JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
46, 1, pp. 85-107, Warsaw 2008

APPROACH TO EVALUATION OF CRITICAL STATIC LOADS OF ANNULAR THREE-LAYERED PLATES WITH VARIOUS CORE THICKNESS

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The evaluation of computational results of annular, three-layered plates with a soft foam core of different thickness under lateral compressive loads acting in facings planes has been conducted in this work. The values of critical static loads of plates and the forms of critical buckling corresponding with them have been the examined results of calculations. The calculations have been carried out using two approximation methods: finite difference method and finite element method. The solution to the problem of static stability of plates presented in both methods concerns the general problem of the loss of plate stability corresponding to possible circumferential wave forms of plate critical deformations. The plates with a thick core have been analysed in detail. Evaluating the results, it has been noted that the observed significant decrease in the critical static loads of plates with thick cores. This observation has been made by carrying out calculations with the help of the finite element method for plate models differing in the condition of the plate layers connection. The observed regions of good consistency and of essential differences of the results obtained using the assumed numerical methods for plates with middle and thick cores have been the main effect of the examinations. They seem to be practical importance in the modelling of plate structures.

Key words: annular layered plate, critical static loads, buckling form, finite differences method, FEM

1. Introduction

Calculations of critical, static loads of plates are the fundamental stage of examinations of their stability. The minimal loads and forms of critical plate deformations allow for determination of the range of practicable loads, character of plate operation and their supercritical behaviour. Static critical parameters have the essential importance in the evaluation of dynamic stability of structures subjected to time-dependent loads. The plate stability problems are multi-parameter tasks strongly depending on the material and geometrical parameters of the structure. The problem become more complex, when the examined object is a multi-element structure. In this case, not only the mentioned parameters, but also the method of connection of components in the structure and their participation in the operation of the whole object affect the behaviour of such a complex structure. Multi-layer plates and, among them, widely applied three-layer plates with soft cores are typical objects in the field of plate structures. Three-layer annular plates analysed in the range of the loss of its static stability are the subject of the present considerations. The presented solution has a general form enabling analysis both the particular axisymmetric form of plate buckling and circumferential wave forms, too. This solution refers to the solution presented by Pawlus (2006).

The computational results presented in detail eg. by Pawlus (2003a,b, 2004, 2007) concern exactly a special example of a plate, which loses its stability in a regular, axially-symmetrical form. The results presented in these works indicate significant differences in values of critical loads of plates with thick cores modelled with differing assumptions on layer deformation. This sensitivity has not been observed in models of plates with thin or medium-thickness cores. The approach to the evaluation of critical loads found by the presented methods is the task of this examination. Plate models, whose critical deformations could be an the unlimited, radial and circumferential wave forms have been considered.

It can be noticed that the field of undertaken problems for annular, three-layer plates with thick cores is still really limited. There are works considering thick annular plates, but with a homogeneous structure (see, Dumir and Shingal, 1985a,b). Numerous works deal with the dynamic stability of sandwich annular or circular plates. Some of them were presented by Wang and Chen (2003, 2004), Chen *et al.* (2006). The dynamic stability problem was also considered by Pawlus (2005). The structure of the analysed sandwich plate with thin and medium cores was similar to that presented in this work.

2. Problem formulation

An annular, three-layer plate under a load acting in the surface of its facings is the object of the analysis. The examined structure of the plate with double slidable clamped edges is presented in Fig. 1. Examples of plates compressed on outer or inner edges are considered. The schemes are presented in Fig. 2. Thus loaded plate loses its static stability. Forms of the loss of the plate stability, which correspond to the minimal value of critical loads are various, particularly for plates compressed on the outer perimeter. The deformation forms strongly depend on the material and geometrical parameters of the plate.

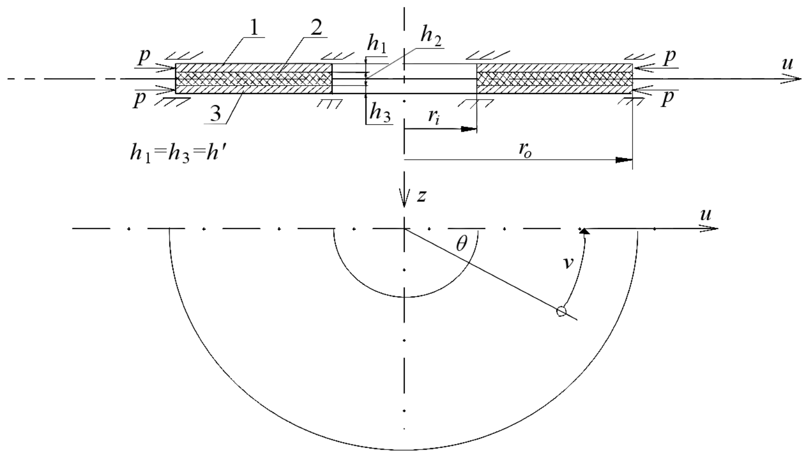

Fig. 1. Scheme of the analysed plate

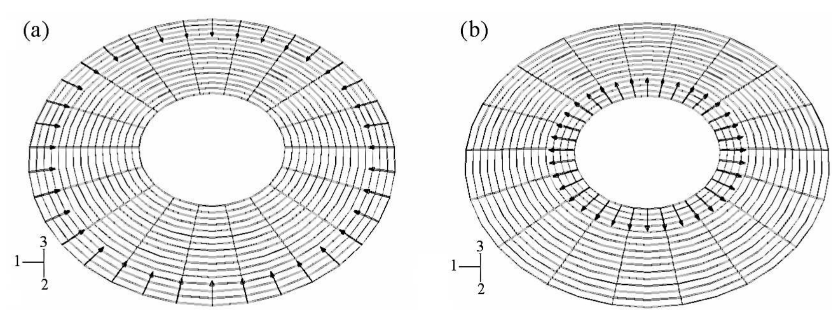

Fig. 2. Scheme of the plate loaded: (a) on the outer perimeter, (b) on the inner perimeter

Particularly, the core thickness has here the significance. The possible forms of loss of plate stability are global or local and basic, which occur for plates with thin or thick core, respectively (Romanów, 1995; Stamm and Witte, 1983). The global form of stability loss is for the global displacement state, which includes the condition of equal deflections of three plate layers. This condition is not in force for other forms of the loss of stability, where the general displacement state exists. Then, the results for critical loads of plates with thick cores are lower than the results obtained for the assumed condition of equal deflections of three plate layers. To some extent, this condition extorts global plate buckling. The introduction of this condition, particularly for plates with thick cores, may essentially disturb the correct final result. Above all, it can influence the increase in values of the critical loads. The evaluation of the effect of taking into account the coupling between layers or not on computational results of critical loads is the main problem considered in this work.

The problem has been solved using two approximation methods: finite difference and finite element. The solution obtained using the finite difference method is exactly based on the condition of equal deflection of layers, whereas, in the finite element method, the layers are connected by a surface contact interaction, which assures continuity of displacement and, additionally, mutual coupling as well. Results of plates with coupled layers and without coupling have been compared with the results obtained by making use of the finite difference method.

