JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
45, 1, pp. 33-40, Warsaw 2007

OCCURRENCE OF STICK-SLIP PHENOMENON

Jan Awrejcewicz
Paweł Olejnik

Department of Automatics and Biomechanics, Technical University of Lodz
e-mail: awrejcew@p.lodz.pl; olejnikp@p.lodz.pl

A self-excited system with unilateral friction contact modelled by a two degrees-of-freedom mechanical system, where the normal force varies during displacement of a block has been studied. The constructed real laboratory rig approximates the investigated system, and it includes feedback reinforcement of the friction force acting on the vibrated block. Some methods of data acquisition and data handling procedures are proposed for experimentally observed results and data collection. A novel static friction force model for both positive and negative velocities of the base is proposed.

Key words: stick-slip dynamics, friction force model, numerical analysis, experiment

1. Introduction

Relative sliding of two solid bodies is a non-equilibrium process, where kinetic energy of motion is transferred into energy of irregular microscopic motion. This dissipative process creates the dry friction phenomenon. The phenomenological laws of dry friction like Coulomb's laws are well-known, and there is well established theory in applied physics (Bowden and Tabor, 1954) related to this subject.

The simplest models describe friction as a function of the velocity difference of sliding bodies. Such models like Coulomb's friction ones are called static models. In fact, Coulomb's dry friction laws simplify very complex behavior which involves mechanical, plastic, and chemical processes (Singer and Pollock, 1992). However, experimentally observed differences in application of Coulomb's law are often found (Awrejcewicz and Delfs, 1990a,b; Galvanetto *et al.*, 1995). Computer simulations of motion of mechanical systems with friction are difficult because of the strongly nonlinear behavior of the friction force near zero velocity and the lack of a universally considered friction model. For rigid bodies with dry friction, the classical Coulomb law of friction is usually applied to engineering contact problems exactly because of its simplicity. It can explain several phenomena associated with friction and it is commonly used for friction compensation (Friedland and Park, 1991). A well known velocity-limited friction model given by Oden and Martins (1985) uses a smooth quadratic function when the sliding velocity is nearly zero. The value of the limiting velocity is very important, but there is no standard method for its estimation.

The problem of modeling of friction forces is not solved, because physical and dynamical effects are not sufficiently understood. There are two main theoretical approaches to model dry friction interfaces: the macro-slip and micro-slip approaches (Feeny *et al.*, 1998). In the micro-slip approach, a relatively detailed analysis of the friction interfaces should be made. In this case, investigations can provide accurate results only when the preload between the interfaces is very high. In the macro-slip approach, the entire surface is assumed to be either sliding or sticking. The force necessary to keep sliding at a constant velocity depends on the sliding velocity of the contact surfaces. With this respect, friction laws of smooth and non-smooth velocity functions have been cited in Awrejcewicz and Olejnik (2003a,b), Popp *et al.* (1996).

There is a lack of works which take into account problems of experimentally observed velocity depending friction force models. The paper by Brandl and Pfeiffer (1999) deals with the measurement of dry friction. A tribometer was developed to identify both sticking and sliding friction coefficients. The so called Stribeck-curve has been determined for any material in the contact zone. Similarly, a multi degrees-of-freedom model of friction was investigated by Bogacz and Ryczek (1997), where an experimentally observed friction characteristic expresses the kinetic friction force as a function of relative velocity of motion.

