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INFLUENCE OF ELASTIC MATERIAL COMPRESSIBILITY ON PARAMETERS OF THE EXPANDING SPHERICAL STRESS WAVE. I. ANALYTICAL SOLUTION TO THE PROBLEM

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We investigated the influence of elastic material compressibility on parameters of the expanding spherical stress wave. The material compressibility is represented by Poisson's ratio ν. The stress wave is generated by pressure created inside the spherical cavity. The isotropic elastic material surrounds this cavity. Analytical closed form formulae determining the dynamical state of mechanical parameters (displacement, particle velocity, strains, stresses, and material density) in the material have been derived. These formulae were obtained for surge pressure $p(t) = p_0 =$ const inside the cavity. From analysis of these formulae it results that Poisson's ratio ν substantially influences the course of material parameters in space and time. All parameters intensively decrease in space together with increase of the Langrangian coordinate r. On the contrary, these parameters oscillate versus time around their static values. These oscillations decay with a lapse of time. We can mark out two ranges of the parameter ν values in which vibrations of the parameters are damped with a different degree. Thus, a decrease in Poisson's ratio in the range $\nu \leqslant 0.4$ causes an intense decay of oscillation of parameters. On the other hand, in the range $0.4 < \nu < 0.5$, i.e. in quasi-compressible materials the damping of parameters vibrations is very low. In the limiting case when $\nu = 0.5$, i.e. in the incompressible material damping vanishes, and the parameters harmonically oscillate around their static values. The abnormal behaviour of the material occurs in the range $0.4 < \nu \leqslant 0.5$. In this case an insignificant increment of Poisson's ratio causes considerable an increase of the parameters vibration amplitude. The specific influence of Poisson's ratio on the parameters of the expanding spherical stress wave in elastic media is the main result of this paper. As we see it, this fact may be the contribution supplementing the description of properties of the expanding spherical stress wave in elastic media.

Key words: expansion of spherical stress wave, isotropic elastic material, dynamic load

1. Introduction

In some theoretical analyses of dynamics of liners driven by explosives, the compressibility of their materials is neglected. On one hand, this assumption makes analytical solution to many boundary value problems possible (Walters and Zukas, 1989; Cole, 1948; Kaliski *et al.*, 1992; Gurney, 1943, 1947; Taylor, 1961; Trębiński *et al.*, 1988a,b, 1989a; Włodarczyk and Zielenkiewicz, 2008). On the other hand, this simplification in a real physical system neutralises the wavy course of the process in this system. On the contrary, results of theoretical analyses (Lambour and Harley, 1965; Vidart *et al.*, 1965; Knoepfel, 1970; Trębiński *et al.*, 1989b) and experimental studies (Gimenez *et al.*, 1985; Derentowicz *et al.*, 1984) show that wave phenomena have substantial quantitative and qualitative influence on the driving process of solids.

Bearing in mind the results of these publications and the needs of explosion mechanics, the influence of the elastic material compressibility on parameters of the expanding spherical stress wave has been theoretically investigated in this paper. The stress wave has been generated by pressure dynamically created inside the spherical cavity. The isotropic linear-elastic material surrounds this cavity. The material compressibility is represented by Poisson's ratio ν.

The paper consists of two parts. An analytical solution to the considered problem and its introductory analysis are placed in the first part. In turn, the vast quantitative and qualitative analysis of mechanical parameters (displacement, particle velocity, strains, stress, and material density) of the expanding spherical stress wave in the compressible isotropic linear-elastic material versus the Lagrangian coordinate r and time t are presented in the second part.

The results of analysis presented in this paper can be used, among other things, to research spherical ballistic casings (Włodarczyk and Zielenkiewicz, 2008). From our point of view, the results of this analysis are a modest contribution of knowledge to the theory of propagation of stress waves in solids.

2. Formulation of the problem

Propagation of the expanding spherical stress wave in an unbounded elastic material is considered. The material is isotropic and compressible. The stress wave has been generated by a time–dependent pressure created inside the spherical cavity (Fig. 1). The symbol r_0 placed in Fig. 1 denotes the initial radius of the cavity.

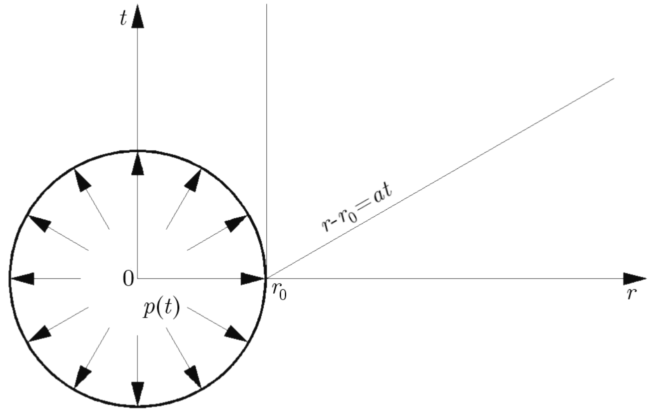

Fig. 1. Physical scheme of the boundary value problem

Taking into account spherical symmetry of the problem, it can be considered as a one-dimensional boundary value problem. Independent variables of the problem are the Langrangian coordinate r and time t. The states of stress and strain in the material are represented by the following components: σ_r – radial stresses, $\sigma_\varphi = \sigma_\theta$ – circumferential stresses, ε_r – radial strain and $\varepsilon_\varphi = \varepsilon_\theta$ – circumferential (tangential) strains.