Calculations have been carried out for the transverse symmetrical structure of plates with the following geometrical parameters: inner radius $r_i = 0.2\,\mathrm{m}$, outer radius $r_o = 0.5\,\mathrm{m}$, facing thickness $h' = 0.0005\,\mathrm{m}$ and $0.001\,\mathrm{m}$, various core thickness: $h_2 = 0.005\,\mathrm{m}$, $0.01\,\mathrm{m}$, $0.02\,\mathrm{m}$ and $0.04\,\mathrm{m}$, $0.06\,\mathrm{m}$. In the analysis, the core thickness kept within the range from $h_2 = 0.005\,\mathrm{m}$ up to $0.02\,\mathrm{m}$ is generally treated as medium. As a thick core, the thickness above $h_2 = 0.02\,\mathrm{m}$ is considered.

3. Numerical calculations

Fundamental numerical calculations have been carried out for plates loaded on the outer edges of their facings. The plates lose their stability in circumferential wave forms. Observation of the forms of plate buckling with the evaluation of minimal critical loads corresponding to them, particularly for plates with thick cores, are the main goal of the undertaken numerical calculations.

The examined plates are built of steel facings and treated as an isotropic polyurethane foam core. The material parameters are the following: Young's modulus of facings $E = 2.1 \cdot 10^5\,\mathrm{MPa}$ and Poisson's ratio $\nu = 0.3$, Kirchhoff's moduli of core materials $G_2 = 5\,\mathrm{MPa}$ (presented in work by Majewski and Maćkowski, 1975) and $G_2 = 15.82\,\mathrm{MPa}$ (presented by Romanów, 1995). Poisson's ratio of the foam material equal to $\nu = 0.3$ is chosen according to the standard specification PN-84/B-03230.

3.1. Calculations using the finite difference method (FDM)

A detailed description of the solution to the formulated problem using the finite difference method was presented in work by Pawlus (2006). The solution is based on the assumption of equal values of layer transverse deflections, the classical theory of sandwich plates with the broken line hypothesis and the normal distribution of plate stresses on the facings and shearing stresses carried by the core (Volmir, 1967). This solution comes down to a solution to the eigen-value problem with determination of the minimal plate load, being the critical static load p_{cr}.

The basic elements of the solution are as follows:

- formulation of the equilibrium equations for each plate layer
- determination of geometrical relations with equations for angles α, β of the circumferential and radial core deformation, respectively
- formulation of physical relations of the layer materials using the relations of Hook's law,
- on the strength of the equations of the sectional forces and moments and suitable equilibrium equations determination of the formulas for the resultant radial Q_r and circumferential Q_θ forces and the resultant membrane radial N_r, circumferential N_θ and shear $T_{r\theta}$ forces determined by means of the introduced stress function Φ
- usage of the equilibrium equations of projections in the z-direction of forces loading the plate layers (Fig. 1), formulation of the basic differential equation describing deflections of the analysed plate, which is in the following form

$$
\begin{aligned}
&k_1 w'_{,rrrr} + \frac{2k_1}{r} w'_{,rrr} - \frac{k_1}{r^2} w'_{,rr} + \frac{k_1}{r^3} w'_{,r} + \frac{k_1}{r^4} w'_{,\theta\theta\theta\theta} + \frac{2(k_1+k_2)}{r^4} w'_{,\theta\theta} + \\
&+\frac{2k_2}{r^2} w'_{,rr\theta\theta} - \frac{2k_2}{r^3} w'_{,r\theta\theta} - G_2 \frac{H'}{h_2}\frac{1}{r}\Big(\gamma'_{,\theta} + \delta + r\delta'_{,r} + H'\frac{1}{r} w'_{,\theta\theta} + H' w'_{,r} + \\
&+H' r w'_{,rr}\Big) = \frac{2h'}{r}\Big(\frac{2}{r^2}\Phi'_{,\theta} w'_{,r\theta} - \frac{2}{r}\Phi'_{,\theta r} w'_{,\theta r} + \frac{2}{r^2} w'_{,\theta}\Phi'_{,\theta r} - \frac{2}{r^3}\Phi'_{,\theta} w'_{,\theta} + \\
&+w'_{,r}\Phi'_{,rr} + \Phi'_{,r} w'_{,rr} + \frac{1}{r}\Phi'_{,\theta\theta} w'_{,rr} + \frac{1}{r}\Phi'_{,rr} w'_{,\theta\theta}\Big)
\end{aligned}
\tag{3.1}
$$

where:

$k_1 = 2D$, $k_2 = 4D_{r\theta} + \nu k_1$

$D = Eh'^3/[12(1-\nu^2)]$, $D_{r\theta} = Gh'^3/12$ – flexural rigidities of the outer layers

w – plate deflection

$\delta = u_3 - u_1, \ \gamma = v_3 - v_1$

$u_{1(3)}$, $v_{1(3)}$ – displacements of points of the middle plane of facings in the radial and circumferential directions, respectively

$H' = h' + h_2$

$\varPhi$ – stress function

- determination of additional equilibrium equations of projections in the radial u and circumferential v directions of forces loading the undeformed outer plate layers
- determination of the boundary conditions described as follows

$$
\begin{aligned}
&w\Big|_{r=r_o(r_i)} = 0 \qquad && w,_r\Big|_{r=r_o(r_i)} = 0\\
&\delta\Big|_{r=r_o(r_i)} = 0 \qquad && \delta,_r\Big|_{r=r_o(r_i)} = 0\\
&\gamma\Big|_{r=r_o(r_i)} = 0 \qquad && \gamma,_r\Big|_{r=r_o(r_i)} = 0
\end{aligned}
\tag{3.2}
$$

- determination of the following dimensionless quantities

$$
\begin{aligned}
&F = \frac{\varPhi}{Eh^2} \qquad && \zeta = \frac{w}{h} \qquad && \rho = \frac{r}{r_o}\\
&\overline{\delta} = \frac{\delta}{h} \qquad && \overline{\gamma} = \frac{\gamma}{h}
\end{aligned}
\tag{3.3}
$$

- assumption that the stress function $\varPhi$ is a solution to the disk state,
- application of the finite difference method for the approximation of derivatives with respect to the dimensionless radius ρ by central differences in discrete points and, after transformation, determination of the following forms of basic equations in the analysed problem

$$
\begin{aligned}
&\mathbf{M}_{AP}\boldsymbol{u} + \mathbf{M}_{AD}\boldsymbol{d} + \mathbf{M}_{AG}\boldsymbol{g} = p^*\mathbf{M}_{AC}\boldsymbol{u}\\
&\mathbf{M}_{ACP}\boldsymbol{u} = \mathbf{M}_{ACD}\boldsymbol{d} + \mathbf{M}_{ACG}\boldsymbol{g}\\
&\mathbf{M}_{P}\boldsymbol{u} = \mathbf{M}_{D}\boldsymbol{d} + \mathbf{M}_{G}\boldsymbol{g}
\end{aligned}
\tag{3.4}
$$

where:

$p^* = p/E$ – dimensionless load

$\boldsymbol{u}$, $\boldsymbol{d}$, $\boldsymbol{g}$ – vectors of plate deflections and differences of the radial u_i and circumferential v_i displacements of facings

