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WAVE-QUASI-PARTICLE DUALISM IN THE TRANSMISSION-REFLECTION PROBLEM FOR ELASTIC WAVES

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Following along the line of recent works in which the notion of quasi-particles associated with surface acoustic waves of different types was introduced via canonical conservation laws, here the emphasis is placed on the possible exploitation of this dualism in the classical problem of the transmission and reflection of waves by a discontinuity between two media (in perfect contact or with possible delamination) and the more general case of a mono-layer or multi-layer sandwiched slab.

Key words: elastic waves, quasi-particle, transmission-reflection

1. Introduction

In recent works (Maugin and Rousseau, 2010a,b; Rousseau and Maugin, 2011), a kind of wave-particle dualism has been constructed for elastic waves in acoustic physics. The idea is close to that of phonons, but we only need to know the analytical wavelike solution to the continuum problem in order to build this dualism. For practical reasons related to a potential use in Non Destructive Evaluation techniques, the emphasis was placed on SAWs (Surface Acoustic Waves) propagating on the top of an elastic half-space with various boundary conditions on the limiting plane and various physico-mechanical couplings (e.g., in electroelasticity). What must be retained from these studies is (i) the exploitation of the notion of *pseudo-momentum* (in the sense of Peierls (1991, pp. 30-42) or *wave momentum* in the manner of Brenig (1955) and in the spirit of the dynamical theory of configurational forces (Maugin, 2011) – and the accompanying application of Noether's theorem for conservation laws, (ii) the fact that the quasi-particle thus associated with the wave-like motion has a momentum obtained by integration of this Brenig momentum (or its canonical generalization in a multi-field theory) over a volume representative of the wave motion (for a SAW, one wavelength in the propagation direction, one penetration depth in depth in the substrate, and one unit laterally), and (iii) only wave *modes* were considered, i.e., independently of any initial-condition or interaction problem. The motion of quasi-particles thus obtained is "Newtonian" and inertial in the absence of dissipative effects. The associated energy is purely kinetic in form, while in the original continuum problem it could be of mixed origin, including, e.g., kinetic, elastic, electromagnetic, and electro-magneto-elastic interactions. Still the "mass" associated with the said quasi-particle accounts for all these origins as well as for the type of boundary condition and depth behaviour.

In the present work, relying on a simple case of the wave – one elastic component of the SH (shear-horizontal) type – and considering the normal incidence of such a wave on the interface between two elastic continua characterised by their own elastic (shear) coefficient and mass density, we revisit the problem of the transmission and reflection of such a wave at this interface in terms of the wave properties (this is only a reminder of well known results) or those of the associated quasi-particle properties (this is the new contribution). The situation considered is that which prevails in the potential exploitation in nanoscale systems (for macro-systems this would be the realm of potentially dangerous seismic waves). In the present case, discarding the variation of amplitude with depth we are satisfied with considering a plane face wave. The representative volume element of the wave modes then is one wavelength in the propagation direction and a unit square in the transverse plane.

2. Reminder on the wavelike picture

For the sake of simplicity, we consider a one-dimensional wave problem along the x-direction for an elastic displacement u that may be a transverse shear-horizontal (SH) mode. The governing wave equation is given by

$$\frac{\partial p}{\partial t} - \frac{\partial \sigma}{\partial x} = 0 \qquad p = \rho \frac{\partial u}{\partial t} \qquad \sigma = \mu \frac{\partial u}{\partial x} \tag{2.1}$$

where ρ and μ are a prescribed matter density and fixed elasticity (e.g., shear) coefficient μ in a given region of space. Space and time partial derivatives are alternately denoted by a comma followed by x or a t. The first of (2.1) follows in a variational format from the Lagrangian density per unit volume

$$L = \frac{1}{2}\rho u_{,t}^2 - \frac{1}{2}\mu u_{,x}^2 \tag{2.2}$$

We consider a harmonic wave motion

$$u = U\cos(kx - \omega t) \tag{2.3}$$

so that from (2.1) we have the trivial dispersion relation

$$D(\omega k) = \omega^2 - c^2 k^2 = 0 \qquad c = \sqrt{\frac{\mu}{\rho}} \tag{2.4}$$