The rest of components of the stress and the strain tensors are equal to zero in the considered coordinate system.

According to the linear elasticity theory (Nowacki, 1970), we have

$$\varepsilon_r = \frac{\partial u}{\partial r} \qquad \varepsilon_\varphi = \varepsilon_\theta = \frac{u}{r} \qquad \upsilon = \frac{\partial u}{\partial t} \tag{2.1}$$

and

$$\begin{aligned}\sigma_r &= 2\mu\varepsilon_r + \lambda(\varepsilon_r + 2\varepsilon_\varphi) = (2\mu+\lambda)\varepsilon_r + 2\lambda\varepsilon_\varphi = (2\mu+\lambda)\frac{\partial u}{\partial r} + 2\lambda\frac{u}{r}\\ \sigma_\varphi &= 2\mu\varepsilon_\varphi + \lambda(\varepsilon_r + 2\varepsilon_\varphi) = 2(\mu+\lambda)\varepsilon_\varphi + \lambda\varepsilon_r = 2(\mu+\lambda)\frac{u}{r} + \lambda\frac{\partial u}{\partial r}\end{aligned} \tag{2.2}$$

where u is the radial displacement, υ is the radial particle velocity, λ and μ are Lame's constants, namely

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)} \tag{2.3}$$

In turn, the symbols E and ν denote Young's modulus and Poisson's ratio, respectively.

For an element of the linear-elastic material, the equation of motion can be written in the form

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_r}{\partial r} + \frac{2(\sigma_r - \sigma_\varphi)}{r} \tag{2.4}$$

where ρ_0 is the initial density of the material.

Through substitution of expressions (2.2) into Eq. (2.4) and simple transformations, we obtain

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial r^2} + 2a^2 \Big(\frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \Big) \tag{2.5}$$

where

$$a^2 = \frac{1-\nu}{(1+\nu)(1-2\nu)} a_0^2 \qquad\qquad a_0^2 = \frac{E}{\rho_0} \tag{2.6}$$

The quantity a denotes the velocity of stress wave propagation.

Equation (2.5) has been solved for the following boundary conditions

$$u(r,t) \equiv 0 \qquad \text{for} \quad r = r_0 + at \tag{2.7}$$

and

$$\sigma_r(r,t) = \begin{cases} (2\mu + \lambda) \dfrac{\partial u(r,t)}{\partial r} + 2\lambda \dfrac{u(r,t)}{r} = -p(t) & \text{for} \quad r = r_0 \quad p(t) > 0 \\ \sigma_r(r,t) \equiv 0 & \text{for} \quad r = \infty \end{cases} \tag{2.8}$$

According to the mass conservation law, the equation of medium continuity can be written as follows

$$\Big(1 + \frac{u}{r}\Big)^2 \Big(1 + \frac{\partial u}{\partial r}\Big) = \frac{\rho_0}{\rho(r,t)} \tag{2.9}$$

where $\rho(r,t)$ is current density of the material.

For small strains, Eq. (2.9) can be transformed into the form

$$1 + \frac{\partial u}{\partial r} + 2\frac{u}{r} = \frac{\rho_0}{\rho} \tag{2.10}$$

The structure of the analytical solution to the problem formulated above has been presented below.

3. General analytical solution to the problem

The general solution of Eq. (2.5) has the following form (Achenbach, 1975; Broberg, 1956; Graff, 1975; Hopkins, 1960; Kaliski *et al.*, 1992)

$$u(r,t) = \frac{\varphi'(r-r_0-at)}{r} - \frac{\varphi(r-r_0-at)}{r^2} \tag{3.1}$$

where the symbol φ' denotes derivative of the function φ with respect to its argument.

For $\varphi(\infty) \neq \infty$ and $\varphi'(\infty) \neq \infty$ solution (3.1) fulfils also boundary condition $(2.8)_2$.

Boundary condition (2.7) and solution (3.1) yield

$$\varphi'(0) = \varphi(0) = 0 \tag{3.2}$$

where $r - r_0 - at = 0$ is the equation of the wave front propagating from the cavity (Fig. 1).