$\mathbf{M}_{AP}$, $\mathbf{M}_{AC}$, $\mathbf{M}_{ACD}$, $\mathbf{M}_{ACG}$, $\mathbf{M}_{D}$, $\mathbf{M}_{G}$ – matrices of elements composed of geometric and material parameters of the plate and the quantity b of the length of the interval in the finite differences method and the number m of buckling waves

$\mathbf{M}_{AD}$ – matrix of geometric parameters and the quantity b

$\mathbf{M}_{AG}$ – matrix of geometric parameters and the number m

$\mathbf{M}_{ACP}$ – matrix with elements described by the quantity $b/2$

$\mathbf{M}_{P}$ – matrix with elements described by the number m

- solution to the eigen-value problem with calculation of the minimal value of p^* as the critical static load p^*_{cr} using the equation in the following form

$$\det\left[(\mathbf{M}_{AP} + \mathbf{M}_{AD}\mathbf{M}_{ATD} + \mathbf{M}_{AG}\mathbf{M}_{ATG}) - p^*\mathbf{M}_{AC}\right] = 0 \tag{3.5}$$

  and $\mathbf{M}_{ATD}$, $\mathbf{M}_{ATG}$ – matrices expressed after some transformations by the matrices $\mathbf{M}_{P}$, $\mathbf{M}_{D}$, $\mathbf{M}_{G}$, $\mathbf{M}_{ACD}$, $\mathbf{M}_{ACG}$, $\mathbf{M}_{ACP}$.

The calculations by means of the finite difference method have been preceded by selection of the number N of the discrete points, which fulfils the accuracy of calculation up to 5% of the technical error. The computational results of critical loads and numbers m of circumferential waves showing the buckling forms of plates with thick cores ($h_2 = 0.04\,\mathrm{m}$ and $0.06\,\mathrm{m}$) for different numbers of discrete points: $N = 11$, 14, 17, 21, 26 are presented in Tables 1 and 2. Exemplary results of plates with medium core thickness were presented by Pawlus (2006). The bold print results indicate the form of plate deformation (the number m of circumferential waves) corresponding to the minimal critical load p_{cr}, which has been calculated for different numbers of discrete points N. The number m does not change with the increase in the number N of discrete points. The number, equal to $N = 14$ has been accepted in the numerical calculations. The results show an essential increase in the number m of circumferential waves with the increase in the plate stiffness. This is exactly presented in Figs. 3 and 4.

Figures 3 and 4 present the distribution of the critical static loads of plates with different forms of their critical deformation expressed by the number m of waves in the circumferential direction for plates with the thickness of facing equal to: $h' = 0.0005\,\mathrm{m}$ and $0.001\,\mathrm{m}$, respectively. The points marked by • in the diagrams correspond to the minimal critical load for the determined form of plate buckling. The presented diagrams show an essential increase in the critical plate loading with an increase in the plate stiffness – it is along with

Table 1. Critical loads p_{cr} [MPa] for different wave numbers m of the plate with parameters: $G_2 = 5\,\mathrm{MPa}$, $h_2 = 0.06\,\mathrm{m}$, $h' = 0.001\,\mathrm{m}$

m \ N	11	14	17	21	26
5	123.49	123.90	124.18	124.43	124.62
6	117.94	118.30	118.54	118.76	118.93
7	114.87	115.18	115.39	115.58	115.74
8	113.44	113.71	113.90	114.06	114.20
9	**113.15**	**113.39**	**113.55**	**113.70**	**113.83**
10	113.72	113.92	114.07	114.20	114.31
11	114.95	115.13	115.25	115.37	115.47
12	116.72	116.88	116.99	117.09	117.17

Table 2. Critical loads p_{cr} [MPa] for different wave numbers m of the plate with parameters: $G_2 = 15.82\,\mathrm{MPa}$, $h_2 = 0.04\,\mathrm{m}$, $h' = 0.0005\,\mathrm{m}$

m \ N	11	14	17	21	26
19	375.49	376.15	376.59	377.00	377.36
20	374.58	375.23	375.68	376.10	376.47
21	373.96	374.62	375.08	375.50	375.87
22	373.62	374.28	374.74	375.17	375.54
23	**373.54**	**374.20**	**374.67**	**375.09**	**375.47**
24	373.71	374.36	374.83	375.26	375.63
25	374.10	374.75	375.21	375.64	376.01
26	374.70	375.34	375.81	376.24	376.61

the increase in the core thickness and Kirchhoff's modulus of the core material. The major number of circumferential waves corresponds to the critical buckling of stiffer plates, too. It is particularly high for plates with thick cores.

For these plates, the critical loads change insignificantly in a wide range of forms of plate deformations, eg. for plates with parameters: $h_2 = 0.06\,\mathrm{m}$, $h' = 0.0005\,\mathrm{m}$, $G_2 = 15.82\,\mathrm{MPa}$, the fluctuation of critical loads for the forms of buckling presented by the number m equal from 24 up to 29 are in the range in 543.54 MPa to 543.37 MPa with a decrease down to the minimal value: $p_{cr} = 542.70\,\mathrm{MPa}$ for $m = 27$. It could be found that these forms of plate buckling are equivalent to each other. The similar plate behaviour is observed for other cases presented in Figs. 3 and 4, particularly for plates with thin $(h' = 0.0005\,\mathrm{m})$ facings.

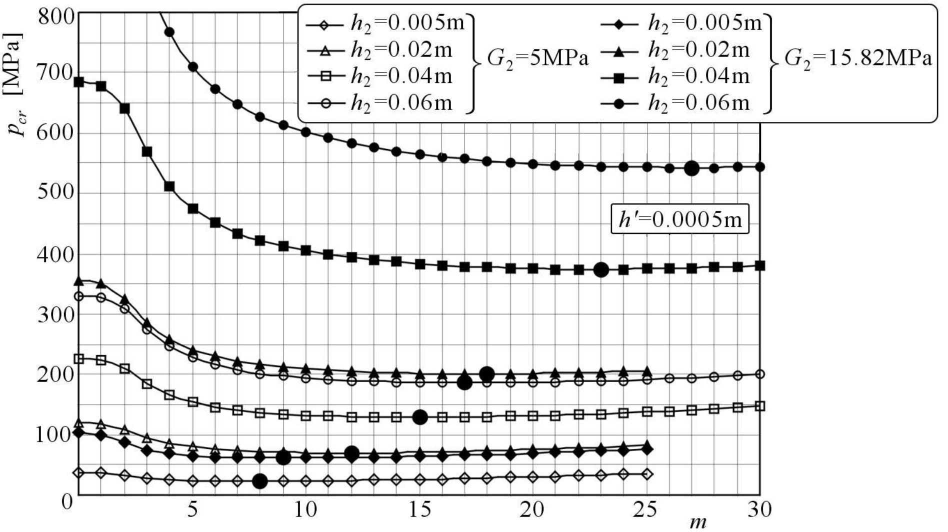

Fig. 3. Critical static load distributions depending on the number of buckling waves for the plate compressed on the outer perimeter with facing thickness $h' = 0.0005\,\mathrm{m}$

