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VERIFICATION OF THE NOMOGRAM FOR AMPLITUDE DETERMINATION OF RESONANCE VIBRATIONS IN THE RUN-DOWN PHASE OF A VIBRATORY MACHINE

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The paper contains an experimental verification of a new nomogram for determination of resonance vibration amplitudes in a vibratory machine driven by means of an inertia vibrator in the run-down phase. Unlike the Katz nomogram, it takes into consideration the interaction between the vibrator and machine body. The verification was performed for a case where the machine body was in curvilinear motion with its trajectory close to circular.

Key words: vibratory machine, transient resonance, run-down

1. Introduction

The problem of determination of the values of resonance amplitudes during the run-up and run-down phases in vibratory machines driven by inertia vibrators was researched in many works in the scientific literature. First attempts were related to analysis of the equation of motion in the form

$$M\ddot{x} + b\dot{x} + kx = f(t) \tag{1.1}$$

describing motion of a vibrating mass M supported in flexible and viscous suspension defined by parameters k and b, excited by a force of the given form. In the simplest case, it was a sinusoidal force having a constant amplitude and linearly increasing (or decreasing) frequency (Lewis, 1932), or having the amplitude directly proportional to the square of vibrator velocity as in the case described by Katz (1947). The results of the above mentioned approach were also presented in form of nomograms (Goliński, 1979) based on the so-called acceleration factor which combines angular acceleration of the vibrator with the square of natural frequency of the vibrating machine. In works by Banaszewski and Turkiewicz (1980), Zeller (1949), Fernlund (1963), Fearn and Millsaps (1967), Irretier and Leul (1993), Leul (1994) and others, one can find formulas for resonance amplitudes obtained analogously; i.e. by empirical approximations of numerically integrated equation (1.1). Based on these works, one could confirm in general that the resonance amplitude at a constant value of the vibrator angular acceleration is inversely proportional to the square root of that acceleration and the resonance frequency values are varying for the run-up and run-down phases and also depend on the value of vibrator angular acceleration. The most accurate presentation for motion of a mass suspended on a flexible-viscous system and excited by a given force is most probably the work done by Markert and Seidler (2001), who solved equation (1.1) in the case when the force was presented as a linear combination of an arbitrarily selected function and its derivatives in time.

However, if the interaction between vibrator motion and the vibrating body of the machine is not considered, it may lead to gross errors. Michalczyk (1994) pointed out this problem, since it shows a significant effect of the vibration moment on behaviour of the resonance and explained the reasons of the vibrator angular velocity breakdown during the run-down phase in relation to the errors created during reading out the Katz nomogram. Michalczyk (1995) derives formula (1.2) based on the energy balance between the vibrator and the machine body allowing one to evaluate the resonance vibration amplitude A_{max} from the top levels down for the run-down phase

$$A_{max} = \sqrt{\frac{J_{zr}}{M_c}} \tag{1.2}$$

where

J_{zr} – moments of inertia of the vibrator and drive shaft reduced to the rotation axis of the vibrator,

M_c – mass of the vibrating part of the machine.

The verification of a new nomogram for determination of the amplitude of resonance vibrations for the machine run-down phase presented in this paper is related to motion during which the body moves along a circular trajectory. Therefore, it is not possible to directly compare the obtained results with the results of studies of other authors. However, according to Section 4, the nomogram can be indirectly compared with the studies related to rectilinear motion of the machine.

The results of comparisons obtained through the application of the nomogram for the body moving along rectilinear and circular trajectories with more accurate computer simulations (Cieplok, 2008) came out very well, thereby it can be also expected that the errors at the test stands are of similar values.