The numerical values of the independent variables r and t are contained within the intervals

$$r_0 \leqslant r \leqslant \infty \qquad\qquad t \geqslant \frac{r-r_0}{a} \tag{3.3}$$

Expression (3.1) and condition $(2.7)_1$ yield

$$\varphi''(x_0) - 2h\varphi'(x_0) + \frac{2h}{r_0}\varphi(x_0) = -\frac{1}{E}\frac{(1+\nu)(1-2\nu)}{1-\nu}r_0 p\Big(-\frac{x_0}{a}\Big) \tag{3.4}$$

where

$$h = \frac{1-2\nu}{1-\nu}\frac{1}{r_0} \geqslant 0 \qquad\qquad x_0 = -at \tag{3.5}$$

Straightforward integration of Eq. (3.4) together with initial conditions (3.2) results in

$$\varphi(x_0) = -\frac{(1+\nu)\sqrt{1-2\nu}}{E}r_0^2\int\limits_0^{x_0} p\Big(\frac{y-x_0}{a}\Big)\mathrm{e}^{hy}\sin(\omega y)\,dy \tag{3.6}$$

where

$$\omega = \frac{\sqrt{1-2\nu}}{(1-\nu)r_0}$$

The function φ and its derivatives φ' and φ'' uniquely determine all parameters of the considered problem.

If the spherical cavity surface is loaded by the pressure $p = p_0 =$ const created in a statical way, then Eq. (2.5) can be written in the form

$$\frac{d^2 u_s}{dr^2} + 2\Big(\frac{1}{r}\frac{du_s}{dr} - \frac{u_s}{r^2}\Big) = 0 \tag{3.7}$$

with boundary conditions

$$\begin{aligned} &\sigma_r(r_0) = (2\mu + \lambda)\frac{du_s}{dr}\Big|_{r=r_0} + 2\lambda\frac{u_s}{r_0} = -p_0 \qquad\qquad p_0 > 0 \\ &\sigma_r(\infty) = 0 \end{aligned} \tag{3.8}$$

The general solution to Eq. (3.7) has the form

$$u_s(r) = Cr + \frac{D}{r^2} \tag{3.9}$$

From relationships (2.3), boundary conditions (3.8) and solution (3.9) it follows that $C = 0$, and $D = (1+\nu)r_0^3 p_0/(2E)$.

Finally, the parameters determining the statical state of the material are defined by the formulae

$$\begin{aligned} &u_s(r) = \frac{1+\nu}{2}\frac{p_0}{E} r_0\Big(\frac{r_0}{r}\Big)^2 \\ &\varepsilon_{rs}(r) = -(1+\nu)\frac{p_0}{E}\Big(\frac{r_0}{r}\Big)^3 \qquad\qquad \varepsilon_{\varphi s}(r) = \frac{1+\nu}{2}\frac{p_0}{E}\Big(\frac{r_0}{r}\Big)^3 \\ &\sigma_{rs}(r) = -p_0\Big(\frac{r_0}{r}\Big)^3 \qquad\qquad \sigma_{\varphi s}(r) = \frac{p_0}{2}\Big(\frac{r_0}{r}\Big)^3 \end{aligned} \tag{3.10}$$

4. Particular solution to the problem for surge pressure inside the cavity

Integration of Eq.(3.6) for $p(t) = p_0 =$ const and differentiation of the function $\varphi(x)$ yield

$$\begin{aligned} &\varphi(x) = -\frac{1+\nu}{2} r_0^3 \frac{p_0}{E}[1 + \sqrt{1-2\nu}\mathrm{e}^{hx}\sin(\omega x) - \mathrm{e}^{hx}\cos(\omega x)] \\ &\varphi'(x) = -(1+\nu)\sqrt{1-2\nu} r_0^2 \frac{p_0}{E}\mathrm{e}^{hx}\sin(\omega x) \\ &\varphi''(x) = -\frac{(1+\nu)(1-2\nu)}{1-\nu} r_0 \frac{p_0}{E}\mathrm{e}^{hx}[\sqrt{1-2\nu}\sin(\omega x) + \cos(\omega x)] \end{aligned} \tag{4.1}$$

where

$$
\begin{aligned}
\omega x &= \frac{\sqrt{1-2\nu}}{1-\nu}\Big(\frac{r}{r_0} - 1 - \sqrt{\frac{1-\nu}{(1+\nu)(1-2\nu)}}\frac{a_0 t}{r_0}\Big) \\
hx &= \frac{1-2\nu}{1-\nu}\Big(\frac{r}{r_0} - 1 - \sqrt{\frac{1-\nu}{(1+\nu)(1-2\nu)}}\frac{a_0 t}{r_0}\Big)
\end{aligned}
\tag{4.2}
$$

In order to simplify the quantitative analysis of the stress wave parameters, the following dimensionless quantities have been introduced

$$
\begin{array}{lll}
\xi = \dfrac{r}{r_0} & \eta = \dfrac{a_0 t}{r_0} & U = \dfrac{u}{r_0} \\
U_s = \dfrac{u_s}{r_0} & V = \dfrac{v}{a_0} & R = \dfrac{\rho}{\rho_0} \\
S_r = \dfrac{\sigma_r}{p_0} & S_{rs} = \dfrac{\sigma_{rs}}{p_0} & S_\varphi = \dfrac{\sigma_\varphi}{p_0} \\
S_{\varphi s} = \dfrac{\sigma_{\varphi s}}{p_0} & P = \dfrac{p_0}{E} &
\end{array}
\tag{4.3}
$$

