfaces parallel to each other (Fig. 5b) of a distance $\beta \in \underline{\beta}$ to the origin of coordiantes $\boldsymbol{U}$. Thus, one reliability Hasofer-Lind index is replaced by the fuzzy sets of indexes. Each of these indexes represents the limit state surface in coordinates $\boldsymbol{U}$ to some degree $\chi_{\underline{\beta}}$. According to the extension rule, each of these indexes represents the limit state to the same degree as each number of fuzzy set $\underline{a_{n+1}}$ represents the parameter a_{n+1}

$$\chi_{\underline{\beta}} = \chi_{\underline{a_{n+1}}} \tag{3.16}$$

After rotation of the $U_1, ..., U_n$ axes around the point of origin into $Z', Z'', ..., Z^{(n)}$ axes so that Z' axis is perpendicular to the limit state surfaces (3.14) and others are parallel to them (Fig. 5b), the probabilities of the fuzzy events $\underline{F}$ and $\underline{-S}$ according to Zadeh can be calculated according to (3.4), and the fuzzy probabilities according to Yager – according to (3.5).

The linear limit state function with at least one variable of $X_1, ..., X_n$ multiplied by the fuzzy number

$$g(\boldsymbol{X}) = a_1 X_1 + ... + a_n X_n + a_{n+1} \qquad a_j \in \underline{a_j} \tag{3.17}$$

describes the fuzzy set of surfaces (line in Fig. 5c) and $\underline{a_j}$ – fuzzy numbers of membership functions $\chi_{\underline{a_j}}$, $j = 1, ..., n+1$. Each of them represents the limit state to some degree

$$\chi_{\underline{g(\boldsymbol{x})=0}} = \max_{g(\boldsymbol{x})=0} \Big(\min_{j=1,...,m} \chi_{\underline{a_j}} \Big) \tag{3.18}$$

The limit state surfaces (Fig. 5d) are not parallel to each other, so the probabilities calculated using the Hasofer-Lind index are approximated.

The nonlinear limit state function of n random variables $X_1, ..., X_n$ and m fuzzy parameters $a_1, ..., a_m$

$$g(\boldsymbol{X}) = g(X_1, ..., X_n, a_1, ..., a_m) \qquad a_j \in \underline{a_j} \tag{3.19}$$

can be approximated by fuzzy sets of linear functions, see Fig. 5e,f ($\underline{a_j}$ – fuzzy numbers of membership functions $\chi_{\underline{a_j}}$, $j = 1, ..., n+1$).

Thus, the structural reliability analysis based on the limit state function with fuzzy parameters is similar to well known methods. In some cases, the Hasofer-Lind indexes can be used.

4. Fuzzy Monte-Carlo methods

Monte-Carlo simulations can also be used in the case of fuzzy events. The probabilities according to Zadeh can be estimated as follows

$$\widehat{P}_{\underline{F}} = \frac{N_{\underline{F}}}{N} \qquad \widehat{P}_{\underline{-S}} = \frac{N_{\underline{-S}}}{N} \tag{4.1}$$

where N is a number of experiments. $N_{\underline{F}}$ and $N_{\underline{-S}}$ are numbers of hitting the fuzzy areas $\underline{F}$ and $\underline{-S}$ in particular trials calculated as follows

$$\begin{aligned} N_{\underline{F}} &= N_{\underline{F}} + \chi_{\underline{F}}(\boldsymbol{u}) \frac{\varphi_n(\boldsymbol{u}, \boldsymbol{0}, \mathbf{I})}{\varphi_n(\boldsymbol{u}, \boldsymbol{U}^*, \mathbf{I})} \\ N_{\underline{-S}} &= N_{\underline{-S}} + \chi_{\underline{-S}}(\boldsymbol{u}) \frac{\varphi_n(\boldsymbol{u}, \boldsymbol{0}, \mathbf{I})}{\varphi_n(\boldsymbol{u}, \boldsymbol{U}^*, \mathbf{I})} \end{aligned} \tag{4.2}$$

The membership functions $\chi_{\underline{F}}(\boldsymbol{u})$ and $\chi_{\underline{-S}}(\boldsymbol{u})$ are equal to degrees of hitting the fuzzy areas $\underline{F}$ and $\underline{-S}$ by the n-dimensional standardised random variables $\boldsymbol{u}$ generated according to *the importance sampling method* (Melchers, 1987; see Fig. 6).

