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ENERGY HARVESTING IN A MAGNETOPIEZOELASTIC SYSTEM DRIVEN BY RANDOM EXCITATIONS WITH UNIFORM AND GAUSSIAN DISTRIBUTIONS

Grzegorz Litak, Marek Borowiec
Technical University of Lublin, Department of Applied Mechanics, Lublin, Poland
e-mail: g.litak@pollub.pl

Michael I. Friswell, Sondipon Adhikari
Swansea University, School of Engineering, Swansea, United Kingdom

A simple magneto-piezoelectric system excited by random forces modelled with a double well potential is considered. System responses for different realizations of noise with uniform and Gaussian distributions are compared. The results show negligible differences in the regions of small and high noise intensity. A more noticeable difference can be seen in the intermediate region of noise just below the transition from isolated single well oscillations to coupled double wells oscillations. Variations in the mechanical displacement in this transition region indicate that the transition between these types of behaviour is broader for a uniform noise excitation. Consequently, the system excited with Gaussian noise tends more clearly to one of the different solutions (i.e. motion in a single well or in both wells) while the uniform noise case demonstrates intermittency with multiple solutions.

Key words: energy harvesting, piezoelectric transducer, random excitation

1. Introduction

In the energy harvesting process, energy is derived from external sources (e.g., solar power, thermal energy, wind or hydro energy, salinity gradients, and also kinetic energy). This energy is captured and stored by autonomous devices, such as those used in wearable electronics and wireless sensor networks. In recent years, energy harvesting has attracted great attention as the energy generated can be used directly or used to recharge batteries or other storage devices, which enhances battery life (Anton and Sodano, 2007).

Many of the proposed devices use the piezoelectric effect as the transduction method (Arnold, 2007; Beeby *et al.*, 2007). These devices are usually implemented as patches on cantilever beams and designed to operate at resonance conditions. The design of an energy harvesting device must be tailored to the ambient energy available. For a single frequency excitation the resonant harvesting device is optimum, provided it is tuned to the excitation frequency (Erturk *et al.*, 2009; Litak *et al.*, 2010; Stanton *et al.*, 2010). One should also note that individual small devices may be combined in arrays to produce a larger and more powerful device.

2. The magneto-piezoelectric harvester

The system of our present investigation consists of a ferromagnetic cantilever beam that is excited at the support (Fig. 1). Two permanent magnets are located symmetrically on the base near the free end, and the static system can have five, three or one equilibrium positions depending on geometry of the system (Erturk *et al.*, 2009; Litak *et al.*, 2010) and, in particular, the distance between the beam and the magnets.

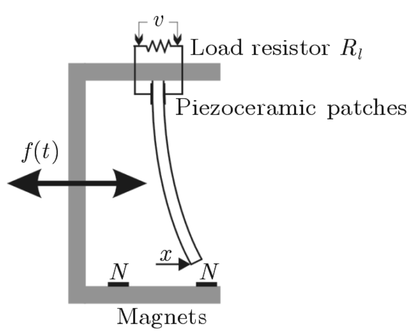

Fig. 1. Schematic of the piezomagnetoelastic device (Litak *et al.*, 2010)

In the present work, we are interested in the case when the system has three equilibrium positions, two of which are stable, and the mechanical system is characterized by the classical double well potential. The non-dimensional equations of motion for this system (Erturk *et al.*, 2009) are

$$\ddot{x} + 2\zeta\dot{x} - \frac{1}{2}x(1 - x^2) - \chi v = F(t) \tag{2.1}$$

and

$$\dot{v} + \lambda v + \kappa\dot{x} = 0 \tag{2.2}$$

where x is the dimensionless transverse displacement of the beam tip, v is the dimensionless voltage across the load resistor, χ is the dimensionless piezoelectric coupling term in the mechanical equation, κ is the dimensionless piezoelectric coupling term in the electrical equation, $\lambda \propto 1/R_l C_p$ is the reciprocal of the dimensionless time constant of the electrical circuit, R_l is the load resistance, and C_p is the capacitance of the piezoelectric material. The non-dimensional excitation $F(t)$ is proportional to the base acceleration on the device, and is assumed to be uniform or Gaussian white noise, with zero mean and specified variance.