The dimensionless variables ξ and η, according to relationships (3.3), (2.6) and (4.3) are contained within the intervals

$$
1 \leqslant \xi \leqslant \infty \qquad \eta \geqslant \sqrt{\frac{(1+\nu)(1-2\nu)}{1-\nu}}(\xi - 1) \tag{4.4}
$$

The above-mentioned formulae determining the stress wave parameters, expressed by dimensionless quantities (4.3), can be written as follows

$$
\begin{aligned}
U(\xi,\eta) &= \frac{1+\nu}{2}\frac{P}{\xi^2}\{1 - [\sqrt{1-2\nu}(2\xi-1)\sin(\omega x) + \cos(\omega x)]e^{hx}\} \\
V(\xi,\eta) &= \frac{P}{\xi}\Big\{\Big[(1-2\nu)\sqrt{\frac{1+\nu}{1-\nu}} - \sqrt{1-\nu^2}\frac{1}{\xi}\Big]\sin(\omega x) + \\
&\quad + \sqrt{\frac{(1+\nu)(1-2\nu)}{1-\nu}}\cos(\omega x)\Big\}e^{hx} \\
\varepsilon_r(\xi,\eta) &= -(1+\nu)P\Big\{\frac{1}{\xi^3} + \Big[\sqrt{1-2\nu}\Big(\frac{1-2\nu}{1-\nu}\frac{1}{\xi} - 2\frac{1}{\xi^2} + \frac{1}{\xi^3}\Big)\sin(\omega x) + \\
&\quad - \Big(\frac{1}{\xi^3} - \frac{1-2\nu}{1-\nu}\frac{1}{\xi}\Big)\cos(\omega x)\Big]e^{hx}\Big\} \\
\varepsilon_\varphi(\xi,\eta) &= \frac{1+\nu}{2}\frac{P}{\xi^3}\{1 - [\sqrt{1-2\nu}(2\xi-1)\sin(\omega x) + \cos(\omega x)]e^{hx}\Big\} \\
R(\xi,\eta) &= \frac{1}{1 + \varepsilon_r(\xi,\eta) + 2\varepsilon_\varphi(\xi,\eta)}
\end{aligned}
\tag{4.5}
$$

$$
\begin{aligned}
S_r(\xi,\eta) &= -\frac{1}{\xi^3}\{1+(\xi-1)[\sqrt{1-2\nu}(\xi-1)\sin(\omega x)+(\xi+1)\cos(\omega x)]e^{hx}\}\\
S_\varphi(\xi,\eta) &= \frac{1}{2\xi^3}\Big\{1+\Big[\sqrt{1-2\nu}\Big(-\frac{2\nu}{1-\nu}\xi^2-2\xi+1\Big)\sin(\omega x)+\\
&\quad -\Big(\frac{2\nu}{1-\nu}\xi^2+1\Big)\cos(\omega x)\Big]e^{hx}\Big\}
\end{aligned}
$$

where

$$
\omega x = \frac{\sqrt{1-2\nu}}{1-\nu}(\xi-1)-\frac{1}{\sqrt{1-\nu^2}}\eta \qquad hx = \frac{1-2\nu}{1-\nu}(\xi-1)-\sqrt{\frac{1-2\nu}{1-\nu^2}}\eta \tag{4.6}
$$

On the cavity surface, i.e. for $\xi = 1$, from these formulae we obtain

$$
\begin{aligned}
U(1,\eta) &= \frac{1+\nu}{2}P\Big[1-\sqrt{2(1-\nu)}\exp\Big(-\sqrt{\frac{1-2\nu}{1-\nu^2}}\eta\Big)\sin\Big(-\frac{1}{\sqrt{1-\nu^2}}\eta+\alpha\Big)\Big]\\
V(1,\eta) &= \sqrt{1-\nu^2}P\exp\Big(-\sqrt{\frac{1-2\nu}{1-\nu^2}}\eta\Big)\sin\Big(\frac{1}{\sqrt{1-\nu^2}}\eta+\alpha_V\Big)\\
\varepsilon_r(1,\eta) &= -(1+\nu)P\Big[1-\sqrt{\frac{2\nu^2}{1-\nu}}\exp\Big(-\sqrt{\frac{1-2\nu}{1-\nu^2}}\eta\Big)\sin\Big(-\frac{1}{\sqrt{1-\nu^2}}\eta+\alpha\Big)\Big]\\
\varepsilon_\varphi(1,\eta) &= U(1,\eta) \qquad\qquad (4.7)\\
R(1,\eta) &= \Big[1-(1+\nu)(1-2\nu)\sqrt{\frac{2}{1-\nu}}P\exp\Big(-\sqrt{\frac{1-2\nu}{1-\nu^2}}\eta\Big)\cdot\\
&\quad\cdot\sin\Big(-\frac{1}{\sqrt{1-\nu^2}}\eta+\alpha\Big)\Big]^{-1}\\
S_r(1,\eta) &= -1 \quad - \quad \text{boundary condition}\\
S_\varphi(1,\eta) &= \frac{1}{2}-\frac{\nu+1}{\sqrt{2(1-\nu)}}\exp\Big(-\sqrt{\frac{1-2\nu}{1-\nu^2}}\eta\Big)\sin\Big(-\frac{1}{\sqrt{1-\nu^2}}\eta+\alpha\Big)
\end{aligned}
$$