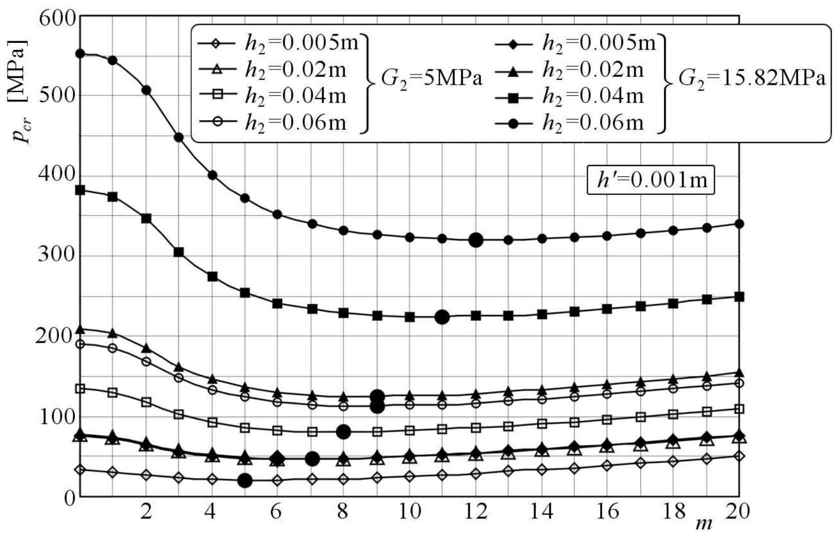

Fig. 4. Critical static load distributions depending on the number of buckling waves for the plate compressed on the outer perimeter with facing thickness $h' = 0.001\,\mathrm{m}$

The distribution of the critical static loads p_{cr} depending on the plate core thickness is shown in Fig. 5. The presented results are for plates loaded both on the inner and outer perimeter of facings. It is obvious that the values of loads p_{cr} increase with an increase in the plate stiffness, and the curve for plates with medium core thickness is near to a linear one. The observed, rather insignificant, decrease in the distribution of the critical loads for plates with thick cores (above $h_2 = 0.02\,\mathrm{m}$) could arise some questions about their excessive values. Therefore, the analysis has been supported by calculations carried out by the finite element method also for plate models without the simplification, which is connected with the assumption of the equal deflection of plate layers.

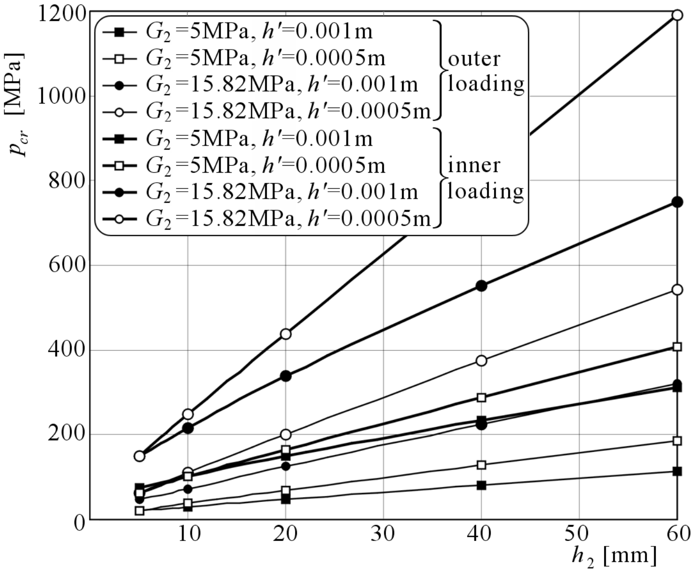

Fig. 5. Critical static load distributions depending on core thickness for plates compressed on the inner and outer perimeter

3.2. Calculations using the finite element method (FEM)

Simulating numerically the plate models built by means of the finite element method, it was possible to observe the essential differences in the critical loads of plates with the thick core. In this observation, the way of the plate layers connection, i.e. having or not having equal deflection was significant.

Calculations using the finite element method were carried out for plate models built of shell and solid elements. The application of the shell elements for the mesh of facings and the solid elements for the core mesh took into account different participation of the layers in carrying the plate loading: normal by the facings and shearing by the core.

The model is composed of 9-node 3D shell elements and 27-node 3D solid elements, which create the facing and core mesh, respectively. The outer surfaces of facing elements are tied with surfaces of the core elements by surface contact interaction. The plate model of the full annulus form supported in slidable clamped edges has been used in the analysis. Some results for the plate loaded on the inner perimeter, have been obtained using a quite simple model built of axisymmetric elements. The structure of such a plate model could be based on axisymmetric elements, because the plate loaded on the inner edge and supported in double slidable clamped edges loses its stability in regular, axially-symmetric form, which corresponds to the minimal value of the critical load. This observation has been confirmed by the general solution to the static stability problem of plates with the wave forms of buckling exactly presented in the work by Pawlus (2006). The structure of this model is the same as the annulus plate model. The facing mesh is built of shell 3-node elements but the core mesh is built of solid 8-node elements. A scheme of both plate meshes is presented in Fig. 6.

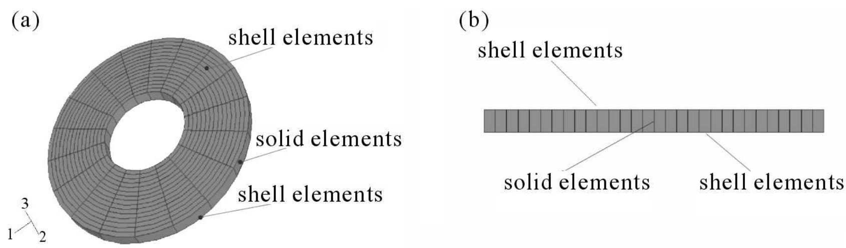

Fig. 6. The scheme of plate mesh: (a) annulus model, (b) model built of axisymmetric elements

The calculations were carried out at the Academic Computer Center CYFRONET-Cracow (MNiSW/SGI3700/Płódzka/016/2007) using the ABAQUS system (ABAQUS, 2000).

Figures 7-10 present results of calculations of critical loads for plates with various core thickness obtained using the Finite Difference Method (FDM) and the Finite Element Method (FEM). The calculations in FEM have been carried out for the full annulus plate model without the condition of equal deflection of the layers. Figures 7 and 8 present results for plates loaded on the outer edge, while Figures 9 and 10 concern plates loaded on the inner edge.

Diagrams presented in Figs. 9 and 10 show the results obtained by FEM for two kinds of plate models: full annulus – denoted by "line 1" (see the legend) and the model built of axisymmetric elements – denoted by "line 2".