2.1. Transmission-reflection problem for a perfect interface

The standard transmission-reflection problem here consists in considering a normally incident wave in medium 1 (properties ρ_1 and μ_1) on the interface at $x = x_0 = 0$ that separates medium 1 from medium 2 (properties ρ_2 and μ_2). The wave solution in medium 1 is sought as the sum of an incident component and a reflected component, while the solution in medium 2 consists in a transmitted component. With an obvious notation, we have thus

$$u_1 = u_I + u_R \qquad u_2 = u_T \tag{2.5}$$

with

$$u_1 = U\cos(k_1 x - \omega t) + R_0 U \cos(k_1 x + \omega t) \qquad u_2 = T_0 U \cos(k_2 x - \omega t) \tag{2.6}$$

where R_0 and T_0 are the reflection and transmission coefficients for this *perfect interface* case for which we assume the continuity of displacement and stress, i.e.

$$u_1 = u_2 \qquad \mu_1 u_{1,x} = \mu_2 u_{2,x} \qquad \text{at} \qquad x = 0 \tag{2.7}$$

The solution of this system is, for any amplitude U, the value of the reflection and transmission coefficients as

$$R_0 = \frac{z_1 - z_2}{z_1 + z_2} \qquad T_0 = \frac{2z_1}{z_1 + z_2} \tag{2.8}$$

where $z_\alpha = \rho_\alpha c_\alpha$, $\alpha = 1, 2$ are impedances. With these we check the conservation of energy flux in the well known form

$$F_0 = 1 - R_0^2 - \frac{z_2}{z_1} T_0^2 \equiv 0 \tag{2.9}$$

2.2. Transmission-reflection problem for an interface with delamination

In view of applications to non-destructive (NDT) evaluation methods, it is of interest to consider the case of an imperfect interface that admits the possibility of delamination and for which matching conditions (2.7) are replaced by the conditions (known as Jones' conditions (Jones and Whittier, 1967))

$$\sigma_1 = \sigma_2 \equiv K[\![u]\!] \tag{2.10}$$

where K is a positive coefficient characterising the degree of delamination and the symbol $[\![\cdot]\!]$ means the jump of its enclosure, i.e., $[\![u]\!] = u_2 - u_1$ at $x = 0$. We must look for *complex* solutions of the type $u = A\exp[\mathrm{i}(kx - \omega t)]$. Conditions (2.10) yield the system

$$z_1(1-R) = z_2 T \qquad z_2 T = -\mathrm{i}\frac{K}{\omega}(1 + R - T) \tag{2.11}$$

where R and T are the *complex* reflection and transmission coefficients corresponding to this imperfect case. The solution to (2.11) reads

$$R = \frac{z_1 z_2 - \mathrm{i}(K/\omega)(z_1 - z_2)}{z_1 z_2 - \mathrm{i}(K/\omega)(z_1 + z_2)} \qquad T = \frac{-2\mathrm{i}(K/\omega)z_1}{z_1 z_2 - \mathrm{i}(K/\omega)(z_1 + z_2)} \tag{2.12}$$

By computing the squares of the moduli of the complex quantities R and T, we check that (2.9) is replaced by

$$F_K = 1 - |R|^2 - \frac{z_2}{z_1}|T|^2 \equiv 0 \tag{2.13}$$

We note that

$$F_K = F_0\Big(1 - \frac{z_1^2 z_2^2}{z_1^2 z_2^2 + (K/\omega)^2(z_1 + z_2)^2}\Big) \tag{2.14}$$

The solution of this imperfect interface case is characterised by the parameter K/ω which shows the role played by the frequency ω. The limit case $K \to \infty$ corresponds to the perfect interface for which (2.9) holds true. The limit case $K \to 0$ corresponds to full delamination (no more transmission and complete reflection: $T = 0$, $R = 1$).

Remark 2.1. It is possible to obtain an estimate of the K coefficient by measuring the amplitude of reflected or transmitted waves. In particular, whenever media 1 and 2 are identical, but K still is not zero, this measure provides a means of determining the presence of an internal delamination in the body.