2. Laboratory rig

A laboratory rig (Olejnik, 2002) designed for observations and experimental research of friction effects including the friction force measurement (Awrejcewicz and Olejnik, 2003a) has been constructed and investigated as well. A general view together with indication of component parts and some connectors like coil springs is schematically shown in Fig. 1. The self-excited system presented in Fig. 1 is equivalent to the real experimental rig (see Fig. 2) in which the block mass m is moving on the belt in the direction x_1, and where the angle body represented by the mass moment of inertia J is rotating around

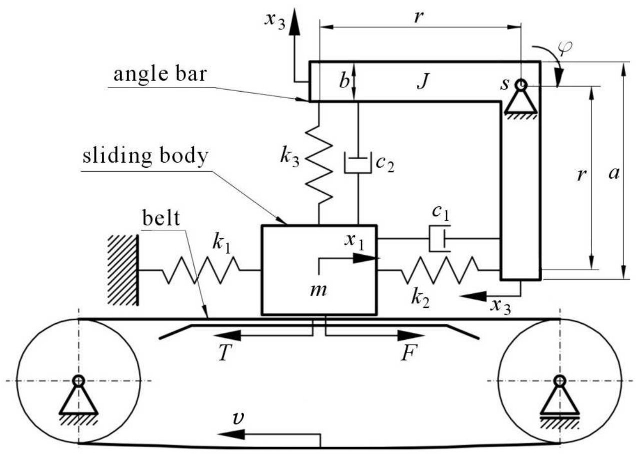

Fig. 1. The analysed 2-DOF system

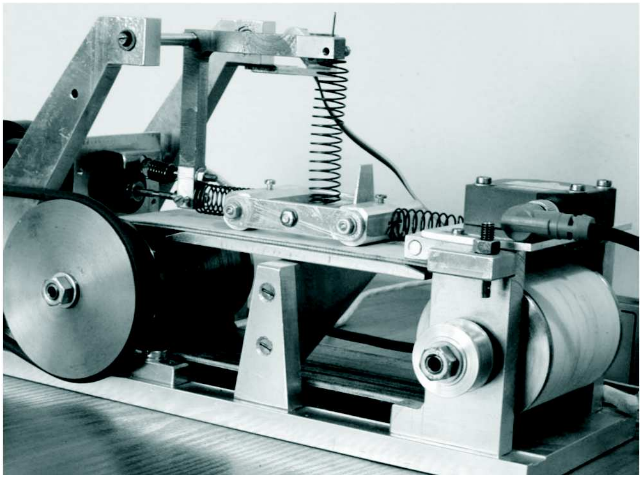

Fig. 2. The laboratory rig

point s with respect to the angle direction. The analysed system consists of the following parts: two bodies are coupled by linear springs k_2 and k_3; the block on the belt is additionally coupled to the fixed base using a linear spring k_1; the angle body is excited only by spring forces; there are no extra mechanical actuators; rotational motion of the angle body is damped using virtual actuators characterizing air resistance (marked by constants c_1 and c_2); damping of the block is neglected; it is assumed that the angle of rotation of the angle body is small and within interval $(+5, -5)$ degrees (in this case, the rotation is equivalent to the linear displacement y_1 of the angle body legs); the belt is moving with a constant velocity v_b and there is no deformation of the belt in the stick-slip contact zone.

Non-dimensional equations governing the dynamics of our investigated system have the following form

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_1 - \alpha_1^{-1}[\eta_1(x_2 + y_2) - y_1 - T(v_w)] \\
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= \alpha_2^{-1}(-\beta_3 y_1 - \eta_{12} y_2 - x_1 - \eta_1 x_2)
\end{aligned}
\tag{2.1}
$$

where: $x = (x_1, x_2)$, $y = (y_1, y_2)$ are state-space variables of the block and the angle body, respectively; ω is the periodicity of the mass m; the friction force $T(v_w)$ is described by Eq. (4.3); $v_w = x_2 - v_b$ is the relative velocity between the bodies $[\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \eta_1, \eta_2, \eta_{12}] = [\omega^2 m, \omega^2 J r^{-2}, k_1 + k_2, \mu_0 k_3, k_2 + k_3, c_1\omega, c_2\omega\mu_0, \omega(c_1 + c_2)]k_2^{-1}$ are remaining parameters.