2. Equations for a symmetrically supported vibratory machine in relative units. Nomogram

Cieplok (2007) analysed a phenomenological model of a vibratory machine illustrated schematically in Fig. 1. The machine body, having a mass M is suspended in a flexible viscous system described by constants k and b. The system is excited to vibration by an inertia vibrator characterised by the static unbalance me. The vibrator inertia moment combined with that of the driving system was reduced to the coordinate of vibrator rotation and is denoted by J_{zr}. The vibrator is exposed to action of the driving moment directed along the coordinate φ of vibrator angular motion. For this model, equations (2.1) were derived

$$\begin{aligned}
&\begin{bmatrix} M_c & 0 \\ 0 & M_c \end{bmatrix} \begin{bmatrix} \ddot{x}_s \\ \ddot{y}_s \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} \dot{x}_s \\ \dot{y}_s \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \end{bmatrix} \\
&\begin{bmatrix} P_x \\ P_y \end{bmatrix} = \begin{bmatrix} me\sin\varphi \\ -me\cos\varphi \end{bmatrix} \ddot{\varphi} + \begin{bmatrix} me\cos\varphi \\ me\sin\varphi \end{bmatrix} \dot{\varphi}^2 \\
&J_{zr}\ddot{\varphi} - me(\ddot{x}\sin\varphi - \ddot{y}\cos\varphi) = M_{el}
\end{aligned} \tag{2.1}$$

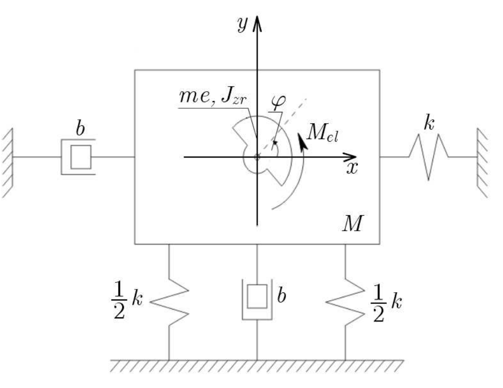

Fig. 1. Model of a vibratory machine

Upon transformation to the coordinate system $0\xi\eta$ rotating with the vibrator angular velocity $\dot{\varphi}$ (Fig. 2), these equations assume the following form

$$
\begin{bmatrix} M_c & 0 & M_c\eta & 0 & 0 \\ 0 & M_c & M_c\xi + me & 0 & 0 \\ 0 & me & me\xi + J_{zr} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} v_\xi \\ v_\eta \\ \omega \\ \xi \\ \eta \end{bmatrix} =
$$

$$
= \begin{bmatrix} 2M_c\omega v_\eta - bv_\xi - (k - \omega^2 M_c)\xi + b\omega\eta + me\omega^2 \\ -2M_c\omega v_\xi - bv_\eta - (k - \omega^2 M_c)\eta - b\omega\xi \\ -2mev_\xi\omega + me\eta\omega^2 + M_{el} \\ v_\xi \\ v_\eta \end{bmatrix} \tag{2.2}
$$

where

$$
M_c = M + m \qquad v_\xi = \frac{d\xi}{dt} \qquad v_\eta = \frac{d\eta}{dt} \qquad \omega = \dot{\varphi} \tag{2.3}
$$

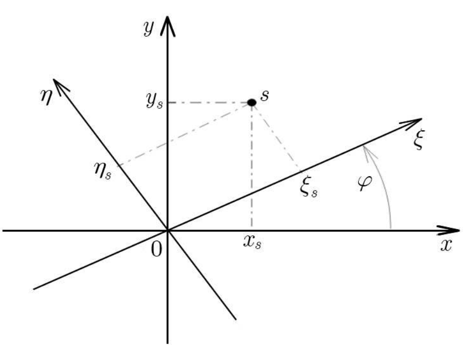

Fig. 2. Position of the machine body mass centre in the coordinate systems $0xy$, $0\xi\eta$