3. Associated quasi-particle picture

With field equation (2.1), there are associated (via the application of Noether's theorem or by direct manipulation) conservation laws of energy and wave momentum in the local form in a homogeneous medium

$$\frac{\partial H}{\partial t} - \frac{\partial Q}{\partial x} = 0 \qquad \frac{\partial P}{\partial t} - \frac{\partial b}{\partial x} = 0 \tag{3.1}$$

where the energy or Hamiltonian per unit volume H, the energy flux Q, the wave momentum P and (here reduced to a scalar) the Eshelby stress b are defined by (see Maugin and Rousseau (2010a) for the canonical definitions in three dimensions)

$$\begin{aligned} H &= E + W = \frac{1}{2}\rho u_{,t}^2 + \frac{1}{2}\mu u_{,x}^2 \qquad & Q &= \sigma u_t = \mu u_{,x} u_{,t} \\ P &= -\rho u_{,t} u_{,x} & b &= -(L + \sigma u_x) \end{aligned} \tag{3.2}$$

The first of these last two follows Brenig's definition. It is a remarkable – but sometimes misleading- fact that in this one-dimensional case there hold the following identities

$$Q = -c^2 P \qquad b = -H \tag{3.3}$$

Now the concept of quasi-particle is introduced by integrating conservation equations (3.1) over a volume that is representative of the present wave motion, i.e., over one wavelength in the x propagation direction and a square section of sides equal to unity in the transverse direction. This introduces averages noted with the symbolism $\langle\cdot\rangle$. For solutions of type (2.3) this procedure yields the following results

$$\langle L\rangle = 0 \qquad \frac{d}{dt}\langle P\rangle = 0 \qquad \frac{d}{dt}\langle H\rangle = 0 \tag{3.4}$$

where, in a homogeneous medium

$$\langle P\rangle = \rho\omega\pi U^2 \equiv Mc \qquad \langle H\rangle = \frac{1}{2}Mc^2 \tag{3.5}$$

where the first of these defined the "mass" $M = \rho k\pi U^2$. Simultaneously, equations $(3.4)_{2,3}$ show that the said quasi-particle has an inertial Newtonian motion. It also satisfies the Lebnizian conservation of kinetic energy (or *vis-viva*). Indeed, while there are both kinetic and elastic energy at the continuum level, the energy of the associated quasi-particle is purely *kinetic* on account of the given definition of "mass". Note that homogeneous equations $(3.4)_{2,3}$ hold good because of the periodicity at the two ends of the integration interval that makes the contributions from Q and b to vanish. Equation $(3.4)_1$ holds true because it is shown in computing the average for solutions (2.3) that the result is proportional to "dispersion" relation (2.4) and therefore vanishes identically, a result known in the kinematic-wave mechanics of Lighthill and Whitham (see, e.g., Whitham, 1974). Note finally that all quantities introduced in (3.4) and (3.5) are proportional to the square of the amplitude of the wave, hence are all of energetic nature.

3.1. Transmission-reflection problem in the quasi-particle picture

Considering first the case of the perfect interface at $x = 0$, we can associate one quasi-particle with each wave component of the problem. With an obvious notation, we have the following "masses"

$$M_I = \rho_1 k_1 \pi U^2 \qquad M_R = \rho_1 k_1 \pi R_0^2 U^2 \qquad M_T = \rho_2 k_2 \pi T_0^2 U^2 \tag{3.6}$$

The corresponding averaged wave momenta are given by

$$\overline{P}_I \equiv \langle P_I\rangle = \rho_1\omega\pi U^2 \tag{3.7}$$

and

$$\overline{P}_R \equiv \langle P_R\rangle = -\rho_1\omega\pi R_0^2 U^2 \qquad \overline{P}_T \equiv \langle P_T\rangle = \rho_2\omega\pi T_0^2 U^2 \tag{3.8}$$

where we account for the fact that the averaged wave momentum $\overline{P}_R$ is oriented towards negative x's.

We note ΔM and $\Delta\overline{P}$ the possible misfits in mass and momentum defined by

$$\Delta M := (M_R + M_T) - M_I \tag{3.9}$$

and

$$\Delta\overline{P} = (|\overline{P}_R| + |\overline{P}_T|) - |\overline{P}_I| \tag{3.10}$$

where the symbolism $|\cdot|$ refers to the absolute value of its enclosure. That is, we are comparing the strengths of the momenta and, therefore, we are not performing a vectorial balance.