where

$$
\tan\alpha = \frac{1}{\sqrt{1-2\nu}} \qquad \tan\alpha_V = \frac{\sqrt{1-2\nu}}{\nu} \tag{4.8}
$$

In the limiting case when $\nu = 0.5$, i.e. for an incompressible material, formulae (4.6) and (4.8) yield

$$
\omega x = -\frac{2}{\sqrt{3}}\eta \qquad hx = 0 \qquad \alpha = \frac{\pi}{2} \qquad \alpha_V = 0 \tag{4.9}
$$

In this case from formulae (4.5) we obtain

$$
\begin{aligned}
&U(\xi,\eta)=\frac{3}{4}P\frac{1}{\xi^2}\Big(1-\cos\frac{2}{\sqrt{3}}\eta\Big)\\
&V(\xi,\eta)=\frac{\sqrt{3}}{2}P\frac{1}{\xi^2}\sin\frac{2}{\sqrt{3}}\eta\\
&\varepsilon_r(\xi,\eta)=-\frac{3}{2}P\frac{1}{\xi^3}\Big(1-\cos\frac{2}{\sqrt{3}}\eta\Big)\\
&\varepsilon_\varphi(\xi,\eta)=\frac{3}{4}P\frac{1}{\xi^3}\Big(1-\cos\frac{2}{\sqrt{3}}\eta\Big)\\
&R(\xi,\eta)=1\\
&S_r(\xi,\eta)=-\frac{1}{\xi^3}\Big[1+(\xi^2-1)\cos\frac{2}{\sqrt{3}}\eta\Big]\\
&S_\varphi(\xi,\eta)=\frac{1}{2\xi^3}\Big[1-(2\xi^2+1)\cos\frac{2}{\sqrt{3}}\eta\Big]
\end{aligned}
\tag{4.10}
$$

The expressions analogous to formulae (4.10) have been obtained for a spherical casing in Włodarczyk and Zielenkiewicz (2008), namely

$$
\begin{aligned}
&U(\xi,\eta)=\frac{3}{4}P\frac{\beta^3}{\beta^3-1}\frac{1}{\xi^2}\Big[1-\cos\Big(\frac{2}{\sqrt{3}}\sqrt{1+\frac{\beta+1}{\beta^2}}\eta\Big)\Big]\\
&V(\xi,\eta)=\frac{\sqrt{3}}{2}P\frac{\beta^3}{\beta^3-1}\sqrt{1+\frac{\beta+1}{\beta^2}}\frac{1}{\xi^2}\sin\Big(\frac{2}{\sqrt{3}}\sqrt{1+\frac{\beta+1}{\beta^2}}\eta\Big)\\
&\varepsilon_r(\xi,\eta)=-\frac{3}{2}P\frac{\beta^3}{\beta^3-1}\frac{1}{\xi^3}\Big[1-\cos\Big(\frac{2}{\sqrt{3}}\sqrt{1+\frac{\beta+1}{\beta^2}}\eta\Big)\Big]\\
&\varepsilon_\varphi(\xi,\eta)=\frac{3}{4}P\frac{\beta^3}{\beta^3-1}\frac{1}{\xi^3}\Big[1-\cos\Big(\frac{2}{\sqrt{3}}\sqrt{1+\frac{\beta+1}{\beta^2}}\eta\Big)\Big]\\
&R(\xi,\eta)=1\\
&S_r(\xi,\eta)=-\frac{1}{\xi^3-1}\Big\{\Big(\frac{\beta}{\xi}\Big)^3-1+\\
&\qquad-\Big[\Big(\frac{\beta}{\xi}\Big)^3-(\beta^2+\beta+1)\Big(\frac{\beta}{\xi}-1\Big)-1\Big]\cos\Big(\frac{2}{\sqrt{3}}\sqrt{1+\frac{\beta+1}{\beta^2}}\eta\Big)\Big\}\\
&S_\varphi(\xi,\eta)=\frac{1}{2(\beta^3-1)}\Big\{2+\Big(\frac{\beta}{\xi}\Big)^3+\\
&\qquad-\Big[\Big(\frac{\beta}{\xi}\Big)^3+2(\beta^2+\beta+1)\Big(\frac{\beta}{\xi}-1\Big)+2\Big]\cos\Big(\frac{2}{\sqrt{3}}\sqrt{1+\frac{\beta+1}{\beta^2}}\eta\Big)\Big\}
\end{aligned}
\tag{4.11}
$$

where $\beta = r_1/r_0$. The symbols r_1 and r_0 denote the outer and inner radii of the casing, respectively.