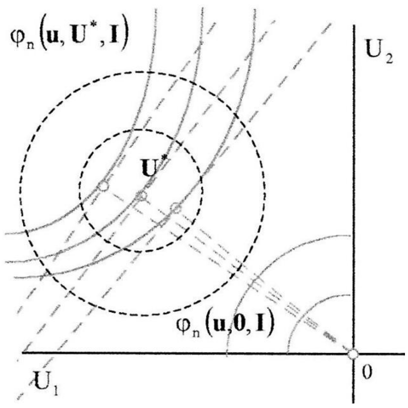

Fig. 6. Generation of random values according to *the importance sampling method*

These functions can be defined as follows

$$\chi_{\underline{F}}(\boldsymbol{u}) = \begin{cases} 0 & \text{for} \quad 0 \leqslant G^0 \\ 1 - \chi_{\underline{G(\boldsymbol{u})=0}} & \text{for} \quad G^0 < 0 \leqslant G^+ \\ 1 & \text{for} \quad G^+ < 0 \end{cases}$$
$$\chi_{\underline{-S}}(\boldsymbol{u}) = \begin{cases} 0 & \text{for} \quad 0 \leqslant G^- \\ \chi_{\underline{G(\boldsymbol{u})=0}} & \text{for} \quad G^- < 0 \leqslant G^0 \\ 1 & \text{for} \quad G^0 < 0 \end{cases} \tag{4.3}$$

where $\chi_{\underline{G(\boldsymbol{u})=0}}$ is the membership function of a fuzzy value of the limit state function $\underline{G(\boldsymbol{u})}$ with fuzzy parameters (Fig. 7)

$$\underline{G(\boldsymbol{u})} = \{\text{about} \quad G^0 \text{ in } < G^-, G^+ >\} \tag{4.4}$$

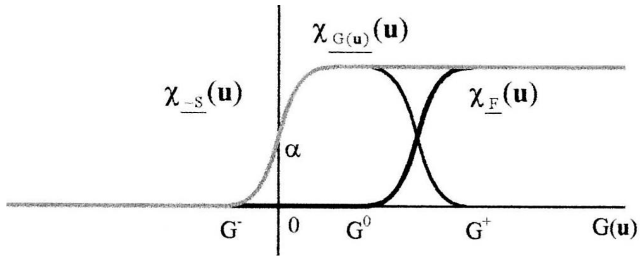

Fig. 7. A fuzzy number as a value of the limit state function with fuzzy parameters

The probabilities according to Yager can be estimated as follows

$$\widehat{P}_{F\alpha} = \frac{N_{F\alpha}}{N} \qquad \chi_{\underline{P_F}} = \alpha$$
$$\widehat{P}_{-S\alpha} = \frac{N_{-S\alpha}}{N} \qquad \chi_{\underline{P_{-S}}} = \alpha \tag{4.5}$$

where $N_{F\alpha}$ and $N_{-S\alpha}$ are the numbers of hitting the α-levels of the fuzzy areas $\underline{F}$ and $\underline{-S}$ calculated as follows

$$N_{F\alpha} = N_{F\alpha} + 1 \cdot \frac{\varphi_n(\boldsymbol{u}, \mathbf{0}, \mathbf{I})}{\varphi_n(\boldsymbol{u}, \boldsymbol{U}^*, \mathbf{I})}$$
$$N_{-S\alpha} = N_{-S\alpha} + 1 \cdot \frac{\varphi_n(\boldsymbol{u}, \mathbf{0}, \mathbf{I})}{\varphi_n(\boldsymbol{u}, \boldsymbol{U}^*, \mathbf{I})} \tag{4.6}$$

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Niezawodność konstrukcji w ujęciu teorii zbiorów rozmytych

Streszczenie

W niniejszej pracy proponuje się uwzględnić w opisie konstrukcji dwa rodzaje niepewności: losowość i nieprecyzyjność. Nieprecyzyjne oceny ekspertów dotyczące konstrukcji proponuje się opisywać za pomocą liczb rozmytych. W rezultacie, niezawodność konstrukcji określać się będzie w oparciu o funkcję stanu granicznego z rozmytymi parametrami. W konsekwencji, awarię konstrukcji lub jej brak traktować się będzie jako rozmyte zdarzenia losowe. Prawdopodobieństwa tych zdarzeń stanowić będą dolne i górne oszacowanie niezawodności konstrukcji. Można je wyznaczać za pomocą metod stosowanych już w niezawodności (np. wskaźnik Hasofera-Linda lub metody Monte Carlo). Mogą one służyć jako podstawa kalibrowania częściowych współczynników bezpieczeństwa w normach projektowych.

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