A non-stretchable 25 mm thick belt (made of hard rubber) is placed on solid uniform shafts and then supported by means of a flat bar at the place, where the examined block vibrates. The propeller shaft is installed in a floating manner to avoid belt's tension, and the rig is essentially equipped with a direct current commutator motor (PZTK 60-46 J suitable to use in cross-feed drives of numerically controlled machines) supplied by voltage 30 V and ultimate current load equal to 4.1 A. Stabilisation and control of motor rotational velocity is additionally accomplished by means of the RN12 regulator (this device works in the system also as an amplifier).

The system variables like displacement of the block and rotation of the equal-leg angle are quantified by using of non-sticking measurement method, where the laser proximity switch is assigned to the displacement measurement of the block; Hall-effect device (principle of operation is based on changes of magnetic field) is used to determine rotation of the equal-leg angle, respectively. It should be emphasized that both the quoted devices have linear characteristics of the measured quantity versus voltage output. Additionally, all construction parts of the described laboratory rig are fixed to a stable frame.

3. Measurement

Analog signals incoming from the measuring devices are processed by dynamic acquisition using test instruments made by National Instruments and cooperated through a PCI card (PCI-6035E with chasis SCXI-1000, -1321 and -1302) with the LabView professional software. The commutator motor is equipped with a rotary-impulse converter whose output is transformed to linear velocity of the belt. Acquisition and data handling is in the LabView environment made thanks to composition of a special block-wire-block diagram. The stored data are indicated on panel-situated scaleable charts.

Disturbances of the whole structure, noise in electrical circuits, and other additional effects have significant influence on accuracy of any measured signal. Therefore, some signals are filtered digitally (e.g. elliptic topology), and then real differentiation preventing formation of high peaks is applied as follows

$$y_n = ay_{n-1} + k_0x_n + k_1x_{n-1} \tag{3.1}$$

where y_n is a value of the derivative at the point n; y_{n+1} denotes the preceding value (the algorithm starts from $n = 1$ when $y_0 = 0$); x_n, x_{n+1} – values of the differentiate function of displacement or velocity, when the computational process is associated with acceleration; a, k_0, k_1 are integration constants.

4. Investigations

Results of measurements are obtained following the methodology described in Section 3. Appropriately transformed equations of motion can be used for calculation of the friction force after a real time measurement of state variables of the investigated system. Characteristics of the friction force T in the nondimensional system versus relative velocity v_w between moving the belt and block for positive and negative velocities of the belt v_b are shown in Fig. 3. In the case of T_+ branch, the equation of friction force dependence describing

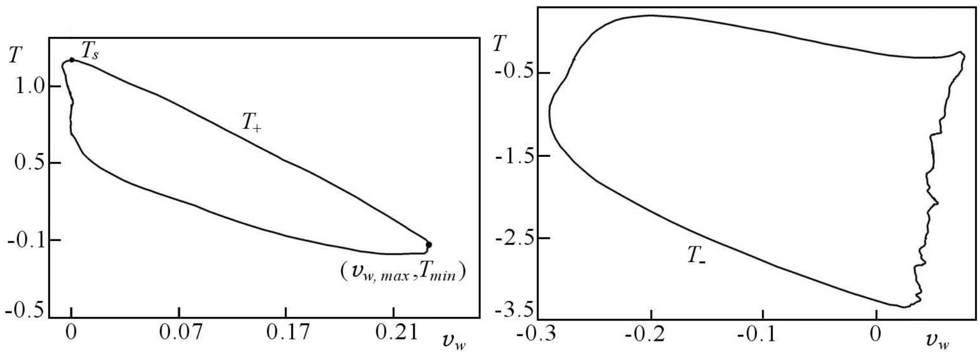

Fig. 3. Experimental characteristics of the friction force

the friction force model for positive relative velocity (see Fig. 3a) have the following form