From the analysis of formulae (4.10) and (4.11) it follows that by their means, for $\beta \geqslant 5$, we obtain comparable numerical results. The differences are smaller than 1%.

5. Introductory analysis of the problem

We consider parameters of the stress wave which has been created by surge pressure $p(t) = p_0 =$ const inside the spherical cavity. From introductory analysis of the formulae derived above it results that all parameters of the stress wave intensively decrease in space together with an increase of the distance from the centre of the system. On the contrary, these parameters oscillate versus time around their static values in the respective spherical sections of the medium.

Exemplary courses of the relative displacement of the cavity surface, $U(1, \eta)/P$, versus the dimensionless time, $\eta = a_0 t/r_0$, for a few values of Poisson's ratio ν are depicted in Fig. 2. It is well known that the quantity ν represents compressibility of the medium. As it can be seen, the parameter ν substantially influences the course of function $U(1, \eta)/P$ versus η.

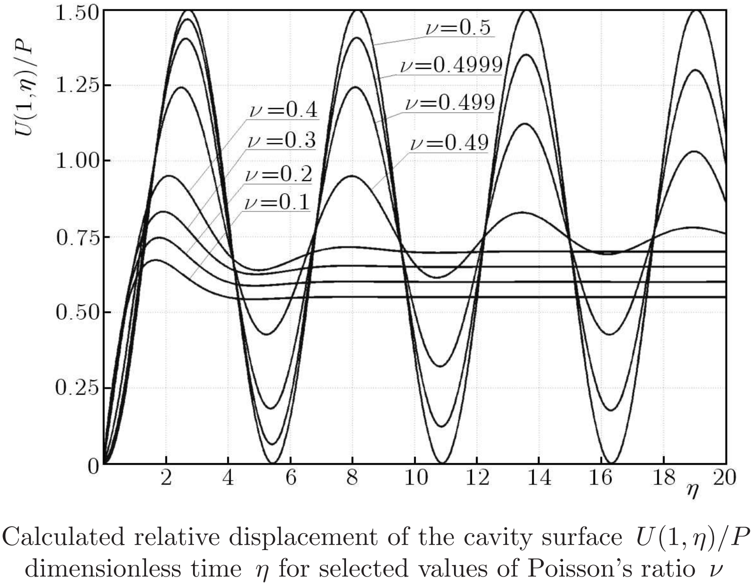

Fig. 2. Calculated relative displacement of the cavity surface $U(1, \eta)/P$ versus dimensionless time η for selected values of Poisson's ratio ν

We can mark out two ranges of the values ν in which vibration of the cavity surface is damped with a different degree. Thus, a decrease of the parameter ν in the range $\nu \leqslant 0.4$ causes an intense decay of the cavity surface vibration. For these numerical values ν, the displacement of the cavity surface attains its statical value, i.e. $U_s/P = (1+\nu)/2$, during one cycle of the vibration (Fig. 2). On the other hand, in the range $0.4 < \nu < 0.5$, i.e. in quasi-compressible media the vibration damping is very low. In the limiting case, when $\nu = 0.5$, i.e. in the incompressible medium, the damping vanishes and the cavity surface harmonically vibrates around its static position $U_s/P = 0.75$ with the constant amplitude $A_U = 0.75$ (Fig. 2).

It is necessary to take into account abnormal behaviour of the materials in the range $0.4 < \nu \leqslant 0.5$. In this case, an insignificant increment of the parameter ν causes a considerable increase of the vibration amplitude of the cavity surface (Fig. 3). For example, an increment of the Poisson's ratio by $\Delta\nu = 0.5 - 0.4 = 0.1$ causes a variation of the relative displacement of the cavity surface by $\Delta U/P = 1.5 - 0.7 = 0.8$.

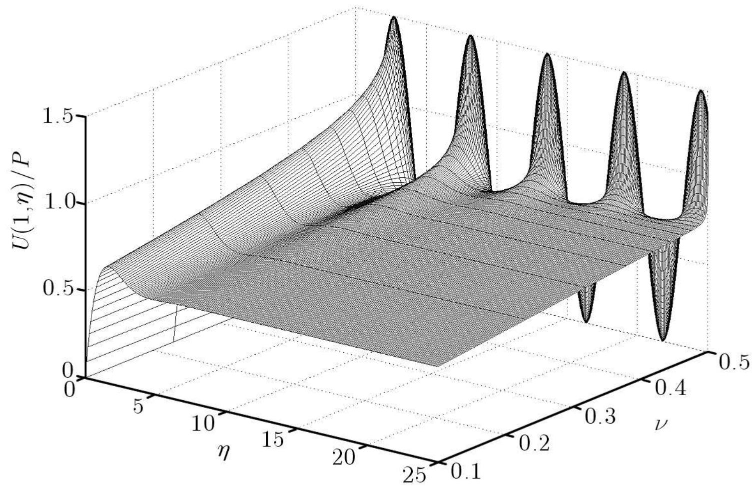

Fig. 3. Calculated relative displacement of the cavity surface $U(1,\eta)/P$ versus dimensionless time η and Poisson's ratio ν

The analysis presented above applies to motion of the cavity surface $\xi = 1$. Respective spherical sections of the material for $\xi > 1$ displace in an analogous way. The displacements of these sections are adequately reduced (Fig. 4).