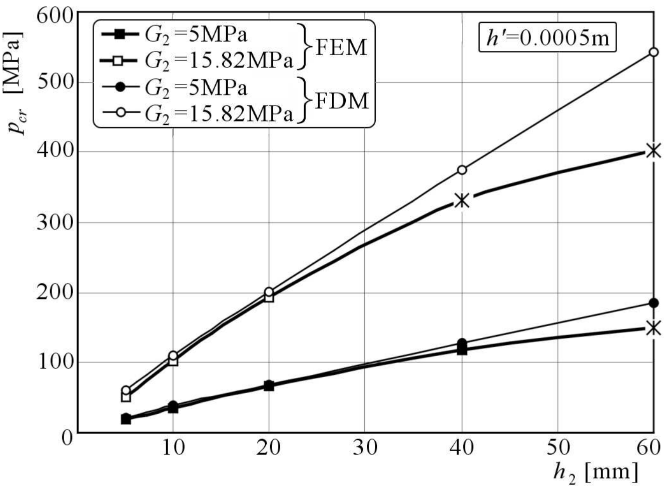

Fig. 7. Critical static load distribution depending on the core for the plate loaded on the outer perimeter with facing thickness $h' = 0.0005\,\mathrm{m}$

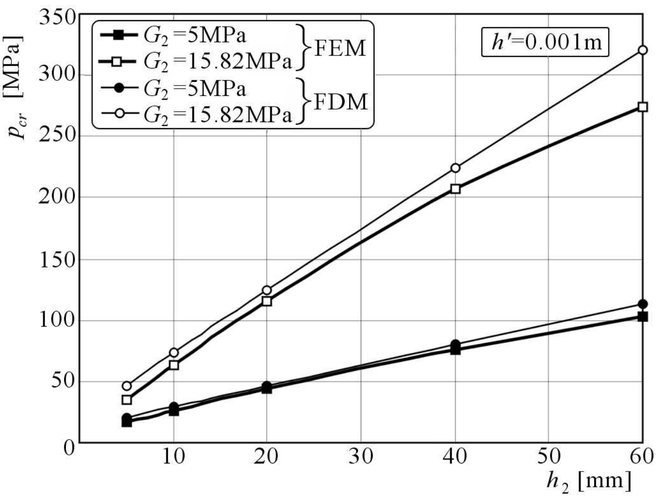

Fig. 8. Critical static load distribution depending on the core for the plate loaded on the outer perimeter with facing thickness $h' = 0.001\,\mathrm{m}$

The consistency of theses curves (lines 1 and 2) informs about the correctness of the mesh structure, and confirms the observations presented in this work. A decrease in the critical static loads of plates with the thick core (above $h_2 = 0.02\,\mathrm{m}$) is observed. Their forms of buckling maintain the global form, or particularly for plates with thin facings $(h' = 0.0005\,\mathrm{m})$ the form are connected with strong layer deformations. Marked by *times* points in Figs. 7 and 9

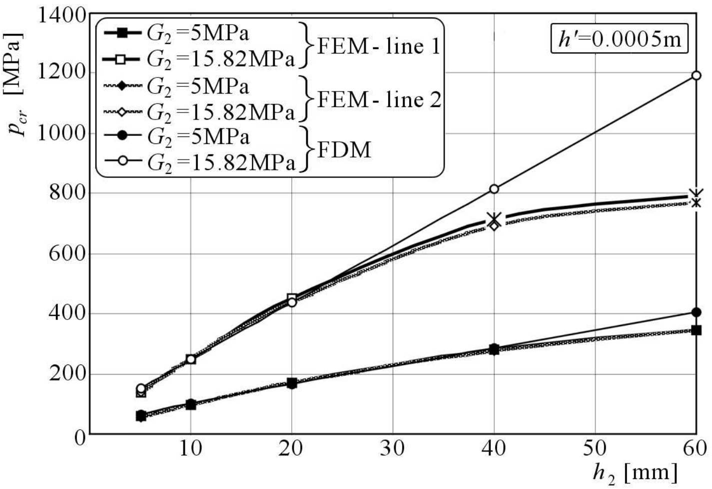

Fig. 9. Critical static load distribution depending on the core for the plate loaded on the inner perimeter with facing thickness $h' = 0.0005\,\mathrm{m}$

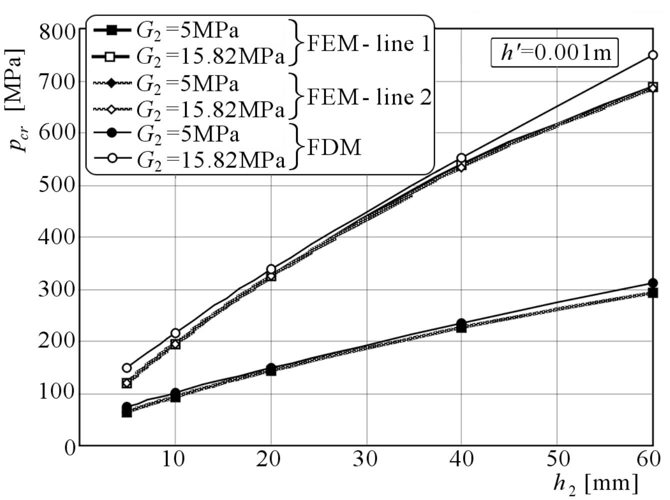

Fig. 10. Critical static load distribution depending on the core for the plate loaded on the inner perimeter with facing thickness $h' = 0.001\,\mathrm{m}$

exactly concern such cases of deformations of plates loaded on the outer and inner edges, respectively. These forms of deformations are shown in Figs. 11 and 12, respectively.

The observed critical buckling of plates loaded on the outer perimeter has a form of circumferential waves concentrated near the inner plate perimeter (see Fig. 11). At the same time, the critical deformation of the plate compressed

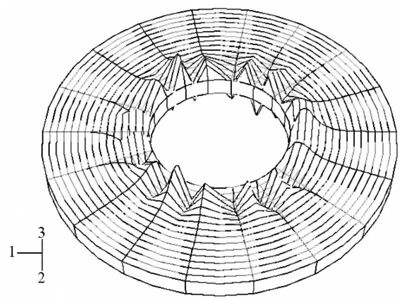

Fig. 11. Critical buckling form of the plate with parameters: $h' = 0.0005\,\mathrm{m}$, $h_2 = 0.06\,\mathrm{m}$ and $G_2 = 15.82\,\mathrm{MPa}$, loaded on outer perimeter

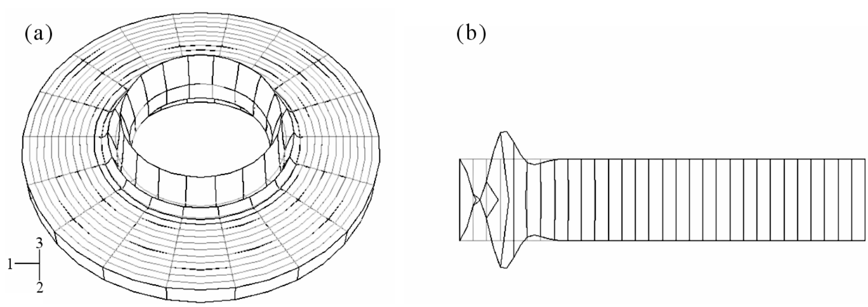

Fig. 12. Critical buckling form of the plate with parameters: $h' = 0.0005\,\mathrm{m}$, $h_2 = 0.06\,\mathrm{m}$ and $G_2 = 15.82\,\mathrm{MPa}$, loaded on inner perimeter and built: (a) as full annulus, (b) of axisymmetric elements

on the inner perimeter has a form of a strongly, radially deformed structure in the region of the loaded edge (Fig. 12) with the observed local form of the loss of stability (see Fig. 12b).