Similarly, for the kinetic energy of the associated quasi-particles:

$$\Delta\overline{H} = (\overline{H}_R + \overline{H}_T) - \overline{H}_I \tag{3.11}$$

We say that a quantity is conserved during the transmission-reflection problem if the corresponding misfit vanishes.

By applying definition $(3.5)_2$, one immediately shows that

$$\Delta\overline{H} = \Big(\frac{1}{2}z_1\omega\pi U^2\Big)F_0 \tag{3.12}$$

where F_0 has been defined in (2.9). But the latter vanishes. Accordingly, $\Delta\overline{H} \equiv 0$: *kinetic energy is conserved* in the transmission-reflection problem seen as a quasi-particle process that may be qualified of Leibnizian (conservation of vis-viva). Historically, it was in fact the collision problem between particles that led the late 17th and early 18th century scientists (prominently Leibniz in 1686; see Smith, 2006) to introduce the vis-viva – twice the actual kinetic energy – as the "physical quantity" to be conserved in such interactions and not Descartes' "quantity of motion" as suggested by a different school (see below for the particle moment in the present problem).

Indeed, "mass" is not generally conserved in the present problem as it is immediately shown that

$$\Delta M = \rho_2 k_2 \pi T_0^2 U^2 \Big(\frac{c_1^2 - c_2^2}{c_1^2}\Big) \tag{3.13}$$

Similarly, on computing $\Delta\overline{P}$ it is obtained that

$$\Delta\overline{P} = \rho_2 k_2 \pi T_0^2 U^2 \Big(\frac{c_1 c_2 - c_2^2}{c_1}\Big) \tag{3.14}$$

This shows that

$$\Delta\overline{P} = \Big(\frac{c_1 c_2}{c_1 + c_2}\Big)\Delta M \tag{3.15}$$

so that $\Delta\overline{P}$ and ΔM always are in the same sign. In particular, $\Delta M > 0$ if $c_1 > c_2$ and $\Delta M < 0$ if $c_1 < c_2$; $\Delta M = 0$ if and only if $c_1 = c_2$, that is, if there is no interface or for the extraordinary case where the two media have elasticity coefficients and densities in the same ratio; $\Delta\overline{P}$ and ΔM also vanish in the same conditions, i.e., we can say that $\overline{P}$ and M are conserved in the same conditions, in particular when media 1 and 2 have the same characteristic elastic speed. This is also true when medium 2 is a vacuum ($\rho_2 \to 0$) or is perfectly rigid ($\mu_2 \to \infty$), cases for which there is perfect reflection ($R_0^2 = 1$) from the wavelike viewpoint or perfect (i.e., elastic) rebound from the particle-like viewpoint.

3.2. Case of an imperfect interface with possible delamination

Remarkably enough, there is no need to redo the computations. It suffices to replace the transmission and reflection coefficients of the perfect case by the moduli of the new complex coefficients. Thus, (3.12) is replaced by

$$\Delta\overline{H} = \Big(\frac{1}{2}z_1\omega\pi U^2\Big)F_K \tag{3.16}$$

and this again vanishes. Similarly, (3.13) and (3.14) hold with T_0^2 replaced by $|T|^2$ while (3.15) remains unchanged, noting that the coefficient $c_1c_2/(c_1 + c_2)$ does not depend on K.

Remark 3.1. In the case when $K \neq 0$ but media 2 and 1 are identical, the presence of the K spring distribution can simulate a homogeneous surface damage. In this case, both ΔM and $\Delta \overline{P}$ vanish so that K is no longer involved. The dependence on K shows only through the value of any of M_R, M_T, $\overline{P}_R$ and $\overline{P}_T$.

Remark 3.2. In the case when $K \neq 0$ but media 1and 2 are different, both ΔM and $\Delta \overline{P}$ do not vanish. This result is not due to the presence of the K spring distribution; it can only be attributed to the difference in characteristic wavelength in both media. These wavelengths depend on the integration length of propagation in each medium. This hypothesis being accepted, the value of K can also be evaluated from ΔM and $\Delta \overline{P}$. As the surface damage increases (i.e., as K decreases), the mass of the reflected quasi-particle increases and that of the transmitted quasi-particle decreases compared to the fully undamaged case.