The transformation also enabled one to create a definition of relative units and parameters for the machine. Hence, by substituting the following

$$
\begin{array}{lll}
\xi_r = \dfrac{\xi}{A_u} & \eta_r = \dfrac{\eta}{A_u} & \omega_r = \dfrac{\omega}{\omega_0} \\
\sigma = \dfrac{m^2e^2}{M_cJ_{zr}} & q = \dfrac{M_{el}}{J_{zr}}\dfrac{1}{\omega_0^2} & \gamma = \dfrac{b}{2\sqrt{M_ck}} \\
\tau = \dfrac{\omega_0}{2\pi}t & A_u = \dfrac{me}{M_c} & \omega_0 = \sqrt{\dfrac{k}{M_c}} \\
v_{\xi_r} = \dfrac{d\xi_r}{d\tau} & v_{\eta_r} = \dfrac{d\eta_r}{d\tau} &
\end{array}
\tag{2.4}
$$

set (2.2) may be expressed in the following form

$$
\begin{gathered}
\begin{bmatrix}
\frac{1}{4\pi^2} & 0 & -\frac{1}{2\pi}\eta_r & 0 & 0 \\
0 & \frac{1}{4\pi^2} & \frac{1}{2\pi}(1+\xi_r) & 0 & 0 \\
0 & \frac{\sigma}{4\pi^2} & \frac{1}{2\pi}(\sigma\xi_r+1) & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\frac{d}{d\tau}
\begin{bmatrix}
v_{\xi_r} \\ v_{\eta_r} \\ \omega_r \\ \xi_r \\ \eta_r
\end{bmatrix}
= \\
=
\begin{bmatrix}
\omega_r^2-(1-\omega_r^2)\xi_r-\frac{\gamma}{\pi}v_{\xi_r}+\frac{1}{\pi}\omega_rv_{\eta_r}+2\gamma\omega_r\eta_r \\
-(1-\omega_r^2)\eta_r-\frac{\gamma}{\pi}v_{\eta_r}-\frac{1}{\pi}\omega_rv_{\xi_r}-2\gamma\omega_r\xi_r \\
-\frac{\sigma}{\pi}v_{\xi_r}\omega_r+\sigma\eta_r\omega_r^2+q \\
v_{\xi_r} \\
v_{\eta_r}
\end{bmatrix}
\end{gathered}
\tag{2.5}
$$

In this way, a set of six physical parameters M_c, me, J_{zr}, M_{el}, k, b required for description of the machine dynamics has been reduced to three parameters σ, γ and q.

Based on set above (2.5), Cieplok (2008) developed the following:

- layer graphs enabling determination of the amplitude multiplication factor for the run-up phase based on values of the parameters σ, γ and q,
- a nomogram (Fig. 3) enabling determination of the amplitude multiplication factor for the run-down phase based on values of the nomogram parameters σ and γ.

3. Verification of the nomogram

In order to verify practical applicability of a new nomogram, an experiment was conducted at the AGH Vibromechanics Laboratory. A machine shown

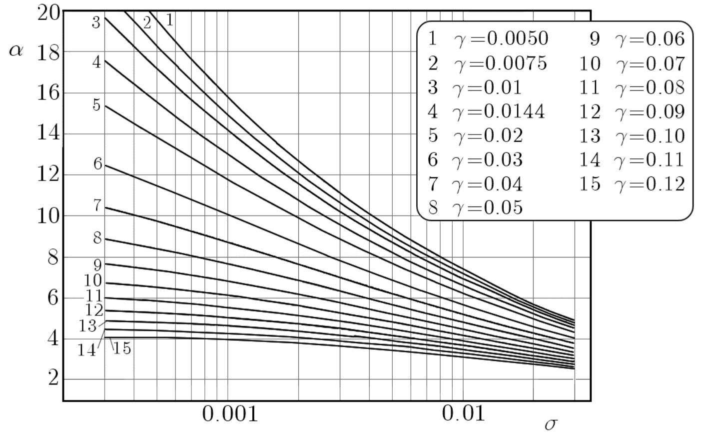

Fig. 3. Multiplication factor α of the machine body vibration amplitude for the run-down phase; $\alpha = A_{max}/A_u$, where A_{max} – resonance amplitude