The material density changes insignificantly in both ranges of Poisson's ratio (Fig. 5). The maximal increment of the relative density ΔR is smaller than 0.2%. From presented analysis it follows that the considered material

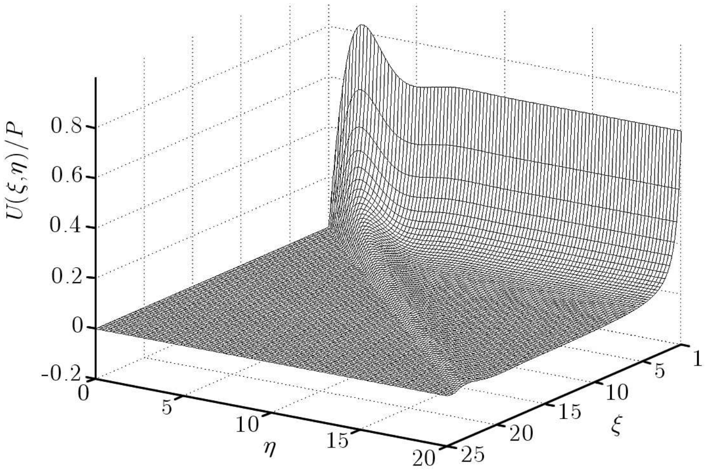

Fig. 4. Calculated relative displacement $U(\xi,\eta)/P$ versus dimensionless variables ξ and η for Poisson's ratio $\nu = 0.3$

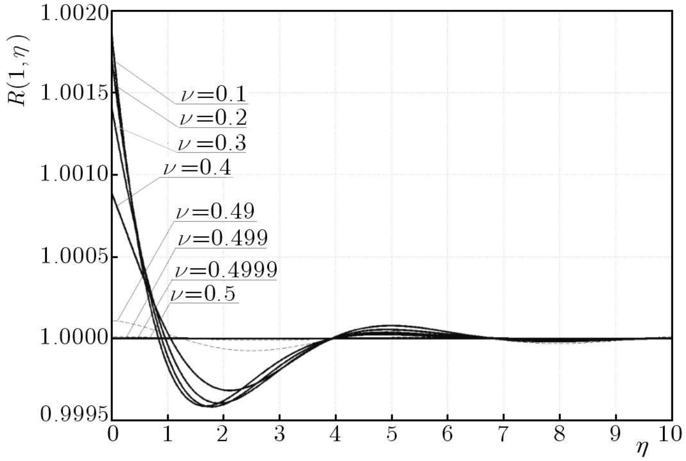

Fig. 5. Calculated relative density R versus dimensionless time η for $\xi = 1$ and selected values of Poisson's ratio ν

is compressible even though its density changes very little. As it is known, the compressibility measure of a linear-elastic material is Poisson's ratio. The density change is not a measure of the material compressibility.

The phenomenon of specific influence of Poisson's ratio ν on the space r and time t variability of parameters of the spherical stress wave expanding in elastic media presented above is the main result of this paper. As we see it, this paper may be a contribution supplementing the description of properties of the spherical stress wave expanding in elastic media and has important significance for technical problems. According to the author's knowledge, this phenomenon has not been described in the available literature yet.

The vast quantitative and qualitative analysis of the mechanical parameters (displacement, particle velocity, strains, stresses and material density) of the expanding spherical stress wave in the compressible isotropic elastic material versus the Langrangian coordinate r and time is presented in the second part.

References

1. Achenbach J.D., 1975, *Wave Propagation in Elastic Solids*, North-Holland Publ. Co., American Elsevier, Amsterdam-New York
2. Broberg K.B., 1956, *Shock Waves in Elastic and Elastic-Plastic Media*, Stokholm
3. Cole R.H., 1948, *Underwater Explosions*, Princeton University Press, Princeton, New Jersey
4. Derentowicz H., Dybek W., et al., 1984, Generation strong shock waves into deuterium by the Voitenki's explosive generator, *Biul. WAT*, **33**, 5 [in Polish]
5. Gimenez P., et al., 1985, EOS of detonation products obtained from symmetrical deflection of liners investigated by laser interferometers techniques, In: *The 8-th International Symposium on Detonation*, Albuquerque
6. Graff K.F., 1975, *Wave Motion in Elastic Solids*, University Press, Oxford
7. Gurney R.W., 1943, The initial velocities of fragments from bombs, shells and grenades, *BRL Report 405*, September
8. Gurney R.W., 1947, Fragmentation of bombs, shells and grenades, *BRL Report 635*, March
9. Hopkins H.G., 1960, Dynamic expansion of spherical cavities in metals, In: *Progress in Solid Mechanics*, Sneddon J.N., Hill R. (Edit.), vol II, North-Holland Publishing Company, Amsterdam
10. Kaliski S., Rymarz C., Sobczyk K., Włodarczyk E., 1992, *Waves*, Elsevier, Amsterdam
11. Knoepfel H., 1970, *Pulsed High Magnetic Fields*, North-Holland Publishing Company, Amsterdam-London