4. Case of a sandwiched slab

Here we consider the case of an elastic slab (medium 2) of thickness d situated between $x = 0$ and $x = d$ and sandwiched between two media of elastic type 1. The propagation is from left to right with the reflection coefficient R in left medium 1 and the transmission coefficient T in right medium 1. We assume that $d \gg \lambda_2$, where λ_2 is the (elastic wave) characteristic wavelength of medium 2, so that the association of quasi-particle properties makes sense in the slab. We need not to reconsider the wavelike solution. We are satisfied with applying the results of the foregoing section to the two interfaces at $x = 0$ (transition $1 \to 2$) and at $x = d$ (transition $2 \to 1$). Just as before M_I, M_R, M_T, P_I, P_R and P_T are masses and momenta granted to the quasi-particles in left and right regions 1. We obviously have (compare (3.6))

$$M_I \propto U^2 \qquad M_R \propto |R|U^2 \qquad M_T \propto |T|U^2 \qquad |R|^2 + |T|^2 = 1 \tag{4.1}$$

and in an obvious notation

$$\Delta M_{11} = (M_R + M_T) - M_I = 0 \tag{4.2}$$

As to the momenta, we have (cf. Section 3)

$$\Delta \overline{P}_{11} = (|P_R| + |P_T|) - |P_I| \equiv 0 \tag{4.3}$$

Within the slab we distinguish between the particle momentum P^+ with mass M^+ associated with the right motion and particle momentum P^- with mass M^- associated with the left motion.

Thus, for the interface $x = 0$ we can write

$$\begin{aligned} \Delta M_{1\to 2} &:= (M^+ + M^- + M_R) - M_I = (M^+ + M^-) - M_T \neq 0 \\ \Delta \overline{P}_{1\to 2} &= (|P^+| + |P^-| + |P_R|) - |P_I| = (|P^+| + |P^-|) - P_T \neq 0 \end{aligned} \tag{4.4}$$

while for the interface $x = d$ we put down similarly

$$\begin{aligned} \Delta M_{2\to 1} &= M_T - (M^+ + M^-) = -\Delta M_{2\to 1} \neq 0 \\ \Delta \overline{P}_{2\to 1} &= |P_T| - (|P^+| + |P^-|) = -\Delta \overline{P}_{1\to 2} \neq 0 \end{aligned} \tag{4.5}$$

Note that

$$(|P^+| + |P^-|) = \Delta \overline{P}_{1\to 2} + |P_T| \tag{4.6}$$

and, obviously

$$R \equiv R_{1\to 2\to 1} \qquad\qquad T = T_{1\to 2\to 1} \tag{4.7}$$

These two coefficients can be computed in terms of the mechanical properties of media 1 and 2. A standard calculation involving the matching conditions at the crossing of the two interfaces separated by the distance d yields

$$\begin{aligned} T &= \frac{1}{\cos(k_2 d) - \frac{\mathrm{i}}{2}\Big(\frac{z_1}{z_2} + \frac{z_2}{z_1}\Big)\sin(k_2 d)} \\ R &= \frac{-\frac{\mathrm{i}}{2}\Big(\frac{z_1}{z_2} - \frac{z_2}{z_1}\Big)\sin(k_2 d)}{\cos(k_2 d) - \frac{\mathrm{i}}{2}\Big(\frac{z_1}{z_2} + \frac{z_2}{z_1}\Big)\sin(k_2 d)} \end{aligned} \tag{4.8}$$

Remark 4.1. It is readily checked that these two formulas comply with the conservation law $|R|^2 + |T|^2 = 1$. Of course we also check that $T = 1$ and $R = 0$ if $d \equiv 0$ (perfect contact). As to the condition $d \ll \lambda_2$, it does not make sense for the introduction of a quasi-particle in the slab.