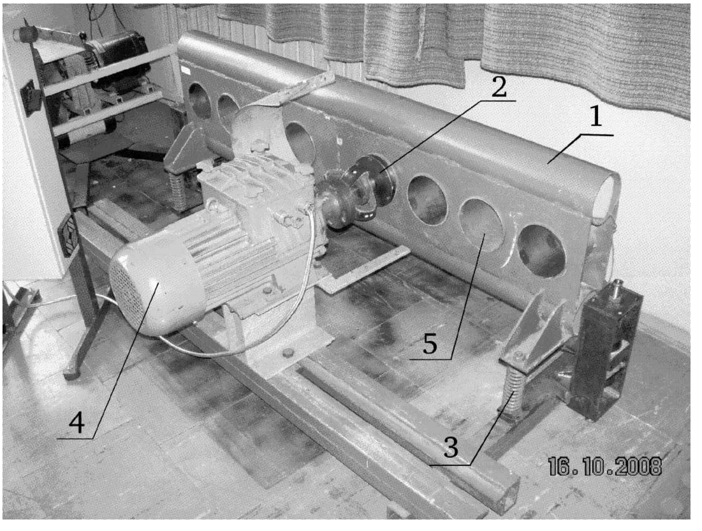

Fig. 4. Test Stand. 1 – machine body, 2 – inertia vibrator, 3 – spiral spring, 4 – electric motor, 5 – hole for an additional mass

in Fig. 4 was selected for testing. It consists of a body supported on four symmetrically spaced spiral springs forced to vibrate by means of an inertia vibrator shown in Fig. 5. The vibrator is driven by a 4-pole asynchronous electric motor ensuring over-the-resonance work of the machine.

The mass of the machine vibrating part M_c was determined based on the change of machine natural vibration frequencies as a result of adding the

Fig. 5. Inertia vibrator used for the purpose of experiment

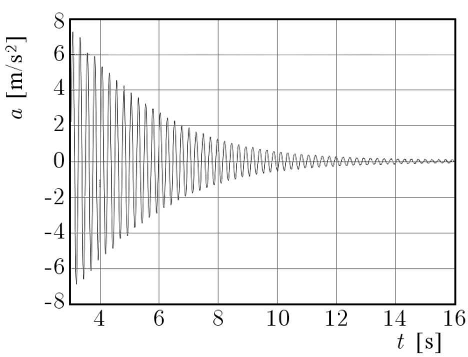

Fig. 6. Graph showing natural vibration acceleration of the machine body recorded during the experiment

mass m_d. Based on Fig. 6 illustrating machine natural vibrations in the vertical direction, its natural period of vibrations was determined

$$T_{01} = 0.248\,\mathrm{s} \tag{3.1}$$

Upon placing additional masses of total 46.5 kg into the open holes shown in Fig. 4, the machine new natural vibrations period was determined to be

$$T_{02} = 0.277\,\mathrm{s} \tag{3.2}$$

Based on the relationship between the vibrating mass natural frequency and the support stiffness, the sought value of mass of the vibrating part was

$$M_c \cong m_d \frac{1}{\left(\frac{T_{02}}{T_{01}}\right)^2 - 1} = 187.8\,\mathrm{kg} \tag{3.3}$$

as well as the equivalent spring coefficient

$$k \cong \left(\frac{2\pi}{T_{01}}\right)^2 M_c = 120545\,\frac{\mathrm{N}}{\mathrm{m}} \tag{3.4}$$

Next, based on the logarithmic decrement δ of vibration damping (Osiński, 1980), the equivalent viscous damping coefficient of the suspension was determined

$$b = \frac{2M_c\delta}{T_{01}} = 136.82\,\frac{\mathrm{Ns}}{\mathrm{m}} \tag{3.5}$$

and then the damping factor was obtained

$$\gamma = \frac{b}{2\sqrt{M_c k}} = 0.0144 \tag{3.6}$$

As now it is possible to notice, the omitting of dissipation in formulas (3.3) and (3.4) does not cause significant errors. The percentage difference between the period of natural undamped and damped oscillations does not exceed

$$(1 - \sqrt{1-\gamma^2}) \cdot 100\% \approx 0.01\%$$

Upon identification of geometry of masses creating the active part of the vibrator, its own mass could be determined