Table 3 presents detailed results of calculations by FDM and FEM for the plate with facing thickness equal to: $h' = 0.001\,\mathrm{m}$ compressed on the outer perimeter.

Exemplary forms of plate buckling are presented in Fig. 13. The results presented in Table 3 show consistency in the critical loads p_{cr} and forms of plate deformations found by numerical methods: FDM and FEM for plates with medium cores. For plates with the core thickness above $0.02\,\mathrm{m}$, one can observe a decrease in the critical loads obtained through the finite element method. For some plates, e.g. characterised by parameters: $h_2 = 0.04\,\mathrm{m}$, $G_2 = 15.82\,\mathrm{MPa}$, it is also observed that the number of circumferential buckling waves is lower for plate models calculated by FEM (the number m is 10) than by FDM $(m = 11)$.

Table 3. Critical loads of plates with facing thickness $h' = 0.001\,\mathrm{m}$ calculated by FDM and FEM

h_2 [m] / G_2 [MPa]	FDM		FEM	
	p_{cr} [MPa]	m	p_{cr} [MPa]	m
0.005 / 5.0	20.52	5	16.48	5
0.005 / 15.82	46.53	6	35.04	6
0.02 / 5.0	46.95	7	43.71	7
0.02 / 15.82	125.11	9	115.10	9
0.04 / 5.0	80.68	8	76.29	8
0.04 / 15.82	224.50	11	207.65	10
0.06 / 5.0	113.39	9	102.86	9
0.06 / 15.82	320.63	12	274.23	12

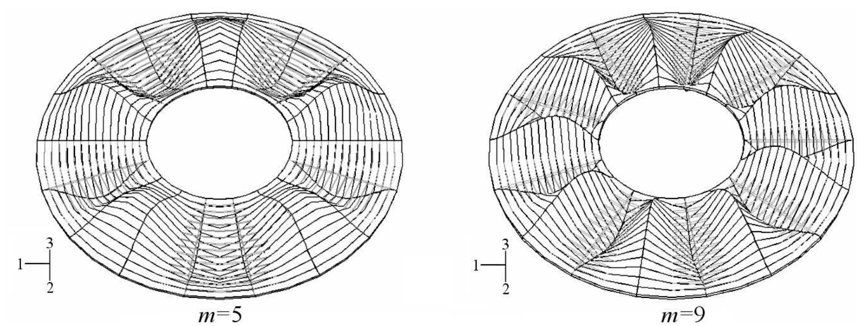

Fig. 13. Exemplary critical buckling forms of plates loaded on the outer perimeters

Table 4 presents the critical loads of plates with thin facings ($h' = 0.0005\,\mathrm{m}$) and medium cores of the thickness $h_2 = 0.02\,\mathrm{m}$. The values are consistent, whereas the forms of buckling found by FEM mostly have a lower number of circumferential waves than in plates solved by FDM.

The results of calculations by FEM of plate models with coupled layers and the results obtained by FDM are presented in Figs. 14-17. Figures 14 and 15 show the results for plates loaded on the outer edge, but Figures 16 and 17 present the results of plates compressed on the inner edge with the facing thickness equal to $h' = 0.0005\,\mathrm{m}$ and $h' = 0.001\,\mathrm{m}$, respectively. The introduction of the condition of equal deflections of three layers essentially changed the critical results of plates with thick cores. Strong critical deformations did not occur and the correspondence of results obtained by the two computational methods really increased.

Table 4. Critical loads of plates with the facing thickness $h' = 0.0005$ m calculated by FDM and FEM

h_2 [m] / G_2 [MPa]	FDM		FEM	
	p_{cr} [MPa]	m	p_{cr} [MPa]	m
0.005 / 5.0	22.37	8	19.16	7
0.005 / 15.82	61.51	9	52.03	9
0.01 / 5.0	38.56	9	35.53	9
0.01 / 15.82	109.75	13	101.72	10
0.02 / 5.0	69.49	12	66.56	10
0.02 / 15.82	200.62	18	193.15	12

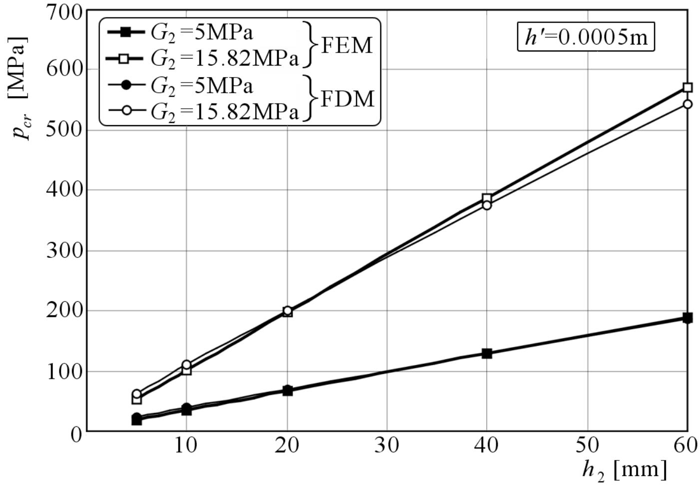

Fig. 14. Critical load distribution depending on the core for the plate with equal layer deflections, loaded on the outer perimeter and with facing thickness $h' = 0.0005$ m

Tables 5 and 6 present the critical loads and critical forms of buckling of plates with coupled layers compressed on the outer perimeter and with facing thickness equal to $h' = 0.001$ m and $h' = 0.0005$ m, respectively.

The presented results indicate a tendency of the number m of circumferential waves for critical buckling of plates calculated with coupled layers to decrease. It is particularly observed for plate models calculated by FEM. Exemplary forms of deformation are presented in Fig. 18 for plates with parameters: $h' = 0.0005$ m, $h_2 = 0.06$ m, $G_2 = 5$ MPa and $G_2 = 15.82$ MPa.