3. The harvester response to random excitation

The system parameters are taken as (Erturk *et al.*, 2009; Litak *et al.*, 2010): $\zeta = 0.01$, $\chi = 0.05$, and $\kappa = 0.5$, while λ was 0.01. The excitation $F(t)$ is stationary uniform or Gaussian white noise with standard deviation σ_F. Equations (2.1) and (2.2) are integrated using the fourth order Runge-Kutta-Maruyama algorithm (Naess and Moe, 2000; Litak *et al.*, 2010). The standard deviations of the displacement x and the voltage v are calculated for a range of excitation noise amplitudes σ_F for both the uniform and Gaussian noise distributions.

Figure 2 shows the signal to noise ratio σ_x/σ_F as a function of noise intensity σ_F for the Gaussian (Fig. 2a) and uniform (Fig. 2b) distributions, respectively. For each value of σ_F depicted in this figure, five different realizations of noise were used. The simulated results for the different noise distributions are similar, and only small differences appear in the regions of small and high noise intensity. However, there is a noticeable difference in the intermediate region of noise just below the transition from isolated single well oscillations (for small σ_F) to coupled double wells oscillations (for large σ_F). The beam displacement in this region indicates that the transition between these types of behaviour is broader in the case of a uniform noise excitation.

The form of the displacement response can be determined from the mean value of displacement (Fig. 3), which shows slightly increased concentration close to the unstable equilibrium point $x = 0$ in the case of a uniform noise distribution (Fig. 3b) above the transition region. The explanation is that the system excited by the uniform noise distribution prefers more frequent hopping between the potential wells. For further clarification, Fig. 4 shows the number of hops (motion from one potential well to the other) between the potential wells for two types of noise. Clearly, the number of hops is zero for lower

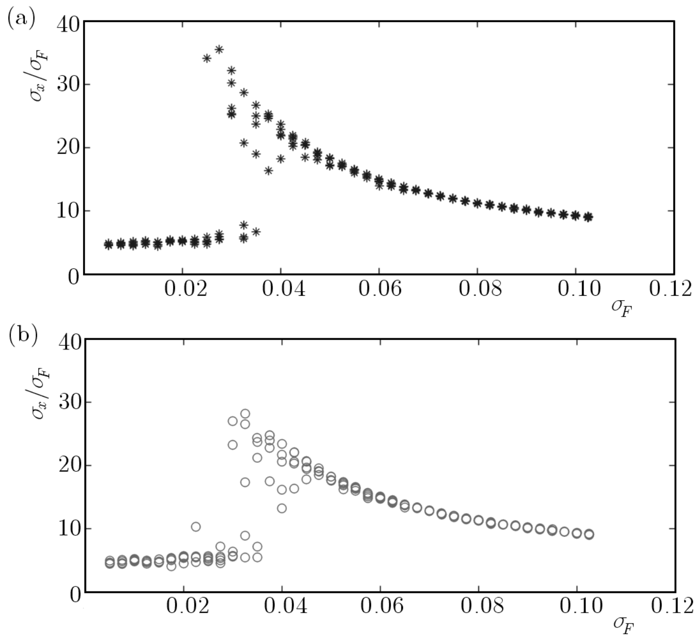

Fig. 2. The displacement signal to noise ratio σ_x/σ_F versus noise intensity σ_F, where σ_x is the standard deviation of the beam displacement and σ_F is the standard deviation of the noise excitation for different noise distributions (a) Gaussian and (b) uniform

noise levels. At a certain critical level, the number of hops starts to increase approximately linearly with the noise intensity. The larger the number of hops, the higher the hopping frequency in the response spectrum.

The ultimate aim of the harvester is to generate energy. Figure 5 shows the variance of voltage σ_v^2 for the two noise distributions. Assuming the voltage has zero mean, this will approximate the energy generated. It is clear that the variance of the voltage is not sensitive to different noise distributions.