$$T_+ = T_s - |v_w|\frac{T_s - T_{min}}{v_{w,max}} \tag{4.1}$$

where T_s is a static friction force, $v_{w,max}$ is the maximum positive relative velocity. The T_- branch can be described by a second order exponentially decay function describing the friction force model for negative relative velocity (see Fig. 3b) of the following form

$$T_- = T_s + A_1 \exp\Big(-\frac{|v_w| - v_{w,min}}{t_1}\Big) + A_2 \exp\Big(-\frac{|v_w| - v_{w,min}}{t_2}\Big) \tag{4.2}$$

where $v_{w,min}$ is the maximum negative relative velocity; A_1, A_2, t_1, t_2 are constant values. Therefore, the main multivalued function describing friction force changes occurring in our investigated 2-DOF system with the variable normal force is

$$T(v_w) = \begin{cases} T_+ \operatorname{sgn} v_w & \text{if} \quad v_w > 0 \\ T_- \operatorname{sgn} v_w & \text{if} \quad v_w < 0 \\ |T_s| & \text{if} \quad v_w = 0 \end{cases} \tag{4.3}$$

With respect to the validation of the estimated static friction model, Eq. (4.3), a special numerical integration algorithm has been used after comparisons of experimental results with their numerical counterparts. The based on Hénon's method numerical procedure describing the stick-slip phenomenon in the contact zone, which is extremely useful to locate the stick to slip and slip to stick transitions in non-smooth systems, has been applied (Awrejcewicz and Olejnik, 2002a,b, 2003a,b). Nondimensional parameters of $T(v_w)$ characteristics obtained by both measurement and identification are presented in Table 1.

Tabela 1. Parameters of the nondimensional friction model

	T_s	T_{min}	$v_{w,max}$	A_1	A_2	t_1	t_2
T_+	3.63	0.86	0.27	–	–	–	–
T_-	−5.94	−1.42	−0.28	3.2345	2.8736	0.0342	0.3053

Numerical analysis with the implementation of the derived friction force dependency has yielded the results presented in Fig. 4.

5. Conclusions

The experimental trajectory shown in Fig. 4 is rather rotated and has an irregular sticking phase. Such irregularity describes micro-stick and -slip conditions usually prevailing in the real contact zone of cooperated surfaces. The numerical trajectory is satisfactorily close to its experimental counterpart recorded for the investigated dynamical system. The sticking velocity is almost the same, but only some distinguishable differences are observed in the sliding

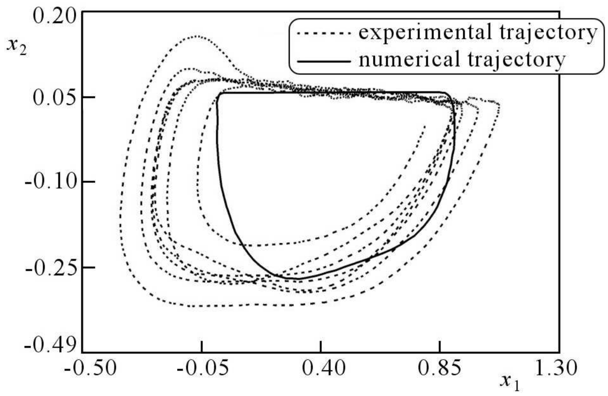

Fig. 4. Similarity of the experimental and numerical phase trajectories of the block for T_{-} friction characteristic

phase. Additionally, the carried out comparison has proved that our analysed system is non-symmetric.

Investigations on the real laboratory rig have been supported by numerical analysis allowing one to model and then to describe the feedback reinforcement of the friction force (model of T_{-} branch). To sum up, the T_{-} friction force model is suggested to be applied in systems, where the normal force acting between cooperated surfaces is fluctuated.