$$m = 4.7\,\mathrm{kg} \tag{3.7}$$

the radius of unbalance

$$e = 0.0156\,\mathrm{m} \tag{3.8}$$

its static unbalance

$$me = 0.073\,\mathrm{kgm} \tag{3.9}$$

and the mass moment of inertia with respect to the mass centre

$$J_{sw} = 0.00112\,\mathrm{kgm}^2 \tag{3.10}$$

Subsequently, based on the determined vibrator value parameters as well as the catalogue value of the drive motor moment of inertia J_r increased by the drive shaft components inertia J_d, one finally finds

$$J_{zr} = J_{sw} + me^2 + J_r + J_d = 0.00112 + 0.00115 + 0.009 + 0.0025 = \\ = 0.0138\,\mathrm{kgm}^2 \tag{3.11}$$

Now, from the coefficient $\sigma = m^2e^2/(M_cJ_{zr}) = 2.1 \cdot 10^{-3}$ one can calculate the amplitude multiplication ratio $\alpha = 10.6$ from Fig. 3 for $\gamma = 0.0144$. It corresponds to the absolute value of resonance amplitude

$$A_{max} = \alpha\frac{me}{M_c} = 4.12\,\mathrm{mm} \tag{3.12}$$

Then, from Fig. 7 showing a recording of the machine body mass centre displacement during the run-down phase, we can read out the resonance amplitude

$$A_{max} = 3.72\,\mathrm{mm} \tag{3.13}$$

The percentage difference between the theoretically determined and measured values was 10.8%.

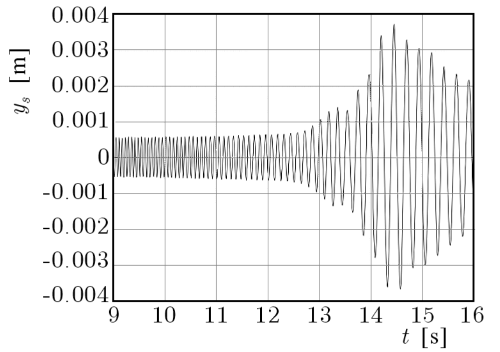

Fig. 7. Graph of the machine body mass centre displacement y_s during the run-down phase. Experiment

4. Conclusions

If we take a practical application into consideration, the obtained result is sufficiently accurate for evaluation of the resonance amplitude for the run-down phase of a machine with the body moving along a circular trajectory.

A better approximation of the resonant vibration amplitude could also be expected for the machine performing a rectilinear trajectory. Although direct comparison of the nomogram with results of other authors is not possible due to a variety of different models assumed for analysis, however, an indirect comparison may still be possible. The possibility of nomogram adaptation to a machine featuring straight linear motion of the body by applying a twice less value of the parameter σ during readout was indicated in Cieplok (2008). In this way, one could read out a value of the vibration amplitude multiplication factor to be $\alpha \approx 13$ for a machine having physical parameters corresponding to the machine used in the experiment and having a rectilinear trajectory of body motion. Its value obtained based on below mentioned methods was:

- formula (1.2)

$$\sqrt{\frac{J_{zr}}{M_c}\frac{M_c}{me}} = 21.6$$

- Katz nomogram – between 7 and 12,
- Fernlund formula – 17.2,
- Markert and Seidler formula – 13.8.

It should be mentioned that for the last three items on the above list, the vibrator angular acceleration was determined based on computer simulation, see Fig. 8, by capturing the breakdown shift of the vibrator angular velocity during its passage through the resonance zone. This simulation was conducted with taking into account idealised friction levels caused by the presence of the machine suspension, which was reflected by a viscous damping effect.

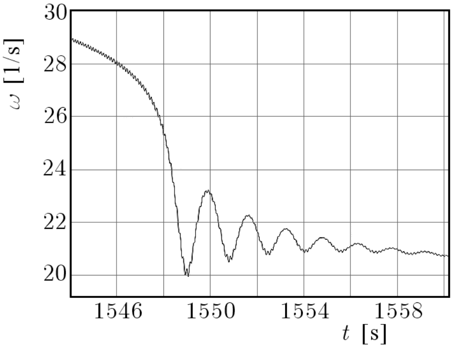

Fig. 8. Graph for vibrator angular velocity during run-down phase in the area of resonant frequency. Computer simulation

However, we cannot generally have the accurate value of the vibrator angular acceleration for the phase of velocity breakdown during resonance and, therefore, the determination of a useful value of the vibration amplitude multiplication factor is not possible.