Remark 4.2. The type of formalism, bookkeeping and "algebra" just introduced can be applied to the more complicated case where the sandwiched slab is made of a number $n-1$ of perfectly elastic layers (each with its own elastic properties) numbered $i = 2,\dots,n$, in perfect contact. That is, the medium left to the slab and the medium right to it are made of the same material (material 1). The estimates in (4.1) still hold good. This is also the case of (4.2). Same as in $(4.4)_1$, we will write

$$\begin{aligned} \varDelta M_{1\to 2} &= (M_2^+ + M_2^- + M_R) - M_I \neq 0 \\ \varDelta M_{2\to 3} &= (M_3^+ + M_3^-) - (M_2^+ + M_2^-) \neq 0 \end{aligned} \tag{4.9}$$

and so on

$$\begin{aligned} \varDelta M_{i\to i+1} &= (M_{i+1}^+ + M_{i+1}^-) - (M_i^+ + M_i^-) \neq 0 \\ &\dots \\ \varDelta M_{n-1\to n} &= (M_n^+ + M_n^-) - (M_{n-1}^+ + M_{n-1}^-) \neq 0 \\ \varDelta M_{n\to 1} &= M_T - (M_n^+ + M_n^-) \neq 0 \end{aligned} \tag{4.10}$$

Here superscripts $+$ and $-$ indicate masses associated with particle motions to the right and left, respectively. On summing over all equations (4.9) through (4.10) we re-obtain (4.2). A similar bookkeeping can be done for the momentum transfers $\varDelta\overline{P}_{11}$ (cf. equation (4.3)) and $\varDelta\overline{P}_{i\to i+1}$. We also note that, globally

$$R = R_{1\to\dots\to 1} \qquad\qquad T = T_{1\to\dots\to 1} \tag{4.11}$$

where the dots stand for the $(n-1)$ layers numbered $(2,\dots,n)$. In principle, global reflection and transmission coefficients (4.11) can be computed in terms of the individual properties of the layers, but this is not of an immediate interest.

5. Conclusion

The above reported analysis shows that, in the transmission-reflection problem within the framework of purely elastic material regions, the resulting mechanics of the associated quasi-particles results in an exchange of linear momentum without any global loss of it while there also is no misfit in "mass", but the masses depend on the region of propagation, i.e., the various propagation properties.

Interestingly enough, there is no misfit in the kinetic energies of the quasi-particles – cf. equations (3.12) and (3.16). This is a direct consequence of the acoustic (wavelike) energy conservation represented by equations (2.9) and (2.13). There is no principle difficulty in dealing with the slab problem and its extension to a multi-layered interface. But there may be computational difficulties. The case of delamination at an interface fits in the picture, and the properties of this delamination (spring coefficient K) can be deduced from the "misfits" in mass and momentum of the associated quasi-particles.

One may wonder what happens if some of the involved media of propagation are viscoelastic. The wavelike problem of an interface between two media was studied a long time ago (see Mackenzie, 1960, and references therein), at least for a small perturbing viscosity. Insofar as the concept of associated quasi-particle is concerned, for a propagation of surface SH waves in an infinite continuum along the propagation direction we have shown elsewhere (Rousseau and Maugin, 2012) the complexity and originality of the result. Not only does the viscous damping result in a friction force as a source in the equation of motion of the associated quasi-particle, but the "mass" itself of this so-called particle is found to vary in time. One can easily imagine the analytical difficulties to be met in treating the case of a viscoelastic slab sandwiched between two identical elastic or viscoelastic media. The limit case of the slab thickness going to zero should provide the case of an absorbing interface of vanishing thickness thus re-instating the notion of coefficient of restitution.

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Dualizm falowo-quasi-korpuskularny w zagadnieniu przesyłania i odbicia fal sprężystych

Streszczenie

Idąc w ślad za ostatnimi publikacjami, w których użyto pojęcia tzw. quasi-cząstek wprowadzonego poprzez zastosowanie kanonicznych praw zachowania w odniesieniu do powierzchniowych fal akustycznych różnego rodzaju, w prezentowanej pracy położono nacisk na możliwe wykorzystanie dualizmu falowo-quasi-korpuskularnego w klasycznym zagadnieniu przesyłania i odbicia fal na nieciągłości pomiędzy dwoma ośrodkami (doskonale połączonych lub zdelaminowanych) i w ogólniejszym przypadku jedno- lub wielowarstwowej płycie typu sandwich.

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