4. Conclusions

This paper has extended the analysis in our previous paper (Litak *et al.*, 2010) by comparing the effects of Gaussian and uniform noise distributions on the harvesting system. For the range of parameters investigated, the beam displacement results only differ in the region of the system response where the system transitions from single well vibrations to vibrations characterized

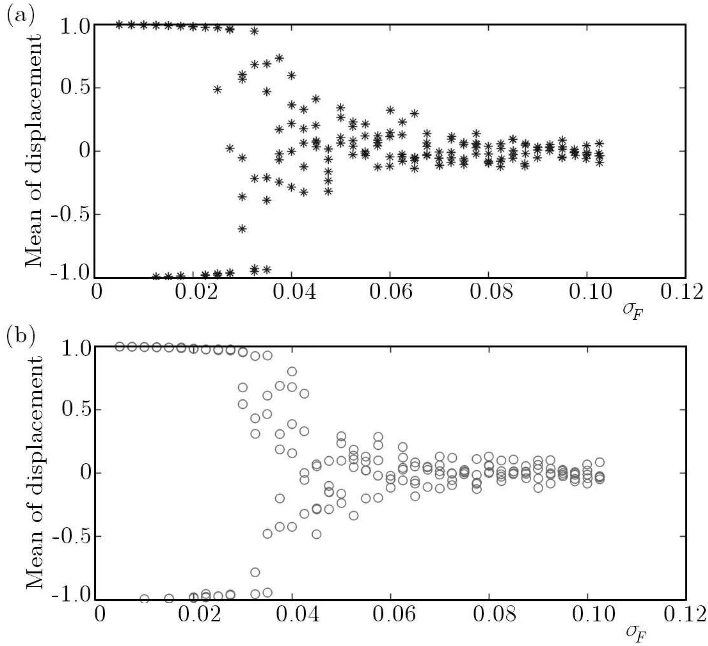

Fig. 3. The mean values of displacements for different noise distributions (a) Gaussian and (b) uniform

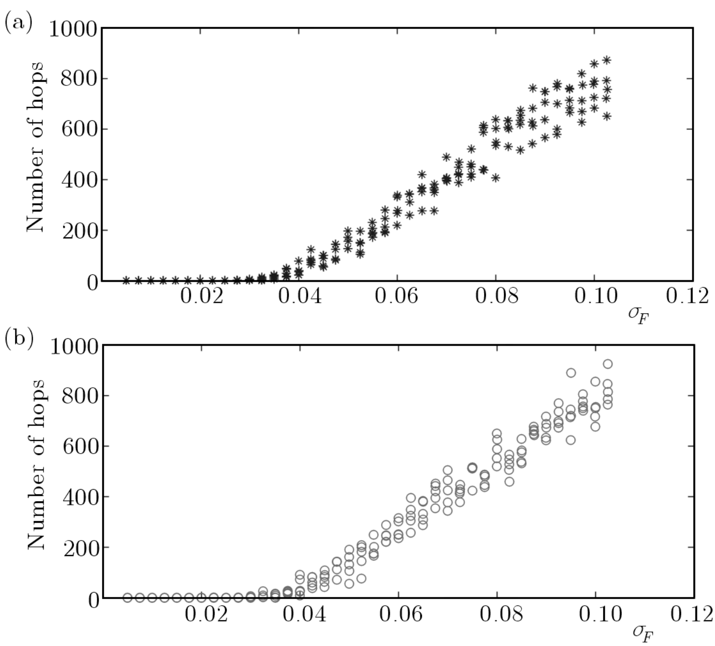

Fig. 4. The number of hops between potential wells for different noise distributions (during the investigated simulation interval) (a) Gaussian and (b) uniform