Detailed computational results for plates compressed on the inner perimeter were presented by Pawlus (2007). This study includes analysis of other

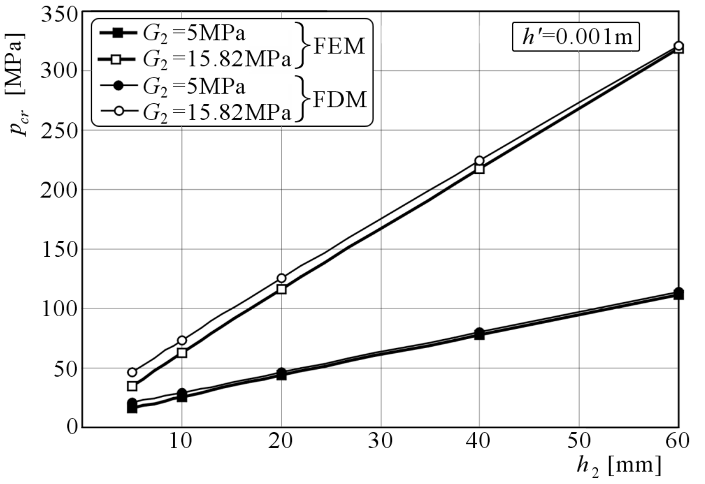

Fig. 15. Critical load distribution depending on the core for the plate with equal layer deflections, loaded on the outer perimeter and with facing thickness $h' = 0.001\,\mathrm{m}$

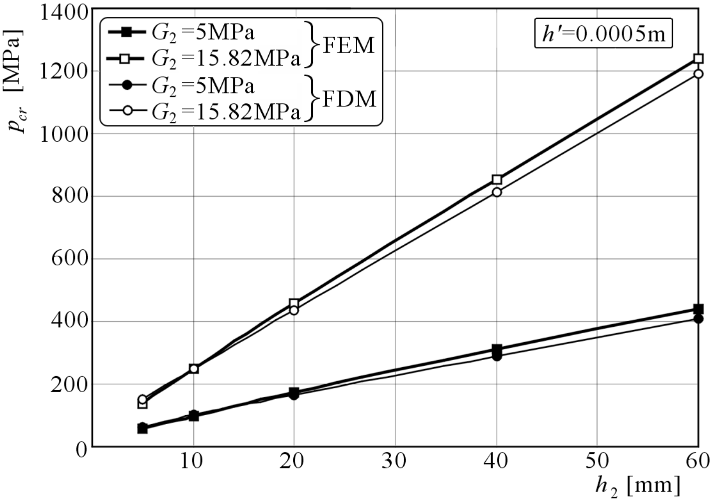

Fig. 16. Critical load distribution depending on the core for the plate with equal layer deflections, loaded on the inner perimeter and with facing thickness $h' = 0.0005\,\mathrm{m}$

than annulus plate models. The models are in the form:

- of an annular sector with a single or double layer of core elements
- of a model built of axisymmetric elements with a single, double or quaternary layer of core elements.

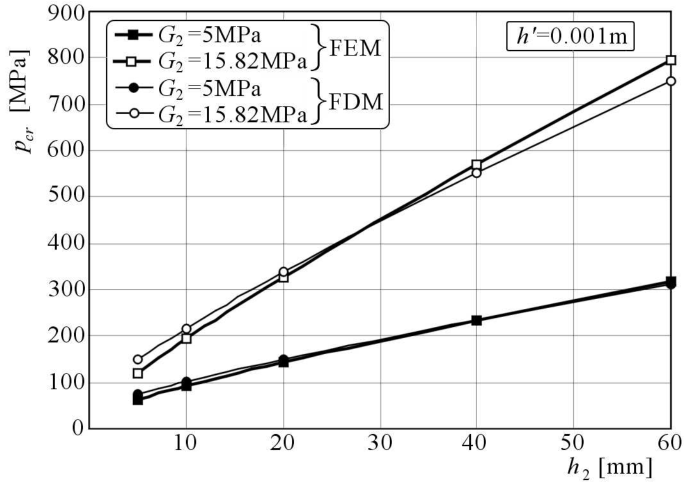

Fig. 17. Critical load distribution depending on the core for the plate with equal layer deflections, loaded on the inner perimeter and with facing thickness $h' = 0.001\,\mathrm{m}$

Table 5. Critical loads of plates with equal layer deflections calculated by FDM and FEM, with facing thickness $h' = 0.001\,\mathrm{m}$

h_2 [m] / G_2 [MPa]	FDM		FEM	
	p_{cr} [MPa]	m	p_{cr} [MPa]	m
0.005 / 5.0	20.52	5	16.49	5
0.005 / 15.82	46.53	6	35.06	6
0.02 / 5.0	46.95	7	43.97	7
0.02 / 15.82	125.11	9	116.31	8
0.06 / 5.0	113.39	9	111.60	8
0.06 / 15.82	320.63	12	318.34	10

The observations of these plates are similar to those of the plates loaded on the outer edge. The global form of critical buckling for the minimal critical load of plates compressed on the inner edge is regular axially-symmetric $(m = 0)$. This form for the full annulus plate model and for the model built of axisymmetric elements, which was used in FEM calculations (presented in Figs. 16, 17 and by line 2 in Figs. 9, 10), is shown in Fig. 19.

Generally, it could be noticed that the examinations indicate quantitative and qualitative compatibility of the results for critical loads of plates with medium core thickness (in the range of h_2 around 0.02 m). For these plates, the introduction of the condition of equal layer deflections or its absence do

Table 6. Critical loads of plates with equal layer deflections calculated by FDM and FEM, with facing thickness $h' = 0.0005\,\mathrm{m}$

h_2 [m] / G_2 [MPa]	FDM		FEM	
	p_{cr} [MPa]	m	p_{cr} [MPa]	m
0.005 / 5.0	22.37	8	19.18	7
0.005 / 15.82	61.51	9	52.10	8
0.02 / 5.0	69.49	12	67.39	9
0.02 / 15.82	200.62	18	198.65	12
0.06 / 5.0	185.68	17	189.45	12
0.06 / 15.82	542.70	27	571.69	14

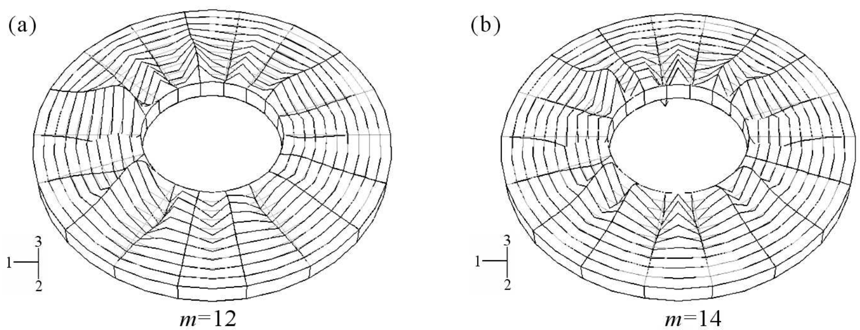

Fig. 18. Critical buckling form of the plate loaded on the outer perimeter with equal layer deflections and with parameters $h' = 0.0005\,\mathrm{m}$, $h_2 = 0.06\,\mathrm{m}$: (a) $G_2 = 5\,\mathrm{MPa}$, (b) $G_2 = 15.82\,\mathrm{MPa}$

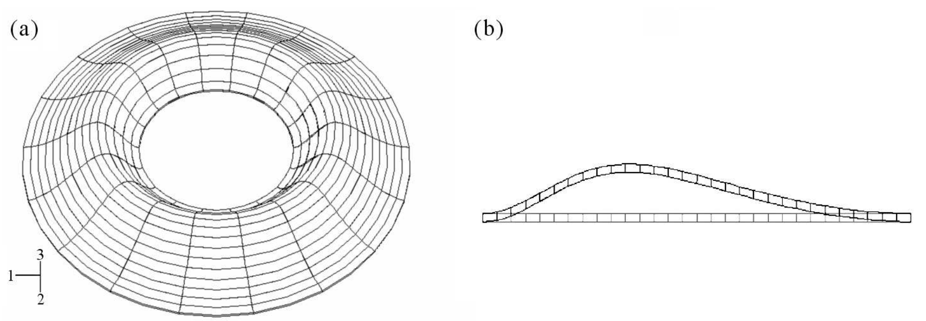

Fig. 19. Axially-symmetric buckling form of the plate compressed on the inner edge: (a) full annulus model, (b) model built of axisymmetric elements

not significantly influence the obtained results. At the same time, this condition really influences the final results for plates with core thickness above $h_2 = 0.02\,\mathrm{m}$.