The nomogram presented in this paper only relates to generally accessible machine physical parameters, thus it is a much more convenient and more accurate alternative than those discussed in previous publications.

References

1. Banaszewski T., Turkiewicz W., 1980, Analiza wzrostu amplitudy drgań przesiewaczy wibracyjnych podczas rozruchu, *Mechanizacja i automatyzacja górnictwa*, **144**, 11
2. Cieplok G., 2007, Quality analysis of symmetrically supported vibratory machine taking into account a rotor stall in resonance, *Machine Dynamics Problems*, **31**, 1, 14-22
3. Cieplok G., 2008, Ampliduda drgań symetrycznie posadowionej maszyny wibracyjnej podczas rezonansu przejściowego, *Czasopismo Techniczne*, **1-M2008**, 37-45, Wydawnictwo Politechniki Krakowskiej
4. Fearn R.L., Millsaps K., 1967, Constant acceleration of an undamped simple vibrator through resonance, *Journal of the Royal Aeronautical Society*, **71**, 680
5. Fernlund I., 1963, Running through the critical speed of a rotor, *Transactions of Chalmers University of Technology*, **277**, Gothenburg, Sweden
6. Goliński J.A., 1979, *Wibroizolacja maszyn i urządzeń*, WNT, Warsaw
7. Hirano I., Matsukura Y., Kiso M., 1968, Behaviour of vibrating system passing through the resonances, *Mitsubishi Denki Ghio*, **42**, 11
8. Irretier H., Leul F., 1993, *Näherungsformeln zur Abschätzung der Maximalantwort transient unwuchterregter Rotoren. SIRM II, Schwingungen in Rotierenden Maschinen*, Vieweg Verlag, Braunschweig/Wiesbaden, Germany, 207-217
9. Katz A.M., 1947, *Forced Vibration During the Resonance Passage*, Inst. Mekh. Akad. Nauk. SSSR Ing. Spornik III/2 [in Russian]
10. Leul F., 1994, *Zum transienten Schwingungsverhalten beim Resonanzdurchgang linearer Systeme mit langsam veränderlichen a Parametern*, Bericht 4/1994 des Instituts fur Mechanik der Universität-GH Kassel, Germany

11. Lewis F.M., 1932, Vibration during acceleration through a critical speed, *Journal of Applied Mechanics*, **54**

12. Markert R., Seidler M., 2001, Analytically based estimation of the maximum amplitude during the passage through resonance, *International Journal of Solids and Structures*, **38**

13. Michalczyk J., 1995, *Maszyny Wibracyjne. Obliczenia dynamiczne, drgania, hałas*, WNT, Warsaw.

14. Michalczyk J., Cieplok G., 1994, Rezonans przejściowy maszyn wirnikowych – przyczyny błędów oszacowań, *Zeszyty Naukowe AGH. Mechanika*, **13**, 1, Cracow

15. Osiński Z., 1980, *Teoria drgań*, PWN, Warsaw

16. Zeller W., 1949, Näherungsverfahren zur Bestimmung der beim Durchlaufen der Resonanz auftretenden Höchstamplitude, *Motortechnische Zeitschrift*, **10**

Weryfikacja nomogramu do wyznaczenia amplitudy drgań rezonansowych dla fazy wybiegu maszyny wibracyjnej

Streszczenie

W pracy poddano weryfikacji doświadczalnej nowy nomogram do wyznaczenia amplitudy drgań rezonansowych maszyny wibracyjnej o napędzie za pomocą wibratora bezwładnościowego dla fazy wybiegu. W odróżnieniu od nomogramu Kaca uwzględnia on sprzężenie pomiędzy wibratorem a korpusem maszyny. Weryfikację przeprowadzono dla przypadku, w którym korpus maszyny wykonuje ruch postępowy o trajektorii zbliżonej do kołowej.

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