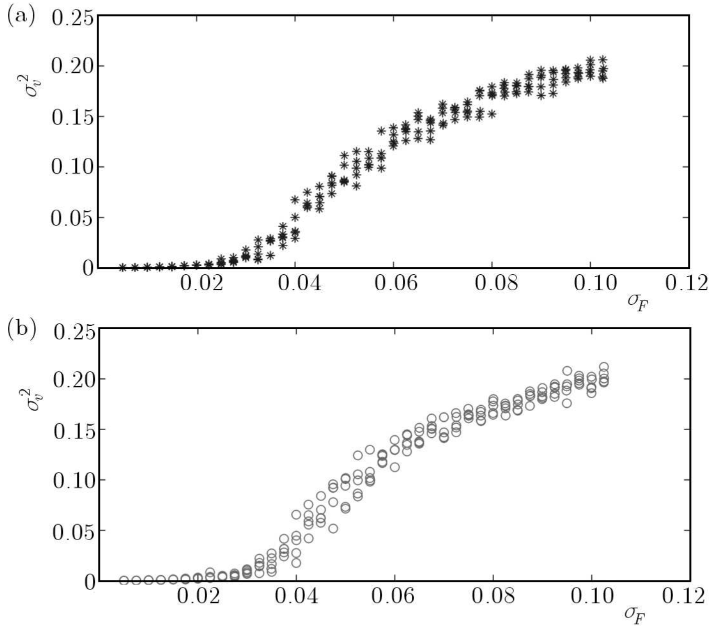

Fig. 5. The variance of the generated voltage, σ_v, for different noise distributions (a) Gaussian and (b) uniform

by hopping between the potential wells. Note that in the uniform excitation case, the escape from the potential well is better defined because the noise is limited to a given band. In contrast, for the Gaussian system, a large amplitude excitation may occur due to the distribution function tails. Furthermore, the lack of tails for the uniform excitation breaks the system ergodicity. In the short time scale, the most important effect is that the noisy force disturbances are usually larger in the case of the uniform noise distribution. Note that the differences between the investigated noise distributions may be larger for different values of λ, which defines the relaxation properties of the electrical part of the system.

Finally, very similar responses were obtained in terms of the voltage output (Fig. 5); this implies that the broadband noise assumption in the previous paper (Litak *et al.*, 2010) is a reliable approach to optimize the system design.

Daqaq (2011) also studied the Gaussian white noise excitation of a bistable inductive generator. He showed that in the limit of higher noise intensity, which corresponds to $\sigma_F > 0.05$ in our work, the shape of the double well potential is not important. High excitation levels lead to a large amplitude system response where the potential barrier is regularly traversed. In contrast, our results consider the crossover between weak and fairly strong levels of noise intensity where the intermittency may play an important role.

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Pozyskiwanie energii w piezo-magnetycznym układzie sprężystym, pobudzanym siłą stochastyczną o rozkładzie jednorodnym i normalnym

Streszczenie

W pracy analizowany jest prosty układ piezo-magnetyczny, pobudzany losowo z potencjałem o dwóch studniach. Porównywane są odpowiedzi układu przy różnej realizacji szumu, o rozkładzie jednorodnym i normalnym (Gaussowskim). Wyniki przedstawiają nieznaczne różnice w obszarach niskiej i wysokiej intensywności szumu. Bardziej zauważalną różnicę można dostrzec w obszarze pośrednim szumu, tuż poniżej przejścia z oscylacji w pojedynczej studni potencjału do oscylacji w dwóch sprzężonych studniach. Zmiany pracy układu w tym obszarze sygnalizują, że obszar przejść pomiędzy takimi typami rozwiązań jest szerszy przy pobudzaniu szumem o rozkładzie jednorodnym. Natomiast układ pobudzany szumem o rozkładzie normalnym wyraźniej wykazuje tendencje do pracy w zakresie jednego z typów rozwiązań. W rezultacie przy szumie Gaussowskim układ dąży do ruchu w obrębie tylko jednej lub dwóch studni potencjału, podczas gdy w obecności szumu jednorodnego, w zachowaniu układu pojawia się zjawisko intermitencji w realizacji dwóch rozwiązań.

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