12. Lambour B.O., Harley J.E., 1965, *Proc the 4-th Symposium on Detonation*, Washington

13. Nowacki W., 1970, *Theory of Elasticity*, PWN, Warszawa [in Polish]

14. Taylor G.I., 1961, Analysis of the explosion of a long cylindrical bomb detonated at one end, In: *The Scientific Papers of Sir Geoffrey Ingram Taylor*, Vol. III, Batchelor G.K. (Edit.), University Press, Cambridge

15. Trębiński R., Trzciński W.A., Włodarczyk E., 1988a, Theoretical analysis of the process of driving a cylindrical liner by the products of grazing detonation, *J. Tech. Phys.*, **29**, 3-4

16. Trębiński R., Trzciński W.A., Włodarczyk E., 1988b, Theoretical analysis of the influence of an external envelope on the process of driving a cylindrical liner by the products of grazing detonation, *J. Tech. Phys.*, **29**, 3-4

17. Trębiński R., Trzciński W.A., Włodarczyk E., 1989a, Theoretical analysis of the influence of the external medium on the launching process of a cylindrical liner by the products of grazing detonation, *J. Tech. Phys.*, **30**, 2

18. Trębiński R., Trzciński W.A., Włodarczyk E., 1989b, Theoretical analysis of the influence of material compressibility of a cylindrical liner on the process of launching it by products of grazing detonation, *J. Tech. Phys.*, **30**, 3-4

19. Vidart A., Beatrix P., et al., 1965, *Proc the 4-th Symposium on Detonation*, Washington

20. Walters W.P., Zukas J.A., 1989, *Fundamentals of Shaped Charges*, Wiley and Sons, New York

21. Włodarczyk E., Zielenkiewicz M., 2008, The dynamics of a thick-walled spherical casing loaded with a time depending internal pressure, *Journal of Theoretical and Applied Mechanics*, **46**, 1

Teoretyczna analiza wpływu ściśliwości ośrodka sprężystego na parametry ekspandującej kulistej fali naprężenia. I. Analityczne rozwiązanie problemu

Streszczenie

Zbadano wpływ ściśliwości materiału, reprezentowanej przez współczynnik Poissona ν, na parametry ekspandującej kulistej fali naprężeń, wywołanej ciśnieniem wewnątrz kulistej kawerny w nieskończonym izotropowym ośrodku sprężystym. Wyprowadzono zamknięte wzory określające dynamiczny stan mechanicznych parametrów (przemieszczenia, prędkości przemieszczenia, odkształcenia, naprężenia oraz gęstości materiału) w ośrodku dla stałego ciśnienia $p(t) = p_0 =$ const przyłożonego

w sposób nagły. Z analizy tych wzorów wynika, że współczynnik Poissona ν wpływa zasadniczo na przebiegi tych parametrów w czasie i przestrzeni. Wszystkie parametry maleją intensywnie wraz ze wzrostem współrzędnej Lagrange'a r. Z drugiej strony, parametry te oscylują w czasie wokół ich wartości statycznych. Oscylacje te z czasem zanikają. Możemy wyróżnić dwa przedziały wartości parametru ν, w których tłumienie oscylacji zachodzi z różną intensywnością. Tak więc dla $\nu \leqslant 0.4$ tłumienie jest bardzo intensywne. Natomiast w przedziale $0.4 < \nu < 0.5$, tzn. w materiałach quasi-ściśliwych, poziom tłumienia jest bardzo niski. W przypadku granicznym dla $\nu = 0.5$, tzn. dla materiałów nieściśliwych, tłumienie zanika i parametry mechaniczne oscylują wokół ich wartości statycznych. Ponadto w przedziale $0.4 < \nu \leqslant 0.5$ można zaobserwować anormalne zachowanie materiału. Niewielki przyrost wartości współczynnika Poissona skutkuje znaczącym wzrostem amplitudy drgań parametrów. Opis szczególnego wpływu współczynnika Poissona na parametry ekspandującej kulistej fali naprężenia jest głównym rezultatem niniejszej pracy. Według autorów może ona być uzupełnieniem opisu właściwości ekspandującej kulistej fali naprężeń w ośrodku sprężystym.

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