Table 7. Comparison of values of critical loads p_{cr} [MPa] obtained for different plate models

h_2 [m] / G_2 [MPa] / h' [m]	**I**	**II**	**III**
0.005 / 5.0 / 0.0005	57.52	57.48	57.84
0.005 / 15.82 / 0.001	119.94	119.92	120.30
0.02 / 5.0 / 0.001	144.16	143.20	143.77
0.02 / 15.82 / 0.001	328.30	324.01	326.41
0.06 / 5.0 / 0.001	317.34	292.68	293.90

I – Plate model built of axisymmetric elements with coupled layers

II – Plate model built of axisymmetric elements without coupled layers

III – Annulus plate model without coupled layers

Additionally, while evaluating the correctness of the plate structure, analysis of critical loads convergent for different numbers of mesh elements has been undertaken. The plate model built of axisymmetric elements has been examined. The results are given for the plate loaded on the inner perimeter. The plate parameters are as follows: $h_2 = 0.02\,\mathrm{m}$, $G_2 = 5\,\mathrm{MPa}$, $h' = 0.001\,\mathrm{m}$. A

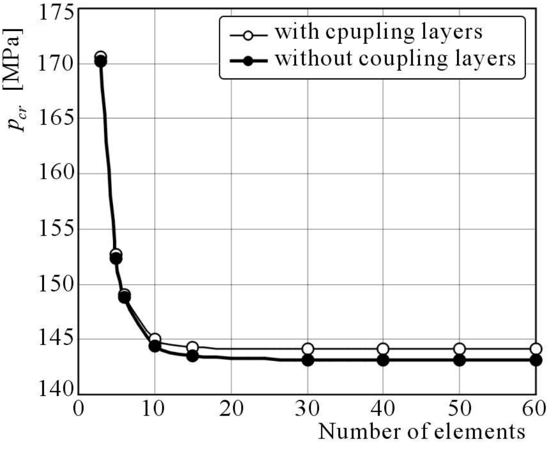

Fig. 20. Convergence of critical loads

diagram presented in Fig. 20 shows results for two analysed plate models: with coupled layers and without this condition. The convergence has been achieved for 15 elements in the radial direction. In the calculations, 30 elements have been accepted. The comparison and good correspondence of the results of plates modelled in the form of full annulus and composed of axisymmetric elements are presented in Table 7. The results are given for the plate loaded on the inner perimeter.

4. Conclusions

Numerical results of the critical static loads of three-layer annular plates with a foam core of medium thickness and a thick core are presented in this paper. The solution to the static stability problem is general. It includes circumferential wave forms of the critical buckling, observed in particular in plates compressed on the outer perimeter. The critical loads of such plates have been specially considered. The calculations have been carried out by means of the finite difference and finite elements method. The model of plate calculated by the finite difference method uses the classical theory of sandwich plates with the broken line hypothesis and the assumption on the equal deflection of three plate layers. The plate models built for the finite element method have essentially an annular plate structure and differ by condition of equal layer deflections. The influence of this condition on the critical loads in thick-core plates is very significant. The results for plates with core thickness above $h_2 = 0.02\,\mathrm{m}$, treated as thick, indicate a great decrease in the critical loads and the possibility of occurrence of different buckling forms than those obtained in models of plates calculated by the finite difference method or models built for the finite element method with coupled layers. It could be stated that the critical loads of plates with thick cores obtained with the use of the finite difference method are too high. Particularly, it is observed for plates with greater Kirchhoff's modulus ($G_2 = 15.82\,\mathrm{MPa}$) and thinner facings ($h' = 0.0005\,\mathrm{m}$). Generally, it could be determined that the presented numerical results show sensitivity of analysed plates to their geometric and material parameters. Here, the thickness of the foam core has the main importance. For thinner cores, the use of the simplifying condition of equal layer deflections in the solution is possible, whereas for thicker cores this simplification may cause incorrect results. It can be taken into consideration in the modeling of sandwich plates. This fact was pointed out by Romanów (1995) who indicated the necessity of implementing a hyperbolic form of deformation of plates with thicker cores.

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Ocena krytycznych obciążeń statycznych pierścieniowych płyt trójwarstwowych o różnych grubościach rdzenia

Streszczenie

W pracy poddano ocenie wyniki obliczeń pierścieniowych płyt trójwarstwowych z miękkim, piankowym rdzeniem o różnej grubości. Płyty obciążano równomiernie rozłożonym ciśnieniem ściskającym wewnętrzny lub zewnętrzny obwód okładzin płyty. Badanymi wynikami obliczeń są wartości krytycznych obciążeń statycznych płyt i odpowiadające im postaci deformacji krytycznych. Analizie poddano płyty o symetrycznej strukturze poprzecznej i utwierdzonych przesuwnie krawędziach. Obliczenia prowadzono dwoma metodami przybliżonymi: metodą różnic skończonych i metodą elementów skończonych. Przedstawione w obu metodach rozwiązanie zagadnienia stateczności statycznej dotyczy ogólnego problemu utraty stateczności płyty, w którym możliwe formy krytycznej deformacji określa liczba m-fal poprzecznych na jej obwodzie. Szczegółowej analizie poddano płyty z rdzeniem traktowanym jako gruby. Oceniając wyniki, zwrócono uwagę na obserwowany znaczący spadek wartości obciążeń krytycznych płyt właśnie z rdzeniem grubym. Spostrzeżenia te ujawniły wyniki obliczeń prowadzone metodą elementów skończonych modeli płyt różniących się wprowadzeniem dodatkowego, upraszczającego warunku wiążącego warstwy płyty założeniem ich jednakowych ugięć. Założenie to należy do formuł opisujących globalny stanu przemieszczeń płyty, której deformacja krytyczna ma postać globalną. Inne spodziewane właśnie dla płyt z rdzeniem grubym formy utraty stateczności występują dla ogólnego stanu przemieszczeń pozbawionego upraszczającego warunku równości ugięć trzech warstw płyty. Wpływ tego warunku na wyniki krytyczne płyt zbadano prowadząc obliczenia numeryczne modeli płyt w obu metodach. Obserwacja obszarów dobrej zgodności oraz istotnych różnic wyników płyt odpowiednio z rdzeniem średniej grubości i grubym jest zasadniczym efektem podjętej analizy, która w zagadnieniach modelowania struktur płyt może mieć istotne znaczenie praktyczne.

Manuscript received May 23, 2007; accepted for print October 15, 2007