Acknowledgement

This work has been supported by the Polish Ministry of Science and Higher Education for years 2005-2008 (grant No. 4 T07A 031 28)

References

1. Awrejcewicz J., Delfs J., 1990a, Dynamics of a self-excited stick-slip oscillator with two degrees of freedom, Part I: Investigation of equilibria, *European J. Mech. A/Solids*, **9**, 4, 269-282
2. Awrejcewicz J., Delfs J., 1990b, Dynamics of a self-excited stick-slip oscillator with two degrees of freedom, Part II: Slip-stick, slip-slip, stick-slip transitions, periodic and chaotic orbits, *European J. Mech. A/Solids*, **9**, 5, 397-418
3. Awrejcewicz J., Olejnik P., 2002a, Calculating Lyapunov exponents from an interpolated time series, *Proc. XX Symposium – Vibrations in Physical Systems*, Błażejewko, 94-95
4. Awrejcewicz J., Olejnik P., 2002b, Numerical analysis of self-excited by friction chaotic oscillations in two-degrees-of-freedom system using Hénon method, *Machine Dynamics Problems*, **26**, 4, 9-20

5. Awrejcewicz J., Olejnik P., 2003a, Numerical and experimental investigations of simple non-linear system modelling a girling duo-servo brake mechanism, *Proc. ASME Design Eng. Tech. Conf.*, DETC2003/VIB-48479
6. Awrejcewicz J., Olejnik P., 2003b, Stick-slip dynamics of a two-degree-of freedom system, *Int. J. Bif. and Chaos*, **13**, 4, 843-861
7. Bogacz R., Ryczek B., 1997, Dry friction self-excited vibrations analysis and experiment, *Eng. Trans.*, **45**, 3/4, 487-504
8. Bowden F.P., Tabor D., 1954, *Friction and Lubrication*, Oxford University Press
9. Brandl M., Pfeiffer F., 1999, Tribometer for dry friction measurement, *Proc. ASME Design Eng. Tech. Conf.*, DETC99/VIB-8353
10. Feeny B., Guran A., Hinrichs N., Popp K., 1998, A historical review on dry friction and stick-slip phenomena, *App. Mech. Rev.*, **51**, 321-344
11. Friedland B., Park Y.-J., 1991, On adaptive friction compensation, *Proc. 30st IEEE Conf. Decision and Control*, 2899-2902
12. Galvanetto U., Bishop S.R., Briseghella L., 1995, Mechanical stick-slip vibrations, *Int. J. Bif. and Chaos*, **5**, 3, 637-657
13. 13 Oden J.T., Martins J.A.C., 1985, Models and computation methods for dynamic friction phenomena, *Comp. Methods App. Mech. Eng.*, **52**, 527-634
14. Olejnik P., 2002, *Numerical and Experimental Analysis of Regular and Chaotic Self-Excited Vibrations in a Two Degrees-of-Freedom System with Friction*, Ph.D. Thesis, Technical University of Lodz
15. Popp K., Hinrichs N., Oestreich M., 1996, *Dynamics with Friction*, World Scientific, London
16. Singer I.L., Pollock H.M., 1992, *Fundamentals of Friction: Macroscopic and Microscopic Processes*, Kluwer Academic Publishers, Dordrecht

O zjawisku typu stick-slip

Streszczenie

Analizie poddany jest układ samowzbudny z jednostronnym kontaktem ciernym, który zamodelowano jako układ mechaniczny o dwóch stopniach z tarciem oraz ze zmienną siłą nacisku wywieraną na podstawę podczas przemieszczenia się badanego ciała sztywnego. Skonstruowane stanowisko doświadczalne przybliża w pewnym stopniu rozważany układ i zawiera sprzężenie ruchu drgającego ciała z siłą normalną pochodzącą od nacisku. Zgodnie z obserwacjami eksperymentalnymi i dla zmierzonych sygnałów podano metody akwizycji danych oraz procedury ich obsługi. Zaproponowano nowy, statyczny model siły tarcia dla odpowiednich dodatnich i ujemnych prędkości przemieszczania się podstawy.

Manuscript received August 3, 2006; accepted for print August